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Improvement of measures of consumption of fixed capital

Different scenarios of estimating the consumption of fixed capital for the government sector with possible impact on gross national income

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Summary

Since consumption of fixed capital (CFC) and capital stocks are generally not available from administrative sources, statisticians use modelling techniques. However, they might face certain challenges, such as insufficiently long series of gross fixed capital formation (GFCF), selection of appropriate survival and depreciation functions, and determination of average service life of fixed assets. In addition to the given challenges, and for transmitting data as of ESA 2010 transmission program, capital stocks and CFC are required at the level of institutional sectors and industries which sometimes might not be available. Accordingly, the aim of this document is to simulate estimations on Croatian data using other depreciation functions than those currently applied, to estimate CFC at different levels of classification, and instead of imputation data, using preliminary GFCF data since 1953 for infrastructure and non-residential buildings in the government sector. Computation methodology is also described using principles of matrix algebra. Differences obtained with the new approaches are put into relation to gross national income (GNI) as its percentage. Results indicate that selection of different depreciation functions and introduction of a longer GFCF time series have an impact on GNI for period 2013-2021.

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I. Introduction and motivation

1. Consumption of fixed capital (CFC) and capital stocks are important variables in national accounts and macroeconomic analysis for several reasons. Capital stocks are important for calculating various capital productivity indicators such as net fixed assets to gross value added, net fixed assets per employed person and net fixed assets per hour worked (Eurostat, 2021). Gross capital stock has been widely used as indicator of the productive capacity of a country or has sometimes been used as a measure of capital input in studies of multifactor productivity (OECD, 2009). Gross capital stock contains all assets surviving from past investments including CFC. In some literature, CFC is called depreciation, meaning a decline during the course of accounting period, in the current value of the stock of fixed assets owned and used by a producer as a result of physical deterioration, normal obsolescence or normal accidental damage (SNA, 2008). Net capital stock is all assets survived adjusted for CFC. While gross and net capital stock are important for various indicators and econometric analyses, CFC is one of the inputs in the compilation of national accounts. CFC is a part of the final consumption expenditure by government and non-profit institutions serving households. Since the final consumption expenditure is the main aggregate on the use side of the supply-use tables, CFC directly affects the GNI. Also, CFC transforms gross measures into net ones, for example, gross income into net terms. Capital stocks and CFC are generally compiled by statistical agencies, which might face certain limitations. Depreciation from administrative sources is generally not aligned with the System of National Accounts methodological guidelines which require a convex depreciation profile. In addition, average service lives of fixed assets in national accounts might significantly differ from average service lives for corporate taxation. Due to these limitations, it is necessary to apply a mathematical model. Statisticians often use a model based on the concept of perpetual inventory, defined with a basic formula

$$\kappa(t) = \kappa(t-1) + \iota(t) - \varrho(t), \quad t \in \mathbb{Z}_+ \qquad (1)$$

where $\kappa(t)$ denotes net dwelling stock at the end of the year t, $\iota(t)$ the gross fixed capital formation (GFCF) during year t, and $\varrho(t)$ is the CFC during year t. Formula (1) can be reformulated and adjusted to (OECD, 2009)

$$\kappa(t) = \kappa(t-1) + \iota(t) - \underbrace{\delta\left(\frac{1}{2}\iota(t) + \kappa(t-1)\right)}_{\varrho(t)}; \quad \delta \in \langle 0,1 \rangle \qquad (2)$$

meaning that net capital stock of year t equals net capital stock of previous year added GFCF of year t adjusted for CFC. The advantage of this method is that it is suitable for shorter GFCF time series and that the depreciation factor is constant over time due to constant growth rate of the geometric function. If the GFCF time series is not long enough, it is necessary to somehow determine the initial capital stock. One way for initial capital stock to be estimated is following the principle (Kohli, 1982):

$$\kappa(t_0) \approx \iota(t_{0-1}) + (1-\delta)\cdot\iota(t_{0-2}) + (1-\delta)^2\cdot\iota(t_{0-3}) + \cdots + (1-\delta)^{q-1} \cdot \iota(t_{0-q}) \qquad (3)$$

meaning that the initial capital stock at some benchmark year t_0 can be written as the cumulative, depreciated GFCF of previous years $\iota(t_{0-q})$, $q \in \mathbb{Z}_+$. Since GFCF is an expenditure aggregate, an assumption could be made about the medium or long-term investment or GDP growth rate λ. By setting

$$\iota(t) = (1+\lambda)\iota(t-1)$$

and inserting this equation into (3), we have

$$\iota(t_{0-1}) + (1-\delta) + \cdots + (1-\delta)^{q-1}\iota(t_{0-q}) = \iota(t_{0-1})[1 + (1-\delta)(1+\lambda) + (1-\delta)^2(1+\lambda)^2 + \cdots + (1-\delta)^q(1+\lambda)^q]$$

and solving the right side for the geometric series, we get

$$\iota(t_{0-1}) + (1-\delta) + \cdots + (1-\delta)^{q-1}\iota(t_{0-q}) = \iota(t_{0-1})\left(\frac{1+\lambda}{\delta+\lambda}\right) \qquad (4)$$

Now, since $\iota(t) = (1+\lambda)\iota(t-1)$, the initial capital stock might be approximated

$$\kappa(t_0) \approx \frac{\iota(t_0)}{\delta + \lambda} \qquad \delta, \lambda \in \langle 0,1 \rangle \quad (5)$$

