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A scientific background on the iterative methods used in Annex 1, Appendix 2, section 1.2 to determine the value of the surface to be used in the determination of coefficient K in ATP isothermal tests

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Introduction

1. The Spanish proposal regarding the iterative calculation of the surfaces of the vehicles (ECE/TRANS/WP.11/2021/03 Amendment to Annex 1, Appendix 2 paragraph 1.2, Testing Method C and ECE/TRANS/WP.11/2021/04, Inclusion of an additional iterative method for tanks to Annex 1, Appendix 2, paragraph 1.2) has its technical support in the Annex attached to this informal document.

2. This Annex is an extract from the document that was presented to the CERTE meeting in April 2021 by the F2I2 Foundation.

3. The Annex contains the topological analysis of the iterative method and a critical review and proposals on the experimental methods used in the ATP Agreement to determine the value of the surface to be used in the determination of coefficient K in ATP isothermal tests for ATP units.

Annex

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A scientific background for the official proposals on the iterative methods to determine the value of the surface to be used in the determination of coefficient K in ATP isothermal tests

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Abstract

This document presents an analysis of the already approved iterative method established in Annex 1, Appendix 2 paragraph 1.2, Testing Method C of ATP for determining the mean surface to be used in the determination of the overall coefficient K, in order to find out its topological properties of convergence and figure out a way to improve the accuracy of the method. This way consists on improving the accuracy of the conductivity assumed in the application of the iterative method.
A second analysis is carried out to support the proposal to use the iterative method on tanks, in a reverse procedure, starting by the measurement of the outer surface.

1. Introduction and background

Heat Transfer is a well established science with a dozen of very good books teaching the same subject (with different flavors, but the same meal). Names as Chapman, Mills, Incropera, Stoecker, Kaviany, Bejan, Jaluria, and others, constitute a fundamental library of thermal engineering, although we could go back two Centuries in History, to find Fourier's "Theorie analyque de la chaleur" as the starting point of our subject. And it is not a minor piece in the bibliography of Science; on the contrary, it is one of the most appreciated books in Mathematical Physics, and it is also truly unique from the viewpoint of Philosophy of Science. It is a peculiar curiosity that he only cites the names of three scientists: Archimedes, Galileo and Newton, but he approaches the subject with a comprehensive view on Heat Transfer, including the Sun-Earth radiation balance. Note that in 1822, year of first edition of that book, nothing serious was known about the origin of the universe and the structure of matter, but Fourier was able to channelize his work through a cartesian organization of facts and coefficients, giving support to his Heat equations. Note also that Fourier is also considered as one of the creators of the concept of physical magnitude and the unquestionable requisite on every equation, of being physically homogeneous. For not to talk about his Transform, his harmonic analysis and series, and his capability to structure the subject in a fully explained cartesian way (a facsimile edition of this book can be found in Cambridge University Press).

In the ATP Agreement, the basic experiment is the so called "isothermal" test, to determine the actual value of the global heat transfer coefficient, K, defined by

$$k = \frac{Q}{S \cdot \Delta T}$$

Where Q is the total heat load rate (thermal power) dissipated in the air inside the box or tank devised for ATP functions, ΔT is the difference in temperature between inner and outer air, and S is the geometric mean surface between the internal and external surfaces.

A main problem in that equation is the difficulty of measuring both surfaces accurately, In particular, the outer surface of a panel van is generally not accessible, and one must rely on information given by the technical project or any other source (not always reliable). The recently approved iterative method opens a way for obtaining a value of the outer surface from the value of the inner surface (actually measured) and an estimate of the conductivity of the main insulator.

Section 2 of this document presents the topological analysis of this method. It is shown that its accuracy critically depends on the accuracy of the assumed conductivity value. Therefore, a preliminary effort should be done for improving said value.

A statistical study of the geometrical problem of boxes will be shown, with very interesting results about the characterization of the state of the art, which is analyzed on the basis of the relation between the size of the body and the ratio between the external over the internal surfaces. It is worth recalling that this ratio depends on the wall thickness, which in turn has a strong effect on coefficient k.

