IO Converter
A framework for
converting supply and use tables
into input-output tables

User Manual
Version 0.0 – September 12, 2022

DISCLAIMER

The IO Converter is under development within the Real Sector Division of the IMF's Statistics Department (STA) to assist compilers of input-output tables. The Converter is intended to be used and distributed through the STA and the IMF's Regional Capacity Development Centers (RCDCs) technical assistance and training programs to member countries.

IO Converter uses standard Microsoft Excel tools and functions available to all users and only requires that users have licensed versions of Microsoft Office. The author and the IMF assume no responsibility to users for support or maintenance and have disclaimed all liability for any errors that may exist in the software or for any other claims relating to the software.

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NOTES ON IO CONVERTER

Version 0.0 of the IO Converter implements the industry by industry input-output tables (IOTs) with fixed product sale structure assumption.

The IO Converter follows recommended practices in the System of National Accounts, 2008; Eurostat Manual of Supply, Use and Input-Output Tables (2008 edition), Handbook on Supply and Use Tables and Input Output-Tables with Extensions and Applications (New York 2018); Miller, Ronald E., and Peter D. Blair. Input-Output Analysis: Foundations and Extensions. Cambridge University Press, 2009

INTRODUCTION

The IO Converter is a computer-based framework that transforms supply and use tables (SUTs) into symmetric (or square?) IOTs. The IO Converter will be able to generate IOTs containing data on the use of domestic production and imports (intermediate consumption, final consumption, capital formation and exports) , at basic prices.

This tool, developed as part of the technical assistance program of the Real Sector Division of the IMF's Statistics Department, is intended to serve compilers of IOTs by way of a spreadsheet-based compilation system. IOTs are analytical tables derived from SUTs.

The main problem the Converter will be able to solve is the transformation of uses from purchaser's to basic prices, split of uses by origin (domestic and imports) and recording of secondary (off-diagonal) output in the IO table. The SUTs to IOTs is not required if no secondary outputs are observed.

The *System of National Accounts, 2008 (2008 SNA)* recommends the compilation of IOTs to benefit from their analytical usefulness. SUTs are primarily intended for compilation purposes, including checking the numerical consistency of definitions and concepts of data sources, and outlining basic economic data in the same structure in which they are gathered. Both SUTs and IOTs can be extended to develop satellite accounts and social accounting matrices.

IOTs and IO analyses serve a wide range of analytical purposes including:

- impact analyses (what-if studies) to assess the effects on the economy of changes in elements which are defined as exogeneous in the model.
- environmental IO analyses to account for pollution generation and abatement associated with inter-industry activities
- structural decomposition analysis to disaggregate total amount of change into contributions made by various components.
- Multi country IOTs in which trade relations between countries are explicated, are used measure Trade in Value Added (TiVA)[1].
- Identification of inconsistencies and weaknesses in SUTs.

[1] Joaquim J.M. Guilhoto, Colin Webb and Norihiko Yamano (2022), Guide to OECD TiVA Indicators, 2021 edition

France is one of the few countries, if not the only one, to compile IOTs directly from data sources (INSEE uses the concept of pure or homogeneous industries except for agriculture where wine is a secondary product, and non-market industries where sales of secondary market output are recorded).

SUTs and IOTs are matrix representations of the interrelationships between producers and consumers in terms of transactions in goods and services. A supply table consists of rows of all products in the economy and columns corresponding to domestic production (organized by industry) and imports at basic prices as well as valuation adjustments to reconcile supply at basic prices with supply at purchasers' prices. A use table covers all products valued at purchaser's prices in the row and various types of uses in the columns. A supply table and a use table are combined into a SUT to describe the values of transactions in the national economy categorized by product group and by industry. SUTs show the structure of costs of production, the flows of goods and services produced within the national economy, and the flows of goods and services between the domestic economy and the rest of the world. IOTs are derived from SUTs to show products in both rows and columns or industry in both rows and columns and row totals match column totals. Symmetric IOTs describes how supply matches uses using a product-by-product classification or an industry-by-industry categorization of output and also the details of intermediate consumption and final use transactions.

Figure 1: A simplified supply and use framework

Supply Table

	Industries	Imports	Trade and transport margins	Taxes minus subsidies on products	Total supply
Products	Output at basic prices	FOB			Purchaser's prices
	GO	M	0	T-S	GO+M+T-S

Use Table

	Industries	Final consumption expenditure	GFCF	CII	Net acquisition of valuables	Exports	Total use
Products	Intermediate consumption						Purchaser's prices
	IC	C	GFCF	CII	V	X	IC+C+GFCF+CII+V+X
	Gross Value added						
	GO						

Figure 2: A simplified industry by industry IOTs

	Producers as consumers	Final uses					Total use
	Industries	Final consumption expenditure	GFCF	CII	Net acquisition of valuables	Exports	
Industries	Intermediate consumption						Gross output
	imports						
	IC	C	GFCF	CII	V	X	IC+C+GFCF+CII+V+X
Gross value added	Compensation of employees						
	Other taxes on production						
	Mixed income and gross operating surplus						
	Gross Output						

Assumptions are required to convert SUTs into IOTs. The underlying assumption of the mid-20th century, supporting many reference manuals including the *SNA 68*, that each industry produces a single product and that each product is produced by a single industry no longer prevails in the modern economy and it is relevant to make a distinction between industries and products. Product by product IOTs require product technology (each product is produced in its own specific way) or industry technology (each industry has its own specific way of production) assumption. Industry by industry IOTs require fixed industry sales structure (each industry has its own specific sales structure) or fixed product sales structure (each product has its own specific sales structure) assumption. The selection of each appropriate model depends on the objective of the study. In particular, industry-by-industry IOTs are well suited for analyses such as tax reform, impact analysis for shocks and macro-fiscal policy. Product-by-product tables are particularly well suited for analyses that assume homogeneous production units, e.g., productivity, comparison of cost structures, employment effects, energy policy and environmental policy.

