## The SUTB function

The SUTB function is intended for balancing Supply and Use Tables (SUTs) using an automated procedure that considers predefined constraints and given reliability values of the input unbalanced data. The function requires 5 input variables:

```
SUTB(SUTu, RT, RC, AM, AdC [optional]):
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SUTU: Unbalanced SUT
$R T$ : Supply and Use Constraints (Product balance by row)
$R C$ : Valuation Adjustment Constraints (Balance by column)
AM: Adjustability coefficients
AdC: Additional Constraints (Optional user-defined constraint)

## Unbalanced SUT - ( SUTu)

The SUTs are composed of the supply matrix and the use matrix. These tables are expected to hold the following accounting equality

$$
\begin{aligned}
& \qquad \text { (Total Supply) }=(\text { Total Use }) \\
& \text { (Trade and transport margins }+ \text { Net taxes on products }+ \text { Output }+ \text { Imports })= \\
& \text { (Intermediate Consumption }+ \text { Final Consumption Expenditure }+ \text { Capital Formation }+ \text { Exports })
\end{aligned}
$$

However, when compiling SUTs, the initial tables will not be balanced. Unbalanced tables are the first input for the procedure. The tables must balance. For instance:


For illustrative purposes the tables in the previous picture include columns for the totals (supply and use). These totals differ, thus the SUT is unbalanced. The procedure can be
performed with or without the column totals, the greyed last row at the bottom of the picture. It depends on the needs of the compiler. This note will assume that the balancing procedure exclude the total, i.e. these are to be generated from the balanced cell within the SUT.

The procedure is performed on the numbers in the tables, so the input for the formula must start in the upper left cell (with the value of 4) and end in the lower right cell (value of zero, where changes in inventories for domestic purchases by non-residents cross)

The "balancing" process requires the users to define the required accounting relationship (constraints) and the explicit "expected value" for each of the constraints. This procedure allows users to include 3 sets of constraints: $R T, R C$, and $A d C$. The latter is optional.

The inclusion of all constraints in this application are carried out in the form of matrix multiplication, with either implicit (RT and RC) or explicit restrictions (AdC).

## Supply and Use Constraints (Product balance by row) - (RT)

The most relevant constraints relate to the product flow. For instance, the secondary product's supply in the table $(73+109-5+2+1,967+55+284=2,484)$ must equal the secondary product's use $(4+34+892+99+408+569+361+3=2,369)$. The implicit nature of the constraint implies that the restriction can be assumed as "the sum of the supply minus the sum of the uses equals zero".

This implicit restriction reads as: the sum of all supply minus the sum of all uses. The explicit binding restriction to this implicit balance is that this difference must equal zero (so that the balance is reached). The RT matrix includes all the row restrictions at once, and the procedure reads each constraint independently.

RT

|  | Supply of products |  |  |  |  |  |  |  |  | Use of products |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Valuation layers |  |  | Output |  |  |  |  |  | Intermediate |  |  |  | FCE |  |  | CF |  |
|  |  |  |  |  |  |  |  |  |  | 흔 W 름 를 |  |  |  | 눈 | $\frac{\frac{n}{n}}{\frac{\omega}{2}}$ | $\begin{aligned} & \underset{\sim}{u} \\ & \hline \end{aligned}$ |  |  |
| Primary |  | 11 |  | 1 | 1 | 1 |  | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| Secondary |  | 1 |  |  |  |  | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| Terciary |  | , | 1 |  | 1 |  |  | , | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| CIF/FOB adjustment |  | 1 |  |  | 1 |  |  | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| Direct purch. abroad by res. |  | 1 |  |  | 1 |  |  | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| Domestic purch. by non-res. |  | 1 |  | 1 | 1 |  |  | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |  |

For simplicity reasons, the constraints (and reliability coefficients) are built using all the cell in question, even if they are zero-valued. For instance, the tertiary activity (column) does not produce primary products (row). But in the example, the constraint includes a 1 in that specific cell.

The procedure is built in such a way that the resulting values of the process will respect the initial zeros. That means that no matter what constraints are imposed on primary product produced by the tertiary activity, the balanced result for that cell will remain in zero.

As a result, the imposition of constraints in RT and RC can include or omit zero values. The examples used in this note express the constraint including the zero-valued cells.

## Valuation Adjustment Constraints (Balance by column) - (RC)

Another set of standard constraints corresponds to the valuation layers in the SUTs. These are adjustments usually carried out to avoid double counting and to account for some differences in the valuation of products in the supply and use. Two of the of these adjustments correspond to margins (trade and transport) and cost, insurance and freight/free on board (CIF/FOB). These values are already included in some transactions but must be reallocated without altering the total values.
In practice, these constraints imply that the product allocation must cancel out throughout the product basket (column). Margin columns require negative adjustments (e.g. margins in the tertiary products) to avoid double counting margins already included in the output matrix. All in all, these adjustments must cancel out ( $4+73-79=>0$ ). This implicit restriction reads as: the sum of margin columns and CIF/FOB adjustment must equal zero.

As in the case or RC, each of the restrictions (columns) are included independently from the others.