It will be shown that the approved “iterative method” devised to obtain a value of the representative S (surface) for calculating the K coefficient, conveys an error that depends on the difference between the actual effective conductivity of the case, and the assumed conductivity for making the iteration process, which currently is 0.025 W/mºC. A proposal is presented to improve the method, either with a direct better knowledge of the conductivity or using a sound statistical evidence of the wall thickness and its relation to the ratio between surfaces.

Section 3 is devoted to analyze the reverse iterative method, going from outside inwards, which will be particularly well suited to tanks, because measuring the inner surface conveys high risks for the technicians in charge of the test, and the working conditions inside the tank are so uncomfortable that the accuracy of the method is too low.

A new iterative procedure, Method D, is proposed for tanks. Its topological analysis also points out the need of having a good estimate of the conductivity of the wall for obtaining a good result with this method. Because of the difference between Methods C and D in the orientation of the process, the values of conductivity taken by default, are slightly different. It is also worth pointing out that both methods reach the exact result if the right value of the wall conductivity is used.

2. A topological analysis of the (already approved) iterative method to determine S to determine K coefficient in ATP tests.

The ATP agreement establishes its basic classification on the bases of the integral coefficient K, which is the heat transfer rate from inside to outside the box or tank, when both inner air and outer air are in some specified dynamic steady state conditions. The equation is

$$K = \frac{Q}{S \cdot \Delta T}$$

S stands for the geometric mean of the inner and outer surfaces. This value could be considered as an input data from the builder, but a test station must determine it by measuring the surfaces in whichever acceptable method. The ATP Agreement does not say much on it. However, according to the experience of many test teams, determining S is the largest source of uncertainty. (In ATP, W is used instead of Q, what is misleading, because W is also the unit, Watt)

Apparently, measuring distances is something that can be done with a high accuracy, but this is not so simple. For instance, measuring the diameter of a round table requires to know where the center of the circle is located. Otherwise, measurements will give a shorter distance, always, than the exact diameter. On the contrary. The distance between two parallel walls will always be measured longer than its actual value, because any deviation from perpendicularity will produce longer lines.

It goes without saying that dilatation effects can be disregarded within the usual limits of ATP transport, although we will come to this topic in relation to air leaks.
It is paradoxical that the major source of uncertainty in applying the ATP Treaty stems from simple geometry. There are several recipes to measure surfaces, particularly when the body is made basically with flat plates. It must be noted that corrugated surfaces and the like, are not accounted for in measuring the surface. In that case, it is only the flat virtual plate what is measured. However, the ATP Treaty does not embody any method to make those measurements. It only states that the total uncertainty of the full process must be lower than 5% (expanded uncertainty).
There was a pending problem in the geometric part of the test, and it is the difficulty for measuring the outer surface of a van, because it is simple to identify what part of the outer skin of the van is acting as an actual surface of the insulator.

2.1. On the iterative method to determine the K coefficient in ATP and a complementary method based on actual statistics.

This section firstly deals with the "iterative method" devised to obtain a value of the representative S (surface) for calculating the K coefficient in ATP tests, starting from internal measurements only. It will be shown that the method conveys an error that depends on the difference between the actual effective conductivity of the case, and the assumed conductivity for making the iteration (for which the method provides a value, stated in the Agreement, of 0.025 W/m·ºC).

The second part of the section is about actual statistics of many cases, looking for a correlation between variables that could be used to confirm if a given embodiment is according to the state of the art, or just escapes from the standard picture. The statistical study can improve the iteration method. Either both methods can be used as complementary, or the statistical method can be applied alone.

1st Part: A TOPOLOGICAL ANALYSIS OF THE ITERATIVE METHOD

The current version of ATP, issued on July 6th 2020, includes a method to determine the effective surface of an insulator, when there is not a clear way to measure one of the surfaces of the insulated container (usually the external one). The method is intended for vans, but it could be applied to any type of unit having such a problem. The method is included in Appendix 2 of Annex 1 of the Agreement, and it also is in the Handbook. It says:

> Method C. If neither of the above is acceptable to the experts, the internal surface shall be measured according to the figures and formulae in method B.
>
> The K value shall then be calculated based on the internal surface area, taking the insulation thickness as nil. From this K value, the average insulation thickness is calculated from the assumption that λ for the insulation has a value of 0.025 W/m·K.
>
> $$d = Si \times \Delta T \times \lambda / W$$
>
> Once the thickness of the insulation has been estimated, the external surface area is calculated and the mean surface area is determined. The final K value is derived from successive iteration.