2. The final solution actually implies that, in order to estimate the initial net capital stock, the first available GFCF $\iota(t_0)$, depreciation rate δ and the long-run growth rate λ are required. Expression (5) can be a critical point of estimation because initial stock relies only on the first available GFCF value in the time series, assuming that growth- and depreciation rates are considered reliable. Although the model can produce decent results, statistical agencies often face additional limitations such as the length of the GFCF time series, reliable determination of average service life, and the choice of appropriate depreciation and survival functions. Potential users of capital stock data are also aware of the limitations. For example, Burda and Severgnini (2008) pointed out in their work that little is known on precision of standard measurement of total factor productivity growth especially when the capital stock is poorly measured. In case the GFCF time series is not of sufficient length, then different imputations are often used. Database on GFCF for the needs of Croatia's national accounts currently contains observations from 1995 onwards while imputation functions are used for backward estimation with constant investment growth rate. However, this might be an issue, as shown by Pionnier, Zinni and Baret (2023) in their study. They found that estimating initial capital stock using constant growth rate might lead to unreliable results. Furthermore, preliminary studies on total GFCF level (thus for all sectors) were carried out as part of an EU project (Motik, 2023) where it was shown that the choice of depreciation function affects the level of capital stock and CFC to a certain extent. All of the above lead to concerns regarding GNI, thus the following three scenarios will be discussed. First scenario is related to CFC which arises from geometric- and linear depreciation functions, where linear depreciation is combined with survival function. The second scenario is dedicated to estimation of CFC at different levels of aggregation because initial net capital stock $\kappa(t_0)$ actually depends on the first available GFCF value as shown in (5). Statistical agencies often do not have data at sufficiently detailed level of aggregation, e.g. the GFCF is missing at government subsectors level or industry level. As mentioned earlier, common problem in estimating CFC and capital stock is insufficiently long GFCF time series, therefore different imputations are used. However, some historical GFCF series can subsequently be reconstructed where significant differences may appear. This is to be the third scenario where GFCF of infrastructure and non-residential buildings are estimated based on historical data. Any of the scenarios can affect the GNI, which is the basis for payments to the EU budget. In this regard, the impact is presented according to the formula

$$\epsilon(t) = \frac{\Delta\varrho(t)}{G(t)}, \quad \Delta\varrho(t) = \left|\varrho_j(t) - \varrho_v(t)\right| \quad (6)$$

where $\varrho_j(t)$ is CFC resulting from the scenarios described above $\varrho_v(t)$ is CFC included in currently valid GNI $G(t)$. The equation (6) refers to the period 2013-2021. Before analyzing the scenarios, data sources and methodology, on which the algorithm for estimating CFC and capital stocks is based, will be described. Mathematical procedures are generalization of stock and flow calculus taking into account limited GFCF time series in terms of their length.

II. Data sources

3. The compilation of GFCF in current prices for government sector, consolidated into database, is carried out from several sources. The main source is the report on income and expenditure (PR-RAS), from which it is possible to extract data on expenditure incurred due to investment in various fixed assets. In addition, there are also items of income resulting from disposal of fixed assets, which is in accordance with the definition of GFCF. Other sources are annual financial statements of entrepreneurs and non-profit organizations, which are classified in government sector. Annual financial statements (GFI) contain data from balance sheets and profit and loss accounts of reporting units. Fixed assets are identified from a separate part of the GFI with the level of asset type being less detailed compared to PR-RAS. Adjustments are applied for re-sectorization, research and development and software. Assets are associated with internal code ensuring unique identification. Concatenating the

internal code and the year of investment, it is possible to create a primary key for joining deflators for previous year's prices (and chain link volumes accordingly) along with deflators for assets' revaluations for CFC and capital stocks computations. This enables the automatic calculation of GFCF in different price concepts being eventually grouped into AN classification as required by ESA 2010. For estimating CFC and capital stocks, revalued GFCF is used, obtained as a product of GFCF in historical prices with a revaluation deflator. Revaluation value $I(k,\tau)$ up to year τ introducing revaluation deflators $\tilde{p}(\tau)$ equals to (adjusted to Van den Bergen et. al., 2009)

$$I(k,\tau) = \iota(k) \odot \tilde{p}(\tau) \quad \text{where} \quad \tilde{p}(\tau) = \prod_{t=k+1}^{\tau} p\,(t) \qquad (7)$$

with $\odot$ being the Hadamard product. The deflators are obtained following

$$\mathrm{p}(t) = \underbrace{\alpha[v_1 p_1(t) + (1-v_1)p_2(t)]}_{DeflatorA} + \underbrace{(1-\alpha)[v_2 p_3(t) + (1-v_2)p_4(t)]}_{DeflatorB} \qquad (8)$$

where $v_1, v_2 \in [0,1]$ denote a share of imported assets (estimated from the SUT tables), otherwise domestic. Also, p_x are indices where x is odd for imported goods and even for domestic goods. In terms of type of assets within a group of assets, α is the weight of an asset A, while $(1-\alpha)$ is the weight of an asset B. It is the GFCF in terms of (7) that will be used in the estimation of CFC and capital stock, which will be explained in the following section.

III. Computation methodology

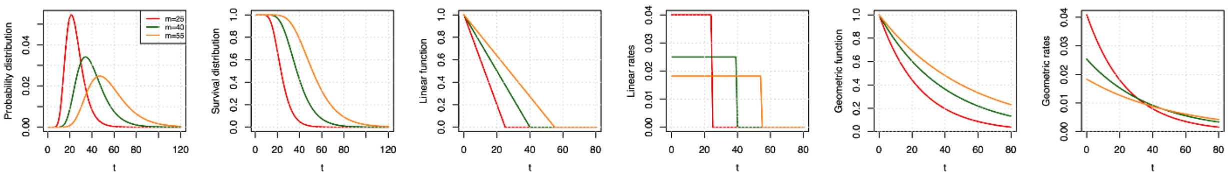
4. As seen in the basic accumulation formula (2), CFC and net capital stock are calculated dependently. However, the calculation becomes more complicated if depreciation other than geometric is used which will be explained in subsection III.B. Before explaining the mathematical concept that will serve as a basis for programming, a brief explanation of the functions used in the estimation of CFC and stocks will first be given in subsection III.A.

A. Functions in the model

5. In the national accounts of Croatia, gross and net capital stock as well as CFC are estimated. As a rule, CFC is derived directly from gross capital stock. Data on gross- and net capital stock are required by the transmission program (Eurostat, 2014). To estimate gross capital stock, survival function of log-normal probability distribution is used (as shown on the second graphics from left in Figure 1). In absence of any solid empirical evidence on the shape, this function seems convenient because only average service life is needed as a parameter.