|  | RC |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Supply of products |  |  |  |  |  |  |  |  | Use of products |  |  |  |  |  |  |  |  |
|  | Valuation layers |  |  | Output |  |  |  |  | Intermediate Consumption |  |  |  |  | FCE |  |  | CF |  |
|  |  |  |  |  |  |  |  |  |  | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \stackrel{U}{u} \\ & \stackrel{\rightharpoonup}{0} \\ & \stackrel{\rightharpoonup}{2} \end{aligned}$ |  |  |  | 亗 | $\frac{\stackrel{N}{N}}{\frac{N}{n}}$ | $\begin{aligned} & \text { U } \\ & \text { U } \end{aligned}$ |  |  |
| Primary | 1 | - | - | - | - | - | 1 |  |  |  | - | - | - | - | - | - | - | - |
| Secondary | 1 | - | - | - | - | - | 1 |  |  |  | - | - | - | - | - | - | - | - |
| Terciary | 1 | - | - | - | - | - | 1 |  |  |  | - | - | - | - | - | - | - | - |
| CIF/FOB adjustment | 1 | - | - | - | - | - | 1 |  |  |  | - | - | - | - | - | - | - | - |
| Direct purch. abroad by res. | 1 | - | - | - | - | - | 1 |  |  |  | - | - | - | - | - | - | - | - |
| Domestic purch. by non-res. | 1 |  | - | - | - | - | 1 |  |  |  | - | - | - | - | - | - | - | $-$ |

## Adjustability coefficients - (AM)

Users can include information about the robustness of these initial estimates. This way, the procedure will consider this information so that more robust data are adjusted relatively less than weaker data. This "robustness coefficients" must be given to the formula, and apply directly on the cells that are allocated to.

The robustness of the input data should come from expert experience -gained during previous balancing exercises-, metrics of relative strength of data sources (samples errors, confidence intervals), and some predefined notion of the reliability of provisional accounts -e.g. proportional historical adjustments. This information must be standardized in a systematic way. The procedure is designed to work with values between 0 and 100. A high value represents high reliability and thus relatively small adjustments (no adjustment if $A M=100$ ). Alternatively, weaker data should have low AM values. These coefficients work in relative terms to other variables involved in the adjustment. If no information is available AM should be a constant for all cells.


In this example, imports and government final consumption expenditure will not be adjusted from their original values; while changes in inventories will be the variable that will be adjusted the most in relative terms.

## Additional Constraints - (AdC)

Compilers may be willing to (and probably will) include user-defined additional constraints. These constraints will require not only the explicit accounting relationship (as in the case of $R T$ and $R C$ ) but will also require the explicit restriction to this relationship. Unlike the previous set of constraints, $A d C$ must be included (and will be interpreted) as a single
constraint. ${ }^{1}$ The same matrix operation approach is used, but an additional piece of information is required.

For instance, certain totals may be binding but the distribution by product may not be binding. If total exports are given (say 520 ) and must be respected, such a restriction must be imposed. Note that the value in the example is $540(14+428+69+29)$. Consider also that the total value added for the primary activity is known to be 45 (initial estimate is $42=89$ 47).

By construction, all the individual constraints are to be stacked vertically. Each individual constraint will include a SUT-sized matrix with the algebraic constraint and an additional cell outside the top right corner of the constraint including the explicit resulting value of this constraint. For instance, to include the previous examples (exports and value added of the primary activity) in the balancing process, the AdC matrix will be:

| AdC |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Products | Supply of products |  |  |  |  |  | Use of products |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  | $\square$ | Intermediate |  |  |  |  | FCE |  |  | cF |  |  |
|  |  |  |  |  |  |  |  |  |  | $\begin{array}{\|l\|} \hline \stackrel{\rightharpoonup}{\Psi} \end{array}$ |  | $\frac{\stackrel{n}{n}}{\frac{\Delta}{2}}$ | 亗 |  |  |  |
| Primary | - - - |  |  |  |  |  |  |  |  |  | - |  |  |  |  |  |  | 520 |
| Secondary | - - - |  |  | - |  | - |  |  |  | - |  |  | - | - | - | - |  |
| Terciary | - - - | - | - | - |  | - |  |  |  | - |  |  | - | - | - | - |  |
| CIF/FOB adjustment | - - |  |  | - | - |  |  |  |  | - | 1 | - | - | - | - | - |  |
| Direct purch. abroad by res. | - - |  | - | - | - |  |  |  |  | - |  |  |  |  |  | - |  |
| Domestic purch. by non-res. | - - |  |  |  |  |  |  |  |  |  | 1 |  |  |  |  | - |  |
| Primary | - - | 1 |  | - |  |  | - |  |  |  |  |  |  |  |  |  | 45 |
| Secondary | $\cdots \cdot$ |  |  | - | - |  |  | 1 |  | - | - | - | - | - |  | - |  |
| Terciary | - - |  |  | - |  |  |  | 1 |  | - | - |  | - | - |  | - |  |
| CIF/FOB adjustment | - - |  |  | - | - |  |  | 1 |  | - | - | - | - | - |  | - |  |
| Direct purch. abroad by res. | - - |  | . | - | - | - |  | 1 | - | - | - | - | - | - |  | - |  |
| Domestic purch. by non-res. | - - |  | . |  | - |  |  | 1. |  |  |  |  | - |  |  | - |  |

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[^0]:    ${ }^{1}$ For RT and RC all the constraints (by row or by column) were included in a single matrix.