The current version of IO Converter implements the industry by industry model with fixed product sales structure assumption. The tool was initially developed to improve the timeliness and geographical coverage of some IOTs-based indicators on the IMF Climate Change Indicators Dashboard https://climatedata.imf.org/pages/re-indicators. Industry by industry model allows better integration with collection of statistics compiled according to market transactions and industrial activities including energy consumption. Fixed product sales structure assumption does not produce negative values in the requirement matrix, which would require *ad hoc* manual adjustment.

The IO Converter seeks flexibility and user-friendliness. It is shared and installed as any MS Excel file. Minimum manual intervention to map data and classification is required to read the SUTs and produce the IOTs. The average time to complete the mapping and obtain an IOTs is a couple of hours. The formula used for the transformation are available across all sheets to enable users to trace all steps and introduce changes if necessary. A balancing method is not implemented as it is assumed that SUTs are balanced.

Conversion of SUTs with industries without output might generate inconsistent IOTs. Features to handle this type of SUTs are under development.

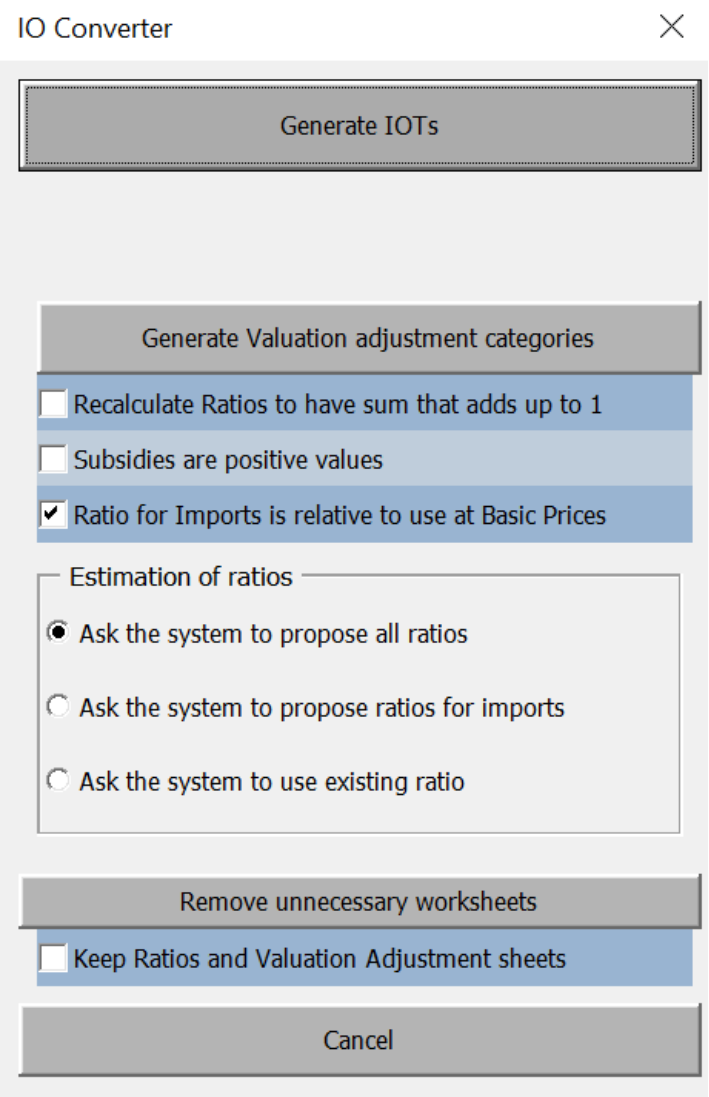
DRY RUN

ENSURE THAT THE SYSTEM CAN FIND THE CODES USED IN 'PARAMETERS' TAB

Unlicensed MS Excel version may request the following check

- Select the 'parameters'
- Run the "find" or "Search" tool (Ctrl+f)
- Find ""IO_P1"
- If you cannot find "IO_P1", then replace all instances of "IO_" with a key word that could be found by your computer (example: "HIO_") (**replace in "Parameters" and "Code_Mapping" tabs**)
- Go to 'Parameters' Tab and click on the arrow to run the code as shown in the form below.