This method requires some explanation, because it induces error in the value of the effective surface and therefore in the K value. Additionally, the first sentence of the second paragraph is misleading and unnecessary. Should the thickness of the insulator be taken as nil, its conductivity would have to be taken as zero as well, because the thermal resistance would be nil otherwise, and K would tend to infinity.
In fact, this pre-step is not needed in the iteration process. This one starts by the only equation included in the method, which can be reformulated simply for the iteration procedure as

$$d_n = \lambda \Delta T S_{n-1}/W$$

d_n= the thickness calculated in n-th step, using the estimate of the effective surface obtained in the previous step. In the first step, the measured internal surface Si must be used.

λ= is the conductivity. The method states that a value 0,025 W/m·ºC must de taken, because there is not any simple way to estimate it.

ΔT= is the difference in temperature between inside and outside the container, during the test, and must be 25ºC or close to it

W= is the measured electric power used in the test for heating the inner volume of the container, so that the former value ΔT is kept along the test.

The main problem of this method is the effect of the conductivity value on the result of the test, expressed by the value of K. Additionally, the iteration process is just a way to solve the problem of finding an accurate value of K, compatible with the insulator thickness, which in turn must be compatible with the effective surface. In the case of simple geometries for the container, as a cubic box or a sphere, there is an explicit algebraic method, which obviously gives the same values as the iteration process, which converges rapidly to the K value that solves the problem formerly explained. Of course, the thickness and the surface also converge to the corresponding values.

The main drawback of the method is that the converged value does depend on the assumed conductivity, λ. It will be shown that λ=0.025 produces overestimated K values in general. The best choice for each case is the actual value of λ, but this is not known. It will be seen in the analysis that values of K are overestimated with this method when the assumed λ is smaller than the actual λ. Values are underestimated when the assumed conductivity is higher than the current one, and it gives the exact value of K if the actual value is used as assumed value, but that value is not known.

In fact, the physical model used in the method does not reflects exactly the physics of the problem, because it considers that the thermal resistance of the heat transfer between inner air and outer air is only the conduction resistance through the wall. Nothing is said about the convection resistance of the inner and outer convection films. It is true that they are not dominant phenomena, but they are not negligible. In an ordinary case, the temperature difference between inner and outer air is 25 ºC, which are distributed as follows:

- 0.5 ºC for the inner film
- 23,5 ºC for conduction through the wall
- 1 ºC for the outer film

The thermal resistances are proportional to the jump in ΔT in each layer. Note that thermal resistance due to convection is around 6% of total, higher than the allowable total error, of 5% (Current version of the agreement does not use the word error, but "expanded uncertainty" which should include all types of inaccuracies).

Nevertheless, it is possible to assimilate the convection resistance into an effective conduction resistance with a conductivity λ, which is the approach followed by the method.

In the following, the iteration method will be analyzed for a simple case: a cubic box. The methodology could be applied to any box, of course. The same can be said about spheric and cylindrical tanks.

The cubic case can also be solved easily by algebraic relations. The results are the same in both methods (with some very small numerical discrepancies (10^{-6}) due to round-off errors),

It will be seen that assumed λ value has an influence on the K value. It will be illustrated with some examples, including one corresponding to the general case, with different values for length, width and height.

This analysis suggests that Method C should at least be completed with an additional prescription on λ, so that some additional reliable information could improve the accuracy.