Figure 1

Survival and depreciation functions with altered average service life



6. Survival function is defined as $S_T(t) = 1 - F_T(t)$, but due to $\lim_{t\to\infty} S_T(t) \to 0$ we introduce a truncated function (e.g., Johnson and Johnson, 1980), that is

$$\mathrm{S_T(x)} = \begin{cases} 1 - \left(\dfrac{1}{\sigma\sqrt{2\pi}}\displaystyle\int_0^{\infty} \frac{1}{t} exp\left[\frac{-(ln(t)-\mu)^2)}{2\sigma^2}\right] dt\right), & t \leq 2m \\ 0, & otherwise \end{cases} \qquad (9)$$

where $\mu = \ln(m) - 0.5\sigma^2$, m is average service life and $\sigma = \sqrt{\ln[1+(s/m)^2]}$ with $s = m/3$. As seen in Figure 1, the log-normal distribution is right-side asymmetric to all years of service life, but differs in shape of peak and tail. Distributions with shorter average service life have more rounded peak after which values decline more significantly, while distributions with longer service life have a less rounded peak after which values decline more slowly. The survival function is a cumulative log-normal distribution function whose values are subtracted from the highest possible probability measure, that is 1. To provide some practical explanation, we take a look at the 40 years service life survival function (green function on the second graphics from the left). It tells that after 40 years of service, around 40% of the fixed assets are still being used. Thus, with survival function we estimate percentage of assets not being discarded from the moment when the assets were pulled into service. As the moment of ownership transfer is important in national accounts, and while the timepoint of assets usage is unknown, the first year is actually considered zero year, meaning that all assets survived. To estimate the CFC and eventually net capital stock, it is necessary to include a depreciation function which is often narrowed to geometric or linear function. Since CFC and capital stock are estimated for a group of homogeneous assets that nevertheless differ in certain properties and may retire in different intervals, it is necessary to achieve a convex or similar pattern (for more details, see the 2009 OECD manual for capital measurement). Since geometric function itself holds a property of convexity, while a linear function does not, the latter is generally combined with survival function. The linear depreciation function $\varphi_l = 1 - (t/m)$ is shown on third graphics from the left with the rates (fourth graphics from the left)

$$\dot{\varphi}_l(t) = \begin{cases} |m^{-1}|, & t \leq m \\ 0, & otherwise \end{cases} \qquad (10)$$

where the rates at $t = 0$ and $t = m$ are adjusted to half amount because, unlike in commercial accounting, the exact month of an asset being pulled into service remains unknown. For this reason, the adjustment was applied halfway through the year, which will still result in the full amount of depreciation due to symmetry.

7. The geometric depreciation function $\varphi_g = (1 - |\dot{\varphi}_l(t)|)^t$ is shown on fifth graphics from the left with the rates (sixth graphics from the left)

$$\dot{\varphi}_g(t) = \begin{cases} |ln\,(1 - |\dot{\varphi}_l(t)|)(1 - |\dot{\varphi}_l(t)|)^t|, & t \leq 2m \\ 0, & otherwise \end{cases} \qquad (11)$$

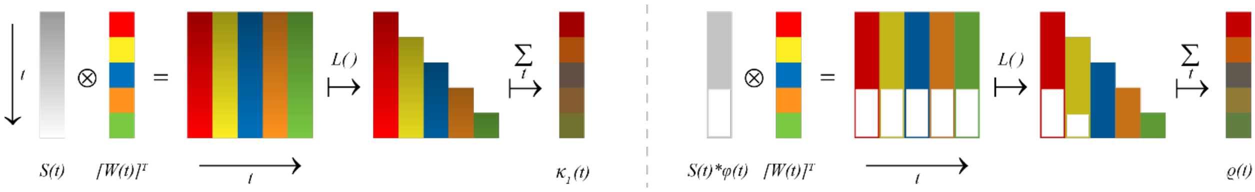
8. Rates are nothing but derivatives of linear and geometric functions and are applied as such when estimating CFC and capital stocks. The following subsection will explain how these functions are integrated into the mathematical model.

B. Mathematics behind the methodology

9. The functions from subsection III.A should be combined in a meaningful way to obtain capital stock and CFC. When deriving the model itself, it is necessary to pay attention to basic concepts and algebra of calculations. Although data in the national accounts are discrete, in the following procedures, all vectors will be viewed as continuous functions. The aim is to mathematically present the procedure sketched in Figure 2.

Figure 2

The concept of deriving CFC from gross capital stock



10. In this context, the main problem to analyze is given

$$\mathcal{T}^{[k]}(t)[I(t)] \mapsto \{\kappa_1(t), \kappa_2(t), \varrho(t)\}, \quad \kappa_1(t), \kappa_2(t), \varrho(t) \in \mathbb{R}^n \qquad (12)$$

11. where $\mathcal{T}^{[k]}(t)$, $k = \{1,2,3\}$, denotes a sequence of operators acting on the revalued GFCF vector $I(t)$ resulting with a set of three new vectors: gross capital stock $\kappa_1(t)$, net capital stock $\kappa_2(t)$ and CFC $\varrho(t)$. With the markings of having two capital stock, it is obvious that $\kappa(t)$ in (1) equals $\kappa_2(t)$. Since the GFCF series has been available since 1995, it is

necessary to apply the imputation function in order to achieve a sufficient level of stock and CFC since 1995 to satisfy the condition

$$d(\kappa_1, \kappa_2) = |\kappa_1(t_0) - \kappa_2(t_0)| \gg 0 \qquad (13)$$

where $d: \mathcal{M} \times \mathcal{M} \to \mathbb{R}_+$ and consequently $\varrho(t) > 0$. Following sample of cumulative historic GFCF data at total levels, capital stock declines backwards at the fixed rate λ following differential equation

$$\dot{\rho}(t) = -\lambda\rho(t) \quad \text{with the initial condition} \quad u(t_0) = \rho_0 \qquad (14)$$

12. The solution is $u(t) = \rho_0 e^{-\lambda t}$ where ρ_0 is capital stock at t_0. The initial stock is adjusted to the solution for initial net stock in (5), meaning for $\delta = 0$

$$\rho_0 \approx \iota(t_0)\lambda^{-1} \qquad (15)$$

13. The imputed GFCF flows are accordingly

$$\iota(t) := \dot{u}(t) = \left|-\lambda\rho_0 e^{-\lambda t}\right|, \quad \rho_0 \approx \int_{t_0-q}^{t_0} |\iota(t)|\,\mathrm{d}t, \quad t \in \left[t_{0-q}, t_0\right] \quad (16)$$