Meaning of options to run the code

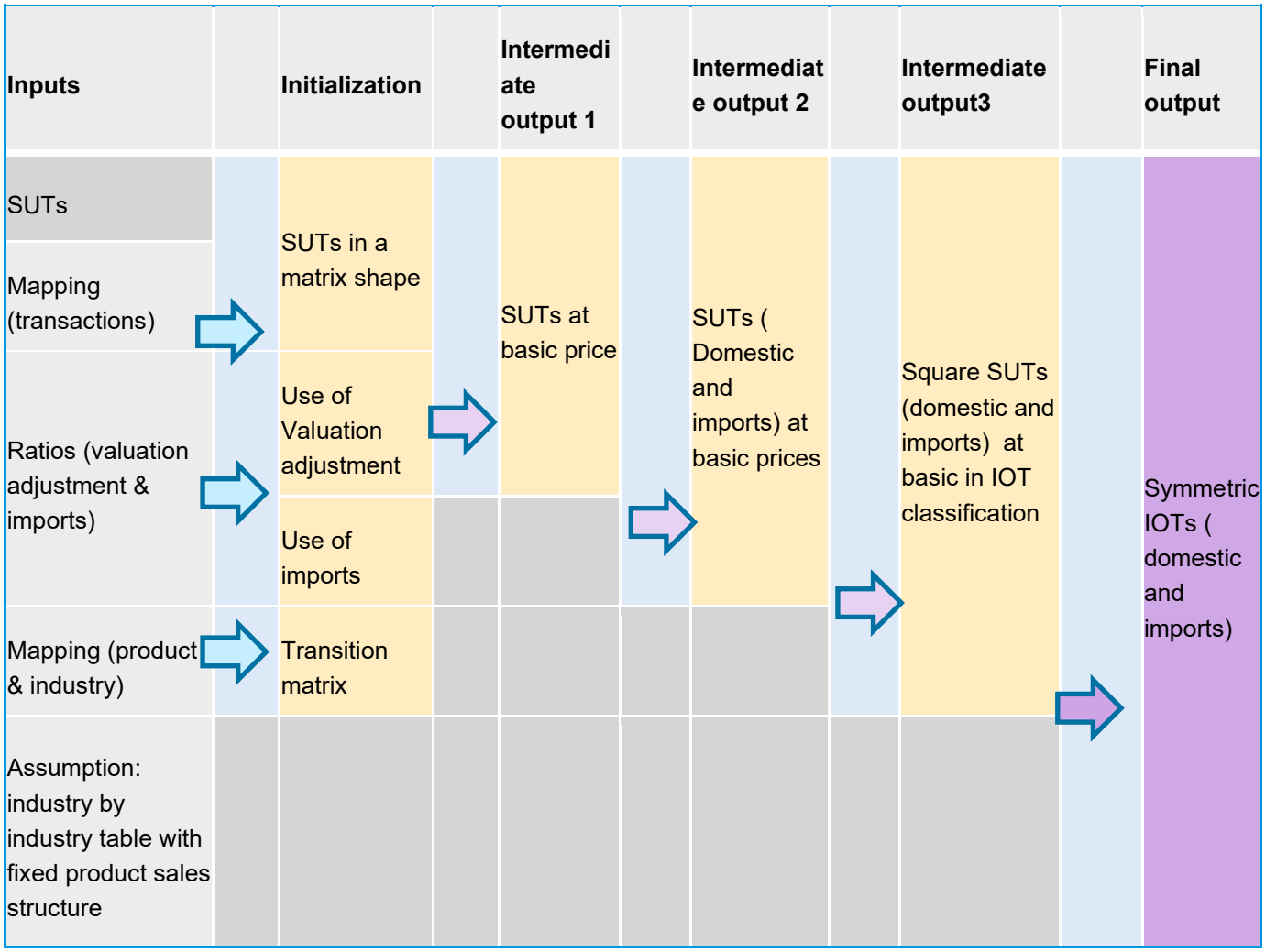


- **Valuation adjustment categories**
  - "*Recalculate Ratios to have sum that adds up to 1*". In the initialization phase, Ratios for valuation adjustment and imports are calculated as shares of use categories relative to total use. Mark this box to request the system to pro rate the given ratios to further ensure that the sum of ratios adds up to one. This is particular useful if the ratios are inconsistent.
- **Estimation of ratios**
  - "*Ask the system to use existing ratios*". Mark this box if specific ratios have been manually proposed to prevent the system from recalculating the ratio. It is recommended to manually adjust the ratios after the system has created a worksheet for ratios
- **Keep valuation adjustment:** Mark this box to prevent the deletion of tabs related to valuation adjustment. This is particular useful if specific ratios or valuation adjustment have been manually entered.

Meaning of actions to run the code:

- **Generate IOTs**: Push this button to generate square SUTs at basic prices and IOTs for domestic and imports
- **Generate valuation adjustment categories**: Push this button to generate ratios and valuation adjustment allocated to use categories.

MAJOR STEPS TO CONVERT SUTS INTO IOTS



The steps includes:

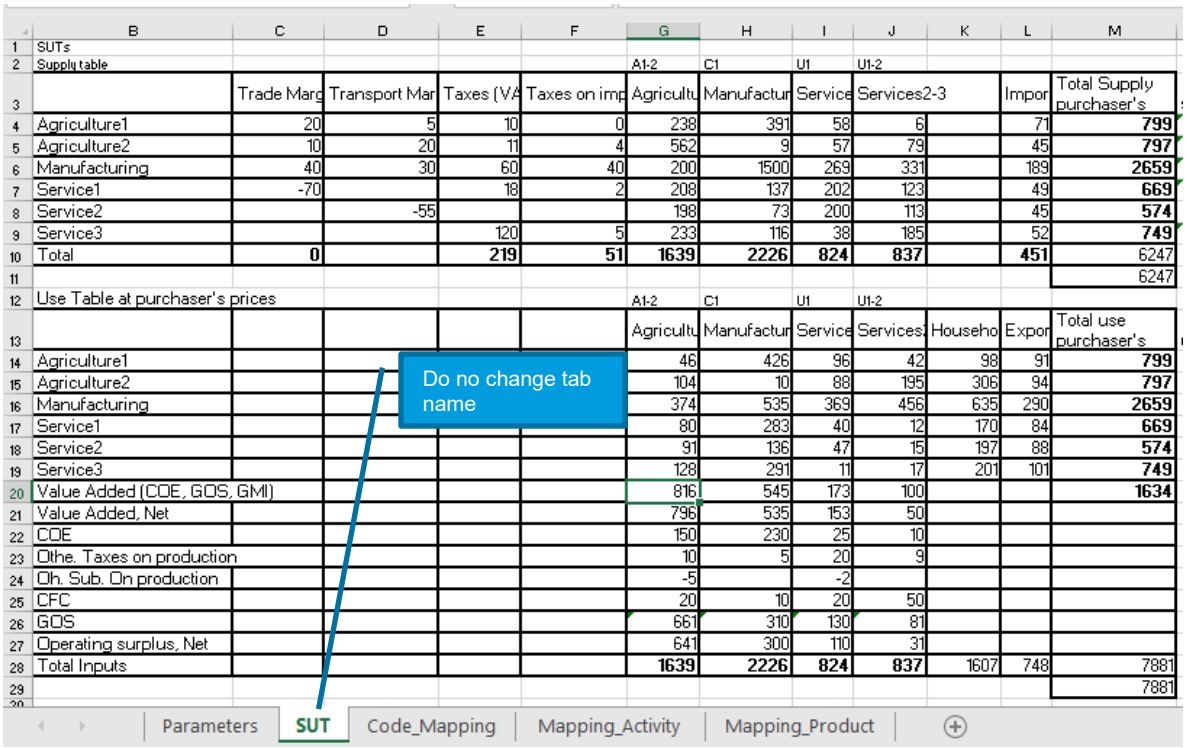
- Calculation of taxes and margins by expenditure categories
- Compilation of SUTs at basic prices (domestic SUT and SUTs for domestic and imported supply)
- Prepare square SUTs at basic prices (use concordance from mapping)
- Prepare symmetric IOTs at basic prices