The iteration process for a cubic box

Let "a" stand for the side of the cube, and therefore the internal surface is

$$S_i = 6a^2$$

Let "d" stand for the thickness of the insulator. S_e will be the external surface and S will be the geometric mean

$$S_e = 6(a+2d)^2 = S_i(1+\frac{2d}{a})^2$$

$$S = \sqrt{S_i S_e} = S_i\left(1+\frac{2d}{a}\right)$$

The iteration equation is

$$d_n = \lambda \Delta T S_{n-1}/W$$

which must be applied recurrently, as expressed here

$$d_1 = \lambda \Delta T S_i / W$$

Then

$$S_1 = S_i \left(1 + \frac{2d_1}{a}\right)$$

$$d_2 = \lambda \Delta T S_1 / W$$

$$S_2 = S_i \left(1 + \frac{2d_2}{a}\right)$$

$$d_3 = \lambda \Delta T S_2 / W$$

And so on. The iteration has a fast convergence, and it correspond to an asymptotic value of d_a and S_a. The K value of this method is obtained from

$$K = \frac{W}{S_a \, \Delta T}$$

In order to obtain the expression of the asymptotic values, we can call

$$m = \frac{d_1}{a}$$

It is straight forward to obtain the law of the iteration,

$$d_2 = (1 + 2m)d_1$$

$$d_3 = \big(1 + 2m(1 + 2m)\big)d_1 = (1 + 2m + 4m^2)d_1$$

It must be noted that the expression within brackets in the last equation is a geometric series with a reason 2m that is smaller than 1. So, the addition of the infinite terms tends to 1/(1-2m). So we end up with an asymptotic solution, namely

$$d_a = \frac{d_1}{1 - 2m}$$

$$S_a = S_i \left(1 + \frac{2d_1}{a - 2d_1}\right)$$

An example can be useful to see the convergence pace. Let us have a=2 meters, actual d=0.1 meters; actual λ=0.04 W/m·ºC. Let us assume a ΔT=25 ºC and 264 W of heating power.

Note that S_i=24 m^2 and S_e=29.04; so S=26.4 and the result is a value of K=0,4 W/m^{2}ºC. This value of K can also be computed as

$$K = \frac{\lambda}{d} = \frac{0.04}{0.1} = 0.4$$

The foregoing values are the reference for the calculation to be done after Method C.

In next table, the iteration process is shown, with the conductivity prescribed by said method (0,.025 W/m·ºC) not the actual one.

D1=	0,05681818
d2=	0,0600463
d3=	0,06022971
d4=	0,06024013
d5=	0,06024072
d6=	0,06024076
d7=	0,06024076

Last value has converged with an accuracy of 10^{-8}, and therefore can be considered the result of the method. The same value is obtained if the asymptotic expression is used. Such a thickness leads to S_e= 26.9786 and S=25.4457; and finally K=0.415. So, the result overestimates the actual value of K, which is 0.4.

What happens if the iteration (actually, the asymptotic) method is used with different assumed values of conductivity?. Next table gives the answer (Numerically for this case, but the trend is general):

W/m·ºC	d1	m	assymp. D	asymp. S	K (ATP)	appr K/real
0,02	0,04545455	0,02272727	0,04761905	25,1428571	0,42	1,05
0,025	0,05681818	0,02840909	0,06024096	25,4457831	0,415	1,0375

0,03	0,06818182	0,03409091	0,07317073	25,7560976	0,41	1,025
0,035	0,07954545	0,03977273	0,08641975	26,0740741	0,405	1,0125
0,04	0,09090909	0,04545455	0,1	26,4	0,4	1
0,045	0,10227273	0,05113636	0,11392405	26,7341772	0,395	0,9875
0,05	0,11363636	0,05681818	0,12820513	27,0769231	0,39	0,975
0,055	0,125	0,0625	0,14285714	27,4285714	0,385	0,9625
0,06	0,13636364	0,06818182	0,15789474	27,7894737	0,38	0,95

There is an evident relation between conductivity and K, which seems to be a paradox, because K decreases as conductivity increases. Of course, this is not a true physical effect; this is just produced by the iteration topology, which is very easy to describe, following the equations of the convergence towards the asymptotic value.