14. The parameter $\lambda = 0.03$ is set to avoid shocks at t_0 due to inflation in prices in 1990s. The time series taken for the estimation of capital stock and CFC will then consist of the imputed non-revalued part and the revalued GFCF available from the database. We can write this as

$$W(t) = \iota(t) \cup I(t), \quad W(t) = \left(\iota_{t_0-q}, \iota_{t_0-(q-1)}, \dots, I_{t_0}, I_{t_0+1}, I_{t_0+2}, \dots\right) \qquad (17)$$

15. As can be seen from Figure 2, the first step is calculation of the gross capital stock, which consists of the accumulated GFCF belonging to different vintages as indicated by squares of different colors and the survival function as indicated by gradually fading gray color. The formula for gross capital stock is given in the literature as (Biorn, 1989)

$$\kappa_1(t) = \int_0^\infty S\,(q)W(t-q)\mathrm{d}q \qquad (18)$$

16. However, this expression could be challenging for programming, therefore, the solution to the problem will be transformed to concepts of matrix algebra. For the purposes of statistical computing, various operations between matrices are nicely summarized in Gentle (2017) and may be of assistance when deriving procedures. Each GFCF vintage, starting from $t_0 - q$ is to be multiplied with the survival function and arranged in columns that will eventually be transformed into the lower triangular matrix to have accumulation of the flows. To achieve this, we introduce

$$\kappa_1(\tau) = \sum_i L_{j=t-1}\left[S_T(t) \otimes W(t)^\top\right], \quad t \in [t_0 - q, t_0 + h] \qquad (19)$$

where $\otimes$ is tensor product, and $L_{j=t-1}[\cdot]$ is a column-wise lag operator. The period $t_0 + h$ is referred to the last GFGF realization in our database. Starting with

$$S_T(t) \otimes W(t)^\top = \begin{pmatrix} \widetilde{w}_{t_0-q} & \widetilde{w}_{t_0-(q-1)} & \widetilde{w}_{t_0-(q-2)} & \cdots & \widetilde{w}_{t_0-(q-h)} \\ \widetilde{w}_{t_0-(q-1)} & \widetilde{w}_{t_0-(q-2)} & \widetilde{w}_{t_0-(q-3)} & \cdots & \widetilde{w}_{t_0-(q-(h+1))} \\ \widetilde{w}_{t_0-(q-2)} & \widetilde{w}_{t_0-(q-3)} & \widetilde{w}_{t_0-(q-4)} & \cdots & \widetilde{w}_{t_0-(q-(h+2))} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \widetilde{w}_{t_0-(q-h)} & \widetilde{w}_{t_0-(q-(h+1))} & \widetilde{w}_{t_0-(q-(h+2))} & \cdots & \widetilde{w}_{t_0-(q-(h+z))} \end{pmatrix}$$

(20)

we get the square matrix and where $h, z \in \mathbb{Z}_+$. Applying the lag operator, we get survived vintages $\widetilde{\mathbf{w}}(\mathbf{t}) := L_{j=t-1}[S_T(t) \otimes W(t)^\top]$, that is

$$\widetilde{\mathbf{w}}(\mathbf{t}) = \begin{pmatrix} \widetilde{w}_{t_0-q} & 0 & 0 & \dots & 0 \\ \widetilde{w}_{t_0-(q-1)} & \widetilde{w}_{t_0-(q-1)} & 0 & \dots & 0 \\ \widetilde{w}_{t_0-(q-2)} & \widetilde{w}_{t_0-(q-2)} & \widetilde{w}_{t_0-(q-2)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \widetilde{w}_{t_0-(q-h)} & \widetilde{w}_{t_0-(q-h)} & \widetilde{w}_{t_0-(q-h)} & \cdots & \widetilde{w}_{t_0-(q-h)} \end{pmatrix} \qquad (21)$$

17. The lag operator transforms the matrix (20) into a lower triangular matrix, ensuring that each GFCF vintage is placed in the accompanying row, otherwise the GFCF from year t would be recorded in the year $t-1$, which is impossible situation. Finally, adding up by rows i, the gross capital stock $\kappa_1(t) \in \mathbb{R}^n$ is obtained

$$\kappa_1(t) = \sum_i \widetilde{w}\,(i) \qquad (22)$$

18. The next step is to estimate the CFC by combining the survival function with the rates of depreciation function and multiplying such vector with GFCF. Mathematically, one way to achieve this is through

$$\varrho(t) = \sum_i L_{j=t-1}\left[\left(S_T(t) \odot \varphi(t)\right) \otimes W(t)^{\top}\right] \qquad (23)$$

19. Since $S_T(t), \varphi(t) \in \mathbb{R}^n$, and with Hadamard product involved, the procedure is the same as for the gross capital stock. The lower triangular matrix for CFC is then

$$\boldsymbol{\rho}(\mathbf{t}) = \begin{pmatrix} \rho_{t_0-q} & 0 & 0 & \cdots & 0 \\ \rho_{t_0-(q-1)} & \rho_{t_0-(q-1)} & 0 & \cdots & 0 \\ \rho_{t_0-(q-2)} & \rho_{t_0-(q-2)} & \rho_{t_0-(q-2)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{t_0-(q-h)} & \rho_{t_0-(q-h)} & \rho_{t_0-(q-h)} & \cdots & \rho_{t_0-(q-h)} \end{pmatrix} \qquad (24)$$

20. The CFC is finally obtained as

$$\varrho(t) = \sum_i \rho\,(i) \qquad (25)$$

21. This procedure is given on the right side of Figure 2. The elements of matrix (24) are flows since the rates of depreciation functions are used. If $\varphi(t)$ is linear depreciation then $S_T(t)$ in (9) is applied and if $\varphi(t)$ is geometric depreciation, then $S_T(t) = (1,1,\ldots,1)$ is applied. In other words, only linear depreciation is combined with a log-normal survival function to achieve an effect similar to geometric depreciation. For net capital stock, an accumulated $\varrho(t)$ will be needed to satisfy the equation

$$\kappa_2(\tau) = \sum_i \left(L_{j=t-1}[\mathbf{1}(t) \otimes W(t)^{\top}] - \int_{t_0-q}^{\tau} \boldsymbol{\rho}_{\mathbf{j}}(\mathbf{t})\,\mathrm{d}t \right), \quad \tau = t_0 - (q-h) \qquad (26)$$