The steps in chart are further described in the following sections and matrix calculations are outlined in the Appendix.

INPUT DATA

Input data include balanced SUTs and concordance table between the SUT classification and the IOT classification. Instructions to use the converter:

- Clear contents of the 'SUT' tab
- Copy a SUTs for a given year and paste in the 'SUT' tab (ensure that values are available [i.e., nonnumerical characters such as – /*' where a value is expected will generate error values])
- Position your SUTs anywhere on the worksheet
- Ensure that your SUTs is balanced (total use=total supply by product, sum of margins=0, etc.)
- Ensure that use categories (intermediate consumption, final consumption expenditure (FCE), fixed capital formation (FCF), exports) are available
- Details for transaction are accepted (e.g., FCE broken down by institutional sector, etc.)
- **Do not change tab name.**

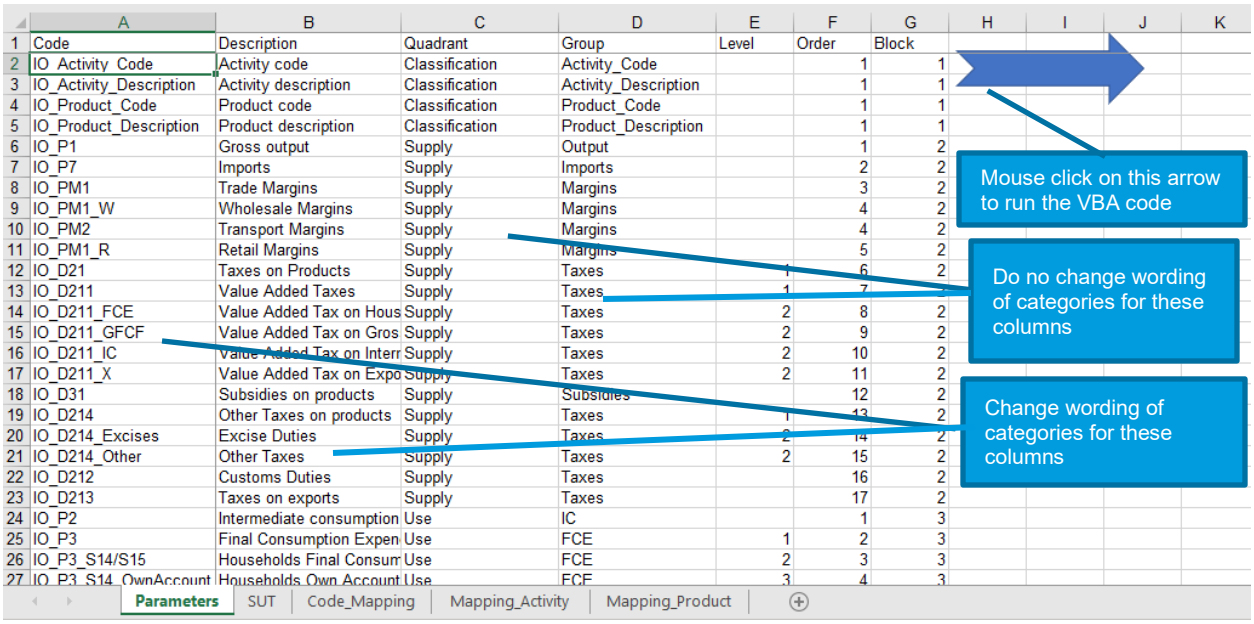


	B	C	D	E	F	G	H	I	J	K	L	M
1	SUTs											
2	Supply table					A1-2	C1	U1	U1-2			
3		Trade Marg	Transport Mar	Taxes (VA	Taxes on imp	Agricultu	Manufactur	Service	Services2-3		Impor	Total Supply purchaser's
4	Agriculture1	20	5	10	0	238	391	58	6		71	**799**
5	Agriculture2	10	20	11	4	562	9	57	79		45	**797**
6	Manufacturing	40	30	60	40	200	1500	269	331		189	**2659**
7	Service1	-70		18	2	208	137	202	123		49	**669**
8	Service2		-55			198	73	200	113		45	**574**
9	Service3			120	5	233	116	38	185		52	**749**