The definition of K applied to a cubic box is

$$K = \frac{W}{S\Delta T} = \frac{W}{S_i\left(1+\frac{2d_a}{a}\right)\Delta T}$$

Where

$$d_a = \frac{d_1}{1-2\frac{d_1}{a}}$$

We can call K_i to the ratio

$$K_i = \frac{W}{S_i\Delta T}$$

And the following equations lead to the final expression for the cubic box

$$K = \frac{K_i}{\left(1+\frac{2d_1}{a(1-2\frac{d_1}{a})}\right)} = \frac{K_i}{1+\frac{2m}{1-2m}} = (1-2\mathrm{m})K_i$$

Note also that

$$d_1 = \frac{\lambda}{K_i}$$

And we arrive at

$$K = K_i - 2\frac{\lambda}{a}$$

It is worth pointing out that K equals K_i when λ=0, as it should be (according to the definition of the method). K_i means that the effective surface of the box is S_i, and the thickness of the insulator in this case vanishes. However, the value of K is λ divided by the thickness, and K would be zero unless λ=0 (for having an indetermined value 0/0). This relation is depicted in the following graph (fig. 1)

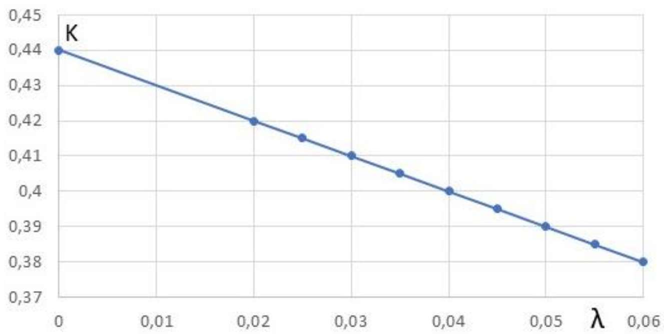

Figure 1. Dependence of K on the conductivity assumed in the iteration process

It is worth pointing out that the slope of the line of results is

$$\frac{d\text{K}}{d\lambda} = -\frac{2}{\text{a}}$$

which is larger for smaller sizes of the box. (It can be recalled that the method was specifically devised for vans, where the error is larger).

The most salient feature of the topology of the method is that it works well when the actual conductivity is used as assumed value. The problem, of course, is that the actual conductivity is not known. Nevertheless, some idea about its value is available since the very beginning of the project, and therefore, a good estimate could be obtained by refining that value. For instance, in the ATP Treaty it is said that λ=0.025 (in SI units) but this is too low a value for a practical insulator. Note that for this assumed value, K will be 1.0375 times the actual value. This implies an error close to 5%, which was the maximum allowable for determining K. In the current version, the error is not used as such in the definition of the ATP qualifications. Now, uncertainty is the parameter, with the same value of 5% (of expanded uncertainty) but the error demonstrated in this technical memorandum is not a question of uncertainties. It is an error introduced by the method itself.

Next graph shows the result for a worse insulator, with an actual K= 0,7 W/m^{2o}C, a thickness of 0,1 m, and a conductivity of 0,07 W/mºC. It is for the same cubic box (and it has therefore the same slope. This means that the result can be tricked by assuming a very high conductivity, which leads to smaller K.

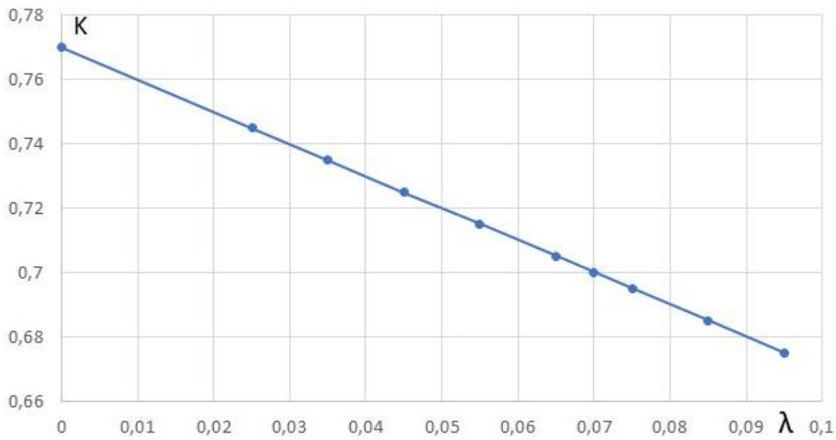

Figure 2. K vs conductivity in the case of a worse insulator

When the box is not cubic, the analytical work is much more cumbersome, because of using 3 independent variables to define the geometry of the box. For instance, a box can be studied numerically, with the following size for the inner surface