22. If we integrate flow matrix $\boldsymbol{\rho}(\mathbf{t})$ over columns, we get

$$\int_{t_0-q}^{\tau} \boldsymbol{\rho}_{\mathbf{j}}(\mathbf{t})\,\mathrm{d}t$$

$$= \begin{pmatrix} \rho_{t_0-q} & 0 & 0 & \cdots & 0 \\ \int_{t_0-q}^{t_0-(q-1)} \rho\,(t)\mathrm{d}t & \rho_{t_0-(q-1)} & 0 & \cdots & 0 \\ \int_{t_0-q}^{t_0-(q-2)} \rho\,(t)\mathrm{d}t & \int_{t_0-q}^{t_0-(q-2)} \rho\,(t)\mathrm{d}t & \rho_{t_0-(q-2)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \int_{t_0-q}^{t_0-(q-h)} \rho\,(t)\mathrm{d}t & \int_{t_0-q}^{t_0-(q-h)} \rho\,(t)\mathrm{d}t & \int_{t_0-q}^{t_0-(q-h)} \rho\,(t)\mathrm{d}t & \cdots & \rho_{t_0-(q-h)} \end{pmatrix} \qquad (27)$$

23. Now, since $\mathbf{w}(\mathbf{t}) := L_{j=t-1}[\mathbf{1}(t) \otimes W(t)^{\top}]$, we have

$$\mathbf{w}(\mathbf{t}) = \begin{pmatrix} w_{t_0-q} & 0 & 0 & \cdots & 0 \\ w_{t_0-(q-1)} & w_{t_0-(q-1)} & 0 & \cdots & 0 \\ w_{t_0-(q-2)} & w_{t_0-(q-2)} & w_{t_0-(q-2)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ w_{t_0-(q-h)} & w_{t_0-(q-h)} & w_{t_0-(q-h)} & \cdots & w_{t_0-(q-h)} \end{pmatrix} \qquad (28)$$

24. For net capital stock the matrix adjusted for CFC accumulation is needed

$$\boldsymbol{\eta}(\mathbf{t}) := \mathbf{w}(\mathbf{t}) - \int_{t_0-q}^{\tau} \boldsymbol{\rho}_{\mathbf{j}}(\mathbf{t})\,\mathrm{d}t$$

$$= \begin{pmatrix} \eta_{t_0-q} & 0 & 0 & \cdots & 0 \\ \eta_{t_0-(q-1)} & \eta_{t_0-(q-1)} & 0 & \cdots & 0 \\ \eta_{t_0-(q-2)} & \eta_{t_0-(q-2)} & \eta_{t_0-(q-2)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \eta_{t_0-(q-h)} & \eta_{t_0-(q-h)} & \eta_{t_0-(q-h)} & \cdots & \eta_{t_0-(q-h)} \end{pmatrix} \quad (29)$$

25. Finally, the net capital stock is

$$\kappa_2(t) = \sum_i \eta\,(i). \qquad (30)$$

26. These procedures can serve for programming solution in a high-level programming language, such as Python or R. For example, Schmalwasser and Schidlowski (2006) used Visual Basic for Applications within Excel spreadsheets in their work. In our case, a language and interface for statistical computing R (R Core Team, 2023) has been used in regular compilation and is used for the accompanying simulations presented in the following section.

IV. Simulations with possible impact on the gross national income

27. This part of the paper is, in a way, a continuation of the case study within the EG20-CFC project where CBS was a beneficiary. In this section, several scenarios will be presented with respect to different assumptions in the model and different levels of availability in the database. Assumptions in the model are related to the selection of depreciation function, while different levels of availability in the database are primarily related to industries and institutional subsectors. GFCF refers only to the government sector, as any change in the GFCF or assumptions may have a certain impact on GNI. In the case study of the EG20-CFC project, the impacts were presented only visually along with qualitative interpretations, without calculating the impact to any of the aggregate.

Table 1
The type of assets available in Croatian national accounts

ESA	*ASSET TYPE*	*AVG SERVICE LIFE*
AN.111	Dwellings	80 (70)
AN.112	Non-residential buildings (business and industrial)	50
AN.112	Other structures (infrastructure)	55
AN.112	Land improvements	55
AN.11O	Metal products	20
AN.11O	General machinery and equipment	20
AN.11O	Other special purpose machinery	20
AN.11O	Machinery for agriculture	20
AN.1132	Computers and peripherals	6
AN.11O	Electrical equipment	15
AN.1132	Communication equipment	6

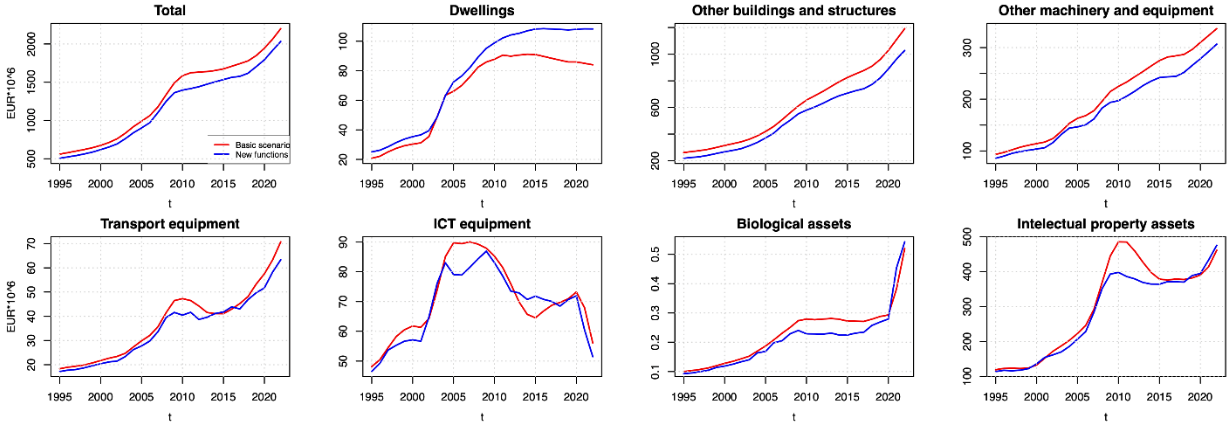
ESA	*ASSET TYPE*	*AVG SERVICE LIFE*
AN.11O	Medical equipment	10
AN.11O	Furniture and interior fittings	15
AN.1131	Passenger cars	7
AN.1131	Trucks and commercial vehicles	12
AN.1131	Other means of transport	25
AN.115	Perennial plantations	15
AN.115	Herd	(-)
AN.117	Research and development	10
AN.117	Mineral exploration	10
AN.117	Software and database	5
AN.117	Literary and artistic originals	7
AN.11O	Other tangible assets	10
AN.117	Other non-tangible assets	7
AN.11O	Weapons system	25