10	Total	**0**		**219**	**51**	**1639**	**2226**	**824**	**837**		**451**	6247
11												6247
12	Use Table at purchaser's prices					A1-2	C1	U1	U1-2			
13						Agricultu	Manufactur	Service	Services	Househo	Expor	Total use purchaser's
14	Agriculture1					46	426	96	42	98	91	**799**
15	Agriculture2					104	10	88	195	306	94	**797**
16	Manufacturing					374	535	369	456	635	290	**2659**
17	Service1					80	283	40	12	170	84	**669**
18	Service2					91	136	47	15	197	88	**574**
19	Service3					128	291	11	17	201	101	**749**
20	Value Added (COE, GOS, GMI)					816	545	173	100			**1634**
21	Value Added, Net					796	535	153	50			
22	COE					150	230	25	10			
23	Othe. Taxes on production					10	5	20	9			
24	Oh. Sub. On production					-5		-2				
25	CFC					20	10	20	50			
26	GOS					661	310	130	81			
27	Operating surplus, Net					641	300	110	31			
28	Total Inputs					**1639**	**2226**	**824**	**837**	1607	748	7881
29												7881

PREPARE MAPPING BETWEEN TRANSACTIONS & CLASSIFCATION AND SUT

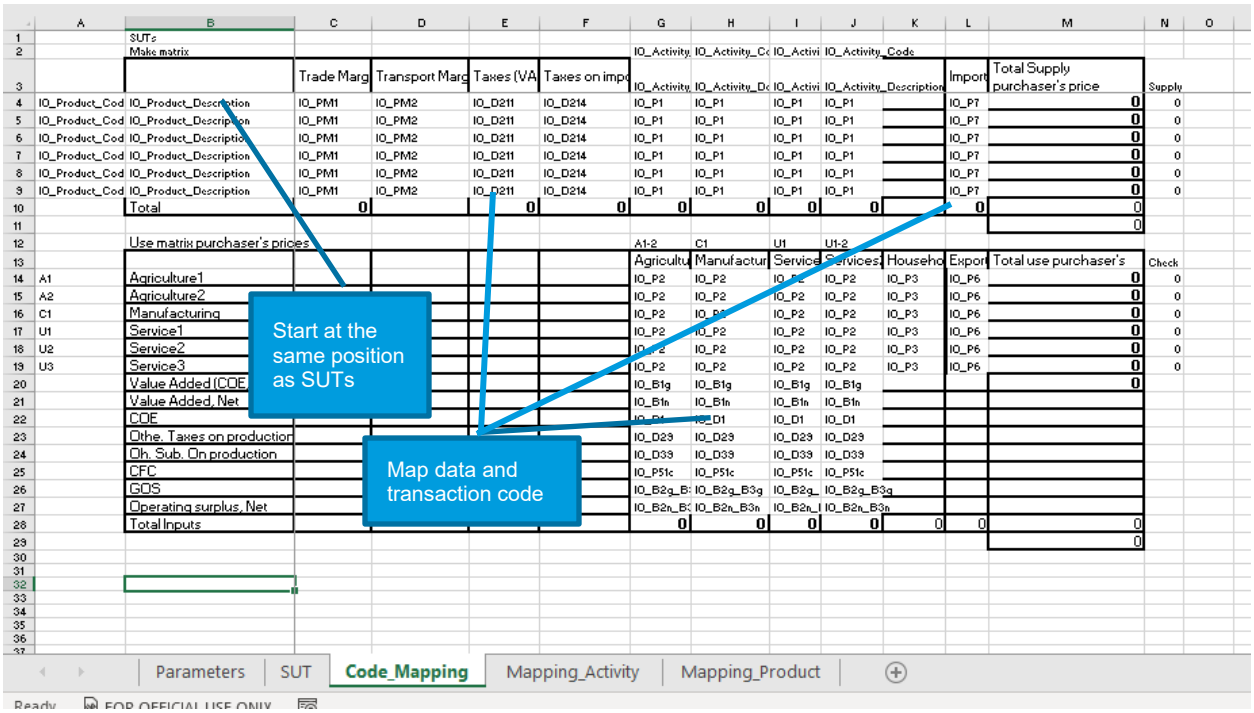
Use information to map the SUT, transactions and classification as follows:

- Update '**Parameters'** tab if need be:
  - change or add codes and description to reflect your data
  - **do not change categories/contents of 'Quadrant' and 'Group'**
  - use 'Order' to indicate how you want to outline you IOTs (e.g. for use category from 'Quadrant', intermediate consumption, followed by FCE, GFCF, changes in inventories, net acquisition in valuables, exports)
  - **Do not change tab name.**