- Length a= 10 m
- Height b=2,5 m
- Wide c= 2,2 m

The inner surface is thus

$$S_i = 2(ab + ac + bc) = 105\ m^2$$

And the external surface

$$S_e = 2(a'b' + a'c' + b'c') = 117\ m^2$$

Where a'=10.2; b'=2.7 and c=2.4. The geometric mean to determine K is obviously
$S = \sqrt{S_i S_e}$= 110.8377 m^2.
Thickness follows in the iteration the same definition as in the cubic case.

$$S_{n-1} = \sqrt{S_i S_{e,n-1}}$$

$$d_n = \lambda \Delta T S_{n-1}/W$$

$$S_{e,n} = 2\left(a'_n b'_n + a'_n c'_, + b'_n c'_n\right)$$

With

$$a'_n = a + 2d_n$$
$$b'_n = b + 2d_n$$
$$c'_n = c + 2d_n$$

The result of the iteration method is shown in figure 3.

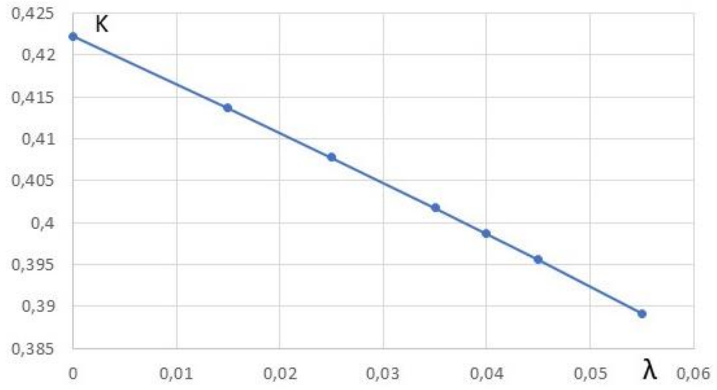

Figure 3. K vs conductivity for a long box

The topology of the equation is the same as before, all results in a straight line, but in this case the slope is not so steep, because the box is much larger. It is worth repeating that this is not a physical effect, but a relation produced by the method. In the cases on a cubic box, the slope was -1. In this case of a larger box, the slope decreases, what is good for the accuracy of the method, but it is an error anyway.

It is true that the error can be reduced if there is a good indication of the actual value of the conductivity. By the way, in the case of a larger box, round off errors have more importance than those of the cubic box. In the latter, K value was exact. On the contrary, in the larger box, K is 0.3987 instead 0,4 (corresponding to 0,03 %).

In summary, Method C should be used when the actual value of the conductivity could be assumed with some accuracy. The λ value recommended to be used by the ATP is too small and produces K estimates higher than the real one. Of course, there can be the temptation to use much higher λ values for the K results to be smaller. So, the problem with this method is its dependence on the starting λ value.

In next section, a statistical survey indicates that some improvements can be made by a better knowledge of the state of the art. Some complementary accuracy can be derived from the comparison between the ratio of surfaces found in the state of the art, and the ratio obtained with the iterative method. This double test can be of some guarantee for better acceptance of these calculations.

2nd part. A STATISTICAL STUDY OF THE SURFACES OF ATP CARRIAGES

A usual way to feature the state of the art is by means of statistics. It is obvious that there is a tendency, in any industrial sector, to improve things for getting customer, as well as a second tendency, which is producing cheaper. The result of the trade-off is that most of the makers in a given industrial sector follow the so called mainstream policy, and do items which are not very different from those of the major part of the producers. So, statistics can provide some complementary insight into the features of ATP builders. The main problem for such a presentation is to identify relevant variables with some inherent correlation between them. In our case, there is a good correlation between the size of the ATP container and the ratio between their outer and inner surfaces, and the most direct way to represent the size is its inner surface.