28. First and foremost, the difference between the base scenario and some new approach must be obtained. The base scenario is the one currently in use, referring to the application of geometric depreciation for dwellings and linear depreciation in combination with the survival function for all other assets. In addition, the base scenario includes the estimation of CFC and capital stock at the level of institutional sectors, types of assets and industries. Type of assets are kept fixed in all cases while different aggregations will be carried out on industries and sub-sectors, which are discussed in more detail below. The level of detail for fixed assets is shown in Table 1 and is used as such in all scenarios. Along with the impacts, a graphical comparison of the results will be shown at the level in the column named ESA. The level of detail for fixed assets is shown in the column ASSET TYPE and will be used as such in all scenarios. It should be noted that for the herd, CFC is equal to zero, which means that the gross stock is equal to the net stock. Average service life for dwellings in the household sector is estimated at 80 years, and 70 years for dwellings in other sectors. This is because average service life for household dwellings was estimated on the basis of census, while for other sectors the recommendations of the Eurostat Task Force for Fixed Capital were followed.

A. Altering depreciation functions

29. As mentioned earlier, for estimations of capital stock and CFC, it is recommended to use a convex shape function instead of solely linear function which is normally used in commercial accounting. This is because the estimation procedures are not undertaken by individual assets but for cohorts of assets of similar ages and characteristics. Individual assets within a cohort will retire at different timepoints but the depreciation profile for a cohort as a whole is typically convex to the origin (SNA, 2008). In this regard, if a linear depreciation function is used, then it needs to be adjusted for survival pattern derived from, at least in our case, log-normal probability distribution. A retirement function combined with a linear depreciation function may produce convex pattern similar to geometric function.

Figure 3
The CFC vectors given from base scenario and new depreciation function



30. Unlike the basic scenario, the alternative one includes the application of linear depreciation for dwellings and geometric depreciation for other assets. As expected, similar vector shapes are achieved, but with pronounced differences at certain intervals, depending on the type of fixed assets. When the results are put in relation to the GNI according to equation (6), the results are obtained as in Table 2.

Table 2
The impact of changes in depreciation functions on GNI (10^6 EUR)

Indicator	*2013*	*2014*	*2015*	*2016*	*2017*	*2018*	*2019*	*2020*	*2021*
$G(t)$	43.876	43.927	45.934	46.441	49.367	51.648	54.604	51.372	58.102
$\Delta\varsigma(t)$	164,61	147,12	139,71	145,56	165,54	161,12	144,31	151,22	147,31
$\varepsilon(t)$%	0,38	0,33	0,3	0,31	0,34	0,31	0,26	0,29	0,25

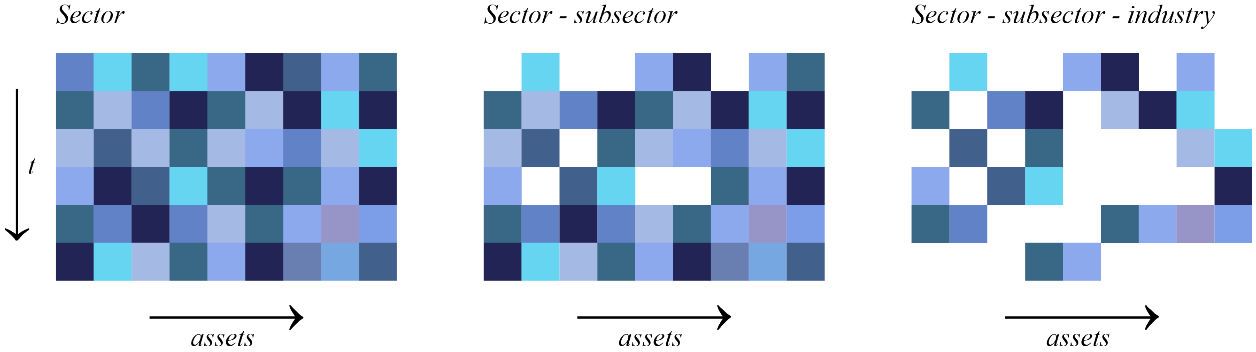
31. A certain impact exists for all years, ranging from 0.25% to 0.38%. If the threshold of 0.1% (document GNIC/283) is taken, application of geometric depreciation for all assets except dwellings, would lead to revision of the time series from 2013 onwards.

B. Different levels of aggregation

32. At some point, statisticians often do not have data at desired classification level. For example, having a satisfactory level of GFCF by type of assets (which we keep fixed anyway), but, for some reason, lacking on the level of institutional subsectors or industries. After some time, data at more detailed levels may become available, and the question is to what extent the resulting change will affect GNI. For statistical agencies that have well-organized registers and sufficiently long GFCF time series, this issue is probably not applicable. Nevertheless, since Croatia lacks sufficiently long time series, this simulation will serve to compare the results in relation to the currently valid estimation levels of CFC and capital stock.