	A	B	C	D	E	F	G
1	Code	Description	Quadrant	Group	Level	Order	Block
2	IO_Activity_Code	Activity code	Classification	Activity_Code		1	1
3	IO_Activity_Description	Activity description	Classification	Activity_Description		1	1
4	IO_Product_Code	Product code	Classification	Product_Code		1	1
5	IO_Product_Description	Product description	Classification	Product_Description		1	1
6	IO_P1	Gross output	Supply	Output		1	2
7	IO_P7	Imports	Supply	Imports		2	2
8	IO_PM1	Trade Margins	Supply	Margins		3	2
9	IO_PM1_W	Wholesale Margins	Supply	Margins		4	2
10	IO_PM2	Transport Margins	Supply	Margins		4	2
11	IO_PM1_R	Retail Margins	Supply	Margins		5	2
12	IO_D21	Taxes on Products	Supply	Taxes	1	6	2
13	IO_D211	Value Added Taxes	Supply	Taxes	1	7	
14	IO_D211_FCE	Value Added Tax on Hous	Supply	Taxes	2	8	2
15	IO_D211_GFCF	Value Added Tax on Gros	Supply	Taxes	2	9	2
16	IO_D211_IC	Value Added Tax on Interr	Supply	Taxes	2	10	2
17	IO_D211_X	Value Added Tax on Expo	Supply	Taxes	2	11	2
18	IO_D31	Subsidies on products	Supply	Subsidies		12	2
19	IO_D214	Other Taxes on products	Supply	Taxes	1	13	2
20	IO_D214_Excises	Excise Duties	Supply	Taxes	2	14	2
21	IO_D214_Other	Other Taxes	Supply	Taxes	2	15	2
22	IO_D212	Customs Duties	Supply	Taxes		16	2
23	IO_D213	Taxes on exports	Supply	Taxes		17	2
24	IO_P2	Intermediate consumption	Use	IC		1	3
25	IO_P3	Final Consumption Expen	Use	FCE	1	2	3
26	IO_P3_S14/S15	Households Final Consum	Use	FCE	2	3	3
27	IO_P3_S14_OwnAccount	Households Own Account	Use	FCE	3	4	3

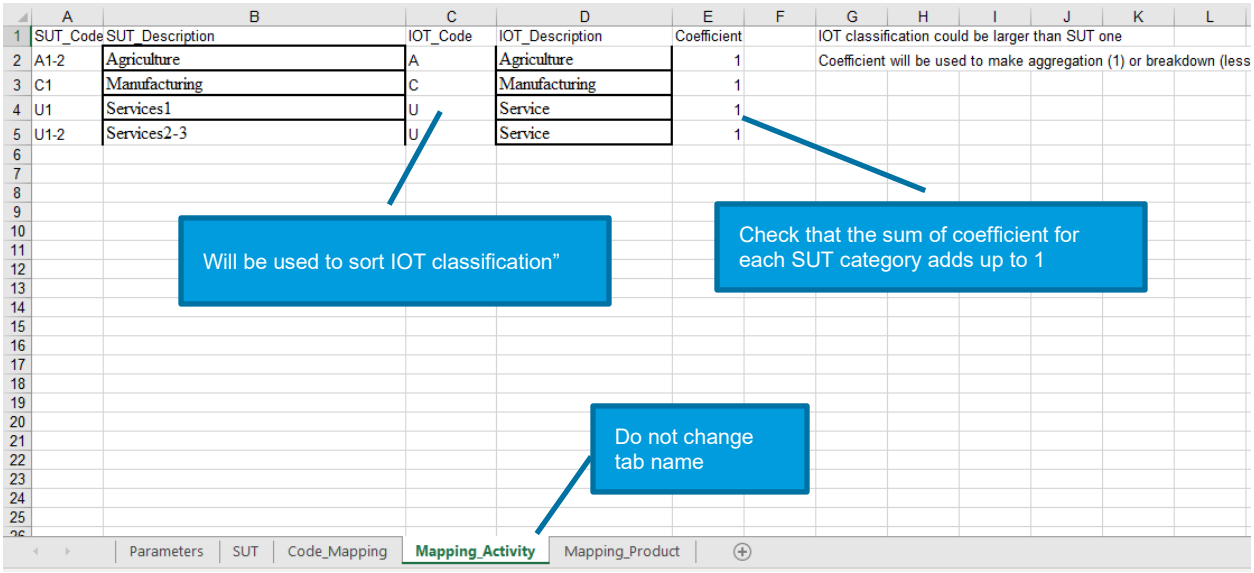
- Copy your SUT worksheet and paste in 'Code_Mapping' tab
  - **Do not change location of data and classification** (data and classification should be in the same range)
  - Use codes from 'Parameters' tab to map data and classification
  - Duplicates are not yet accepted: Do not map "Total" and components (e.g. total supply, total use, etc.)
  - Balancing items are excepted (e.g. gross and net value added (GVA, NVA), gross and net operating surplus (GOS, NOS))
  - Only a single column for "Imports" is accepted (combine Imports of goods and services in single column)



IOT CLASSIFICATIONS

SUT activities and products are mapped to IOT activities and product as follows

- In 'Mapping_Activity' tab
  - Provide SUT code and description in column 'Old Code' and 'Old Description'
  - Provide IOT code and description in column 'New Code' and 'New Description'
  - Provide coefficient to use for mapping: for simple aggregation, coefficient is one. For breakdown, coefficient is smaller than 1
  - New code will be used to sort description in IOTs (i.e., change your code to allow meaning arrangement)
  - Ensure that similar categories are used for both new activities and products to enable symmetric IOTs



- In 'Mapping_Product' tab
  - Provide SUT code and description in column 'Old Code' and 'Old Description'
  - Provide IOT code and description in column 'New Code' and 'New Description'
  - Provide coefficient to use for mapping: for simple aggregation, coefficient is one. For breakdown, coefficient is smaller than 1
  - New code will be used to sort description in IOTs (i.e., change your code to allow meaning arrangement)
  - Ensure that similar categories are used for both new activities and products to enable symmetric IOTs

FIRST RUN

Go to 'Parameters' Tab and click on the arrow to run the code.