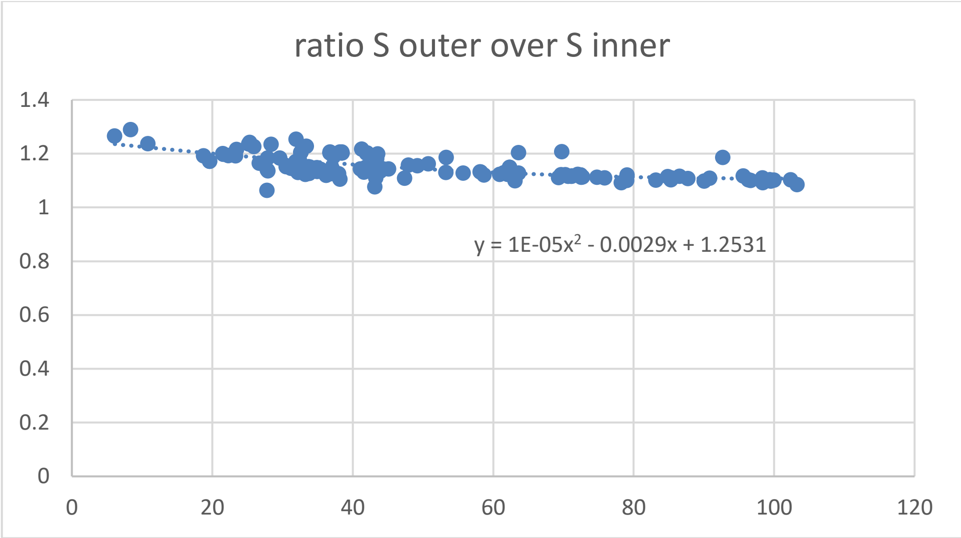

Figure 4. A display of a number of actual cases (sampling last years tests, although the year of construction is in general much older than the year of the test). There is a strong correlation between the ratio of surfaces (outer over inner ones) which is in ordinates, and the inner S, which is in abscissas.

Last figure shows a display of 140 cases sampled from the files of F2i2 Test Station in Getafe (Spain). It is remarkable that there are a large number of carriages very near the line of tendency. There are a few cases that move a little further, but a conclusion could be derived from this figure: the state of the art is well represented by the line of tendency shown in the plot. Of course, this fit can be improved with many more points, but the essential geometric properties of these pieces can be characterized by that equation. Therefore, if the outer surface is very difficult to measure, one can have a good estimate thanks to this statistical approach.

The average error of this set of cases, expressing the error as the difference between the actual measurement and the value given by the fit, is 0,009, so well below 5%. Only 9 cases present a deviation higher than 5% between measurement and estimate (6,5% of the cases). Only two of them are close to, but below 10%.

A potential improvement to this method is to make more precise fits in shorter intervals of the inner surface value. Of course, other samplings from other Test Stations would increase quite a lot the guarantee of this statistical approach, which must be used to help define the characteristics of a given ATP container (of any kind).

In order to use both methods as complementary ones, we need to go back to the equations of the 1st part, particularly in the case of a cubic box, because vans are not far from that geometry. It holds

$$S_a = S_i\left(1 + \frac{2d_1}{a - 2d_1}\right)$$

S_a stands for the asymptotic value of the geometric average surface, and S_i is the inner surface, which has been measured. They combine as follows to find outer S_o:

$$S_o = S_i\left(\frac{1}{1 - 2m}\right)^2$$

Where $m=d_1/a$.

An example can be useful to guide the complementary pace. Let us have a=2 meters, actual d=0.1 meters; actual λ=0.04 W/m·ºC. Let us assume a ΔT=25 ºC and 264 W of heating power. This example was already considered in the first part of this section.

Note that S_i=24 m^2 and we use our correlation, the surface ratio is 1.2083, and therefore S_e=29.04; so S=26.4 and the result is a value of K=0,4 W/m^{2}ºC. (This value of K can also be computed as

$$K = \frac{\lambda}{d} = \frac{0.04}{0.1} = 0.4$$

The foregoing values are the reference for calculating the surface ratio and K at the same time. Next figure shows both variables, versus the effective conductivity.