Figure 4

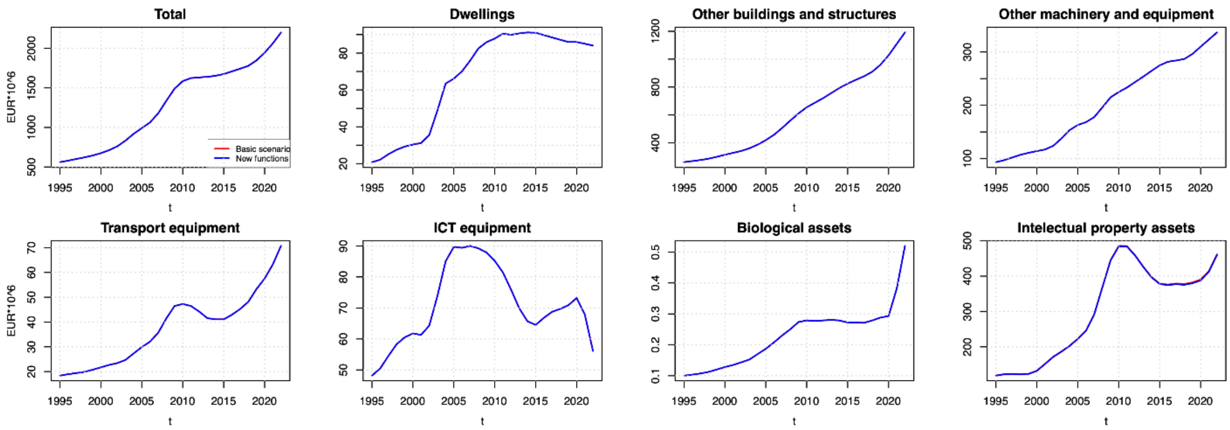
GFCF matrices resulting from different levels of aggregation



33. The main issue is shown in Figure 4. Squares in various blue shades indicate GFCFs different from zero. It is to be expected that at the sector level, the values will differ from zero since institutions in any subsector and industry have invested in some assets. If data is filtered for a sub-sector, zeros will appear in some places because, within that sub-sector, some industries did not invest in certain assets in some years.

Figure 5

The CFC vectors from the sub-sectors aggregation only



34. When some chosen industry is additionally filtered from the subsector, the zeros are more visible, and such a matrix becomes sparse. Sparse matrices on industry-level filtering are not uncommon in the case of Croatia, especially if the zeros are at the beginning of observable time series, i.e. 1995 or 1996. What adjustments are additionally applied due to this issue will be described below as part of the interpretation of impacts. Figure 5 graphically presented results when CFC for all types of assets is estimated only by subsectors, avoiding the level of industries. CFC vectors overlap indicating that almost identical results are obtained or the difference is negligible. The extent to which the difference is negligible is presented as an impact on GNI, as in the previous case.

Table 3

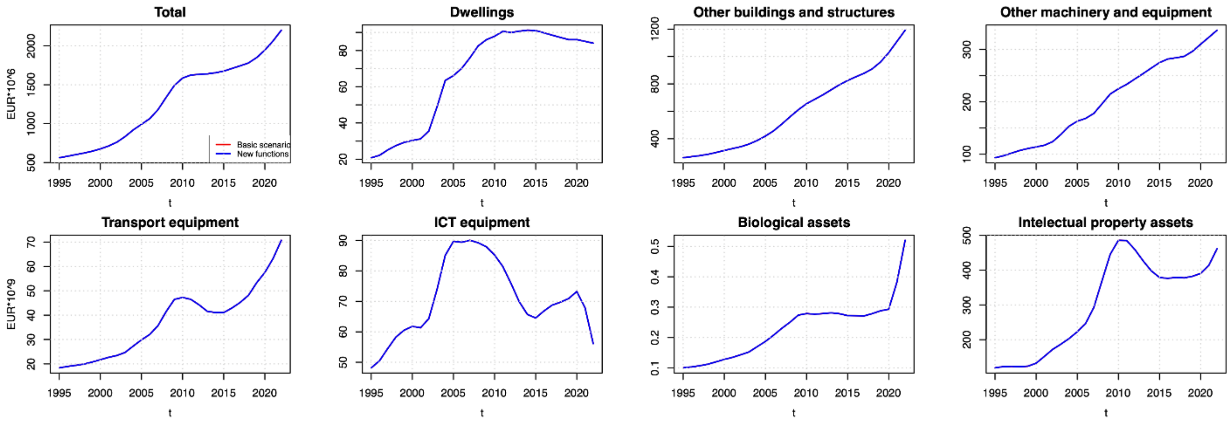
The impact on GNI of estimating CFC at the subsectors level (10^6 EUR)

Indicator	*2013*	*2014*	*2015*	*2016*	*2017*	*2018*	*2019*	*2020*	*2021*
G(t)	43.876	43.927	45.934	46.441	49.367	51.648	54.604	51.372	58.102
Δς(t)	1,08	1,19	1,4	1,67	2,21	2,4	2,51	2,63	2,21
ε(t)%	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

35. In this simulation, the differences resulting from bypassing the industry level and calculating CFC only by subsectors of the general government are negligible. We perform

the same procedure but this time on the government sector, meaning without sub-aggregation of GFCF to sub-sectors and industries. Visually, the results are presented in Figure 6.

Figure 6
The CFC vectors from the sector aggregation only



36. The results are almost identical as in the case of the CFC estimation at the subsector level, the impact is negligible, as can be seen from the graphic where the vectors overlap. Numerical results are shown in Table 4.

Table 4
The impact on GNI of estimating CFC at government sector level (10^6 EUR)

Indicator	*2013*	*2014*	*2015*	*2016*	*2017*	*2018*	*2019*	*2020*	*2021*
$G(t)$	43.876	43.927	45.934	46.441	49.367	51.648	54.604	51.372	58.102
$\Delta\varsigma(t)$	0,25	0,24	0,24	0,23	0,23	0,22	0,21	0,21	0,2
$\varepsilon(t)\%$	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