Click on **Generate valuation adjustment categories** if you want to adjust the allocation of imports and valuation adjustment to use categories. Then mark the box "Ask the system to use existing ratios" and click on "Generate IOTs"

Click on **Generate IOTs** if want to see the first IOTs. You can input self-computed ratios or allocation of valuation adjustment and imports to use categories later on

FUTURE DEVELOPMENTS

Additional checks on classification and consistency (SUTs, intermediary steps, IOTs)

Replicate SUT format including total

Implement the remaining three assumptions

- industry by industry approach using industry sales structure
- product by product IOTs using product technology assumption
- product by product IOTs using industry technology assumption

Add a TiVA module

APPENDIX

TRANSITION MATRICES

This section illustrates how transition matrices have been constructed to transform rectangular SUTs into square SUTs.

A matrix $T = \begin{pmatrix} \begin{bmatrix} A_{11} & \cdots & A_{1w} \\ \vdots & \ddots & \vdots \\ A_{n1} & \cdots & A_{nw} \end{bmatrix} & \begin{bmatrix} D_{11} & \cdots & D_{1k} \\ \vdots & \ddots & \vdots \\ A_{n1} & \cdots & A_{nk} \end{bmatrix} \\ \begin{bmatrix} F_{11} & \cdots & F_{1w} \\ \vdots & \ddots & \vdots \\ F_{m1} & \cdots & F_{mw} \end{bmatrix} & \begin{bmatrix} E_{11} & \cdots & E_{1k} \\ \vdots & \ddots & \vdots \\ E_{m1} & \cdots & E_{mk} \end{bmatrix} \end{pmatrix}$ will be converted as follows:

Let R_i and C_j be row i and column j of T with $1 \leq i \leq n$ and $1 \leq j \leq w$, and $\tilde{R}_h$ be a linear combination of R_i, and $\tilde{C}_f$ be a linear combination of C_j, with $1 \leq h, f \leq l$, then $\tilde{R}_h = \sum_{j=1}^{n} \propto_{hj} R_j$ and $\tilde{C}_f = \sum_{i=1}^{w} \beta_{if} C_f$. If the combination is conducted consecutively, either R_i or C_j represents the new row or column after the first combination.

The conversion will lead to a second matrix

$\tilde{T} = \begin{pmatrix} \begin{bmatrix} \tilde{A}_{11} & \cdots & \tilde{A}_{1l} \\ \vdots & \ddots & \vdots \\ \tilde{A}_{l1} & \cdots & \tilde{A}_{ll} \end{bmatrix} & \begin{bmatrix} \tilde{D}_{11} & \cdots & \tilde{D}_{1k} \\ \vdots & \ddots & \vdots \\ \tilde{D}_{l1} & \cdots & \tilde{D}_{lk} \end{bmatrix} \\ \begin{bmatrix} \tilde{F}_{11} & \cdots & \tilde{F}_{1l} \\ \vdots & \ddots & \vdots \\ \tilde{F}_{m1} & \cdots & \tilde{F}_{ml} \end{bmatrix} & \begin{bmatrix} \tilde{E}_{11} & \cdots & \tilde{E}_{1k} \\ \vdots & \ddots & \vdots \\ \tilde{E}_{m1} & \cdots & \tilde{E}_{mk} \end{bmatrix} \end{pmatrix}$

$\tilde{T}$ is obtained by a linear combination of first n rows and first n columns of T i.e.

Such a conversion is needed for instance to bridge the industry classification of an input and output tables (IOT) and the industry classification of emission intensity. In that situation, the IOT matrix T is $dim(T) = (n + m, n + k)$ and it can be divided into four blocks:

- Block A is a rectangular matrix of intermediate consumption with $dim(A) = (n, w)$ to be transformed into $\tilde{A}$ with $dim(\tilde{A}) = (l, l)$
- Block F is a rectangular matrix of production factors, $dim(F) = (m, n)$ to be transformed into $\tilde{F}$ with $dim(\tilde{F}) = (m, l)$
- Block D is a rectangular matrix of expenditure categories, $dim(D) = (n, k)$ to be transformed into $\tilde{F}$ with $dim(\tilde{D}) = (l, k)$
- Block E is rectangular matrix that might be 0 or contain additional information, with $dim(E) = (m, k)$ to be transformed into $\tilde{E}$ with $dim(\tilde{E}) = (m, k)$

The problem is to construct two transition matrices Ω (TM_Product) and Ψ (TM_Activity) such that $\tilde{T} = \Omega \cdot \mathrm{T} \cdot \Psi$

Transition matrix

$$\Omega = \begin{pmatrix} \begin{bmatrix} \propto_{11} & \cdots & \propto_{1w} \\ \vdots & \ddots & \vdots \\ \propto_{l1} & \cdots & \propto_{lw} \end{bmatrix} & \begin{bmatrix} 0_{11} & \cdots & 0_{1m} \\ \vdots & \ddots & \vdots \\ 0_{l1} & \cdots & 0_{lm} \end{bmatrix} \\ \begin{bmatrix} 0_{11} & \cdots & 0_{1w} \\ \vdots & \ddots & \vdots \\ 0_{m1} & \cdots & 0_{mw} \end{bmatrix} & \begin{bmatrix} \delta_{11} & \cdots & \delta_{1k} \\ \vdots & \ddots & \vdots \\ \delta_{k1} & \cdots & \delta_{kk} \end{bmatrix} \end{pmatrix}$$

$$\Psi = \begin{pmatrix} \begin{bmatrix} \beta_{11} & \cdots & \beta_{1l} \\ \vdots & \ddots & \vdots \\ \beta_{n1} & \cdots & \beta_{nl} \end{bmatrix} & \begin{bmatrix} 0_{11} & \cdots & 0_{1k} \\ \vdots & \ddots & \vdots \\ 0_{n1} & \cdots & 0_{nk} \end{bmatrix} \\ \begin{bmatrix} 0_{11} & \cdots & 0_{1l} \\ \vdots & \ddots & \vdots \\ 0_{k1} & \cdots & 0_{kl} \end{bmatrix} & \begin{bmatrix} \delta_{11} & \cdots & \delta_{1k} \\ \vdots & \ddots & \vdots \\ \delta_{k1} & \cdots & \delta_{kk} \end{bmatrix} \end{pmatrix}$$

Where $0_{ij} = 0$ for any i and j, and $\delta_{ij} = \begin{cases} 1 \; if \; i = j \\ 0 \; elswhere \end{cases}$.