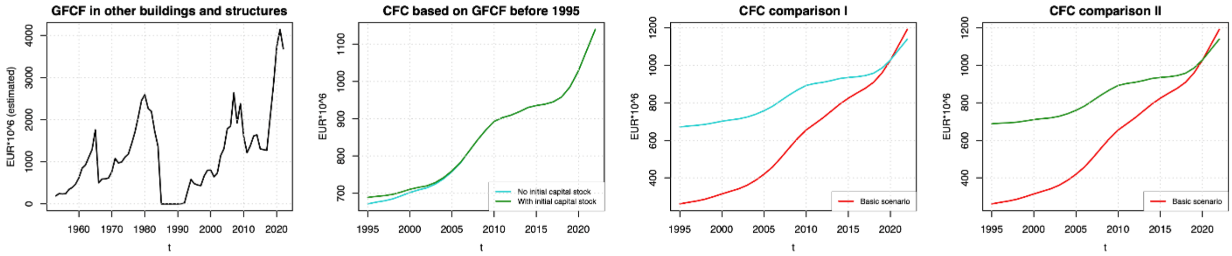
37. Differences are also negligible in terms of percentage of GNI. In both cases, therefore, the estimation of CFC at the sector and sub-sector level has almost no impact on the GNI. Such outcomes are probably expected given the adjustments made in the regular CFC estimation. Namely, according to (15), if the first available GFCF, i.e. for 1995, is equal to zero, then the imputed values will also be zero. When the time series, obtained by filtering sectors, sub-sectors and industries from the GFCF database as shown in Figure 4, contains zeros in the first or first two places, then the initial capital stock is calculated for the first non-zero value. This will result in the CFC also appearing in, for example, the first two years where the GFCF is missing, which may seem unusual at first. However, due to (21) and (24), and assuming that a certain industry within the (sub)sector invested in the assets before 1995, these assets will survive in gross terms and accumulate into the future. The criterion for the GFCF offset by a year or two forwards was the analysis of the time series noticing values other than zero. If the time series contains a majority of zeros, then no offset is applied, and imputation before 1995 is zero. It seems that this approach has an effect in achieving some meaningful level of capital stock and CFC in 1995, at least according to (13).

C. Estimated GFCF time series back to 1953

38. Imputation functions for estimating historical GFCF time series can lead to questionable results, which is especially true for fixed assets with longer average service life. Accordingly, the backward GFCF reconstruction is based on the data from a series of statistical yearbooks (Federal Statistical Bureau of Croatia, SFR Yugoslavia, 1954-1990). However, different concepts of aggregate calculation in a mixed economy system (Croatia was part of SFR Yugoslavia) as well as the denomination of the domestic currency create difficulties in reliable GFCF estimation before 1995. From 1945 to 1990 in Yugoslavia, two ownership systems were constituted: a privileged dominant system of public assets, later

social ownership in which cooperative ownership was gradually included, and a legally and economically limited system of private ownership mainly for arable agricultural land per household (Simonetti, 2010). For these reasons, it has been quite difficult to allocate assets within institutional sectors as requested by ESA 2010 transmission program. Along with the debatable issue of ownership, the problem with currency denomination requires special attention and careful analysis. For this reason, two type of fixed assets that are considered to have been part of government sector in Yugoslavia were taken. It is about infrastructure such as roads, tunnels, streets, squares, bridges, railways, and buildings such as administrative buildings, cultural and educational institutions, health facilities, sports halls and similar facilities. These two sets of data are added up to a single time series and joined with the GFCF from 1995 onwards as shown in the first graph on the left in Figure 7.

Figure 7
The CFC vectors from historic data on public infrastructure (106 EUR)



39. Significant improvements are included in the GFCF. Since statistical yearbooks of Yugoslavia 1954–1995 contain data with different denominations and different currencies, it was difficult to generate meaningful time series. For period 1953–1990, the exchange rate of the German mark (DEM) was used at intervals of denominations and new currencies, where jumps in the exchange rate of the Yugoslav and Croatian dinar versus DEM were observed with the denominations. Since the GFCF data 1991–1994 were in Croatian kuna, they were converted directly to EUR. Data from 1953–1990 were first converted to DEM and eventually EUR using fixed conversion rate of the two. The result of the conversions is shown in Figure 7 with a black graph. It should be taken into account that no revaluation was applied due to significant fluctuations in the price index of materials and wages in construction industry in period 1953-1994. Although the estimate of GFCF 1953–1994 in EUR might not be the most reliable, it could perhaps certainly reflect a more realistic situation compared to the imputation function under (16) which is actually a rescaled exponential function.

40. The black vector in Figure 7 serves as an input to the CFC estimation. Although data for infrastructure and non-residential buildings in the government sector are now preliminary available since 1953, the estimation was approached in two ways; without initial capital stock and with initial capital stock as shown by the cyan and green vectors. As is clearly visible, there is a difference between the two since 1995, but it is gradually disappearing. The inclusion of the imputation function based on GFCF in 1953 has almost no effect, therefore the impact in Table 5 will be given only for the green vector, meaning with no initial stock included.

Table 5
The impact of estimating CFC (based on GFCF back to 1953) on GNI

Indicator	*2013*	*2014*	*2015*	*2016*	*2017*	*2018*	*2019*	*2020*	*2021*
G(t)	43.876	43.927	45.934	46.441	49.367	51.648	54.604	51.372	58.102
Δς(t)	161,97	136,45	110,7	87,04	67,99	48,38	25,94	0,86	0,86
ε(t)%	0,37	0,31	0,24	0,19	0,14	0,09	0,05	0,00	0,00

41. The fourth graph from the left shows a comparison of the green vector based on historical GFCF data with the basic scenario. A significant difference between the two vectors is observed, most pronounced since 1995 and decreasing towards 2020. According

to Table 5, impacts greater than 0.1% for 2013–2017 are observed, while after 2018 the impacts decrease.

V. Conclusion

42. Based on the limited GFCF data that serve as inputs for the CFC estimation and capital stock, a mathematical methodology was presented serving as the basis for programming solution in the statistical computing language R. That is, apart from regular estimation of CFC and capital stock being the base scenario, R also served as a tool for simulating different assumptions in the estimation procedure. The aim was to simulate estimations using different depreciation functions, to estimate the CFC at different levels of classification, and using preliminary real data on GFCF since 1953 for infrastructure and non-residential buildings in the government sector. Results indicate that selection of other depreciation functions and the introduction of longer time series of GFCF impact the GNI for period 2013-2021. Changes in depreciation functions from linear to geometric (and *vice versa* for dwellings) would cause a significant impact for all years from 2013 to 2021, ranging from 0.25% to 0.38%. Introduction of a longer GFCF time series has an impact on GNI, the highest in 2013, with gradual decreasing to be less than 0.1% for years 2019-2021. Regarding the preliminary estimated GFCF for period 1953–1994, a caveat should be taken into account due to historical currency denominations along with the GFCF recording in several currencies through all years. In addition, assets revaluation was not carried out due to galloping price indices in certain periods. Finally, further challenging analyses on the GFCF estimation before 1995, with focus on assets with longer average service life, are certainly needed while this paper is a preliminary step in considering any updates in regular methodological procedures.

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