Indication to prepare Ω: the block $(\propto_{ij})_{1 \le i \le l \; and \; 1 \le j \le n}$ is obtained by extracting $\propto_{ij}$ from $\tilde{R}_i = \sum_{j=1}^{n} \propto_{ij} R_j$ and allocating them to column of row i.

Indication to prepare Ψ: the block $(\beta_{ij})_{1 \le i \le n \; and \; 1 \le j \le l}$ is obtained by extracting β_{ij} from $\tilde{C}_j = \sum_{i=1}^{n} \beta_{ij} C_i$ and allocating them to rows of column j.

Solution

It can easily be shown that

$$\tilde{A}_{ij} = \sum_{h=1}^{n} \beta_{hi} \sum_{j=1}^{n} \propto_{if} A_{fh}$$

$$\tilde{D}_{ij} = \sum_{f=1}^{n} \propto_{if} D_{fj}$$

$$\tilde{F}_{ij} = \sum_{h=1}^{n} \beta_{hi} F_{ih}$$

$$\tilde{E}_{ij} = E_{ij}$$

FIXED PRODUCT SALE STRUCTURE ASSUMPTION

This section illustrates how transition matrices have been constructed to transform rectangular SUTs into square SUTs

Figure 3: Matrix components of an integrated IO framework

	Domestic products	imported products	Industries	Final uses	Total
Domestic products			$U_d = \begin{bmatrix} U_{d_{11}} & \cdots & U_{d_{1n}} \\ \vdots & \ddots & \vdots \\ U_{d_{ln}} & \cdots & U_{d_{ln}} \end{bmatrix}$	$Y_d = \begin{bmatrix} Y_{d_{11}} & \cdots & Y_{d_{1n}} \\ \vdots & \ddots & \vdots \\ Y_{d_{ln}} & \cdots & Y_{d_{ln}} \end{bmatrix}$	$X = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$
imported products			$U_m = \begin{bmatrix} U_{m_{11}} & \cdots & U_{m_{1n}} \\ \vdots & \ddots & \vdots \\ U_{m_{ln}} & \cdots & U_{m_{ln}} \end{bmatrix}$	$Y_m = \begin{bmatrix} Y_{m_{11}} & \cdots & Y_{m_{1n}} \\ \vdots & \ddots & \vdots \\ Y_{m_{ln}} & \cdots & Y_{m_{ln}} \end{bmatrix}$	$m = \begin{bmatrix} m_1 \\ \vdots \\ m_n \end{bmatrix}$
Industries	$V = \begin{bmatrix} V_{11} & \cdots & V_{1n} \\ \vdots & \ddots & \vdots \\ V_{ln} & \cdots & V_{ln} \end{bmatrix}$				$g = \begin{bmatrix} g_1 \\ \vdots \\ g_n \end{bmatrix}$
Gross Value Added			$W = [W_1 \quad \cdots \quad W_n]$		$w = W \cdot \mathbf{1}$
Total	$x^T = [x_1 \quad \cdots \quad x_n]$	$m^T = [m_1 \quad \cdots \quad m_n]$	$g^T = [g_1 \quad \cdots \quad g_n]$	Y	

Where $\mathbf{1} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$

The transformation matrix is $\boldsymbol{D} = \boldsymbol{V} \cdot (\tilde{x}^{-1})$

where $(\tilde{x}^{-1}) = \begin{bmatrix} \tilde{x}^{-1}{}_{11} & \cdots & \tilde{x}^{-1}{}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{x}^{-1}{}_{n1} & \cdots & \tilde{x}^{-1}{}_{nn} \end{bmatrix}$ and $\tilde{x}^{-1}{}_{ij} = \begin{cases} {}^{1}/_{x_i} \; 1 \; if \; i = j \; and \; x_i \neq 0 \\ 0 \; elswhere \end{cases}$.

It is worth noting that $(\tilde{x}^{-1})$ is slightly different from $(x)^{-1}$ as defined in the literature because the former allows to take into account the case of an industry output of zero value.

The domestic intermediate consumption matrix is $\boldsymbol{B_{Dom}} = \boldsymbol{D} \cdot \boldsymbol{U_d}$

The imported and domestic intermediate consumption matrix is $\boldsymbol{B_{Tot}} = \boldsymbol{D} \cdot (\boldsymbol{U_d} + \boldsymbol{U_m})$

The imported intermediate consumption matrix is $\boldsymbol{B_M} = \boldsymbol{B_{Tot}} - \boldsymbol{B_{Dom}}$

The domestic final use matrix is $\boldsymbol{F_{Dom}} = \boldsymbol{D} \cdot \boldsymbol{Y_d}$

The imported and domestic final use matrix is $\boldsymbol{F_{Tot}} = \boldsymbol{D} \cdot (\boldsymbol{Y_d} + \boldsymbol{Y_m})$

The imported final use matrix is $\boldsymbol{Y_M} = \boldsymbol{Y_{Tot}} - \boldsymbol{Y_{Dom}}$