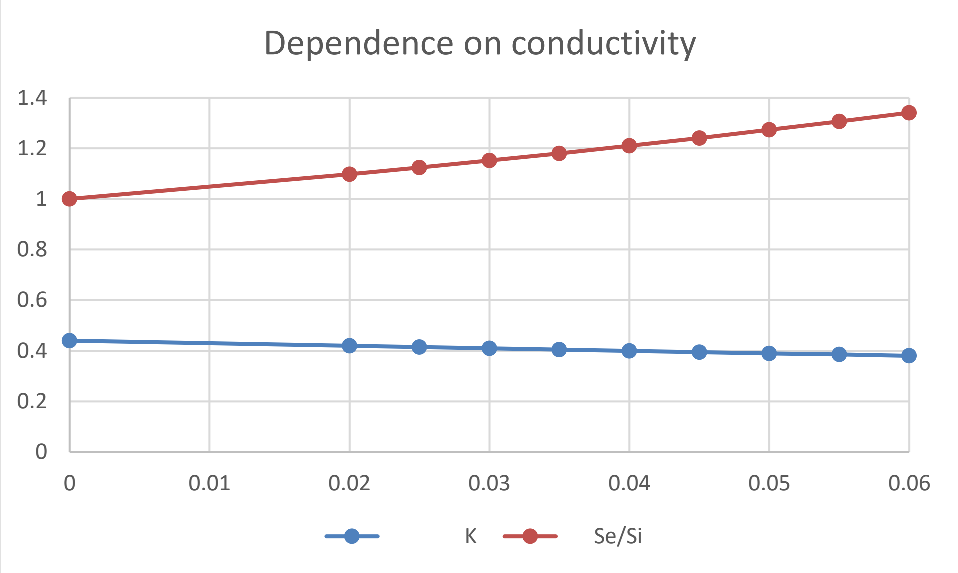

Figure 5. Dependence of K and surface ratio on the value of effective conductivity. They go with opposite slopes. K value goes down, and So/Si goes up. If we look up to the statistical figure previously described, it can be seen that most of the cases with Si around 24m^2 have a ratio around 1.20. If this value is transported onto last figure, it can be seen that K value is 0,4 W/m^{2o}C, which is the theoretical value of this virtual example.

On the contrary, if λ=0.025 is assumed, the corresponding surface ratio is 1.124 which is too low a value for a van with an inner surface of 24 m^2. According to the statistics of the state of the art, **the results of the iterative method are much better if they are guided by the statistic study**.

The methodology can be extended to boxes of any size and tanks, where it is to some extent easy to make measurements of the external surface, proceeding afterwards inwards, for determine K as the best estimate.

Measuring accurately both surfaces is the best way for having the most accurate K value, but this is a real problem for tanks, because some occupational safety regulations in some countries forbid persons to go into tanks. So, the compound method (iterative + statistical) can be used for tanks starting from the outer surface inwards, i.e., in the opposite direction to the method applied to vans.

3. The geometric problem for tanks

Measuring the inner surface of a tank is not an easy task, and it is even forbidden in some conditions, because of occupational safety regulations. It is worth noting that the iterative procedure can be applied to tanks in the opposite way to vans, from the outer surface inwards. And this application seems still more adequate, because errors are smaller for larger bodies.

Although we will use another geometry for cylindrical tanks, we can first study the cubic case. The external surface will be

$$S_e = 6b^2$$

The iteration starts from

$$d_1 = \lambda \Delta T S_e / W$$

From which we have the first value of internal surface

$$S_{i1} = 6(b - 2d_1)^2$$

The relevant surface S_1 (corresponding to the geometric mean) is then:

$$S_1 = S_e \left(1 - \frac{2d_1}{b}\right)$$

$$d_2 = \lambda \Delta T S_1 / W$$

and next step is:

$$S_2 = S_e \left(1 - \frac{2d_2}{b}\right)$$

$$d_3 = \lambda \Delta T S_2 / W$$

And so on. The iteration shows again a fast convergence, and it corresponds to an asymptotic value of d_a and S_a. The K value of this method is obtained from

$$K = \frac{W}{S_a \, \Delta T}$$

In order to obtain the expression of the asymptotic values, we can call

$$n = \frac{d_1}{b}$$

It is again straight forward to obtain the new law of the iteration,

$$d_2 = (1 - 2n)d_1$$
$$d_3 = \big(1 - 2n(1 - 2n)\big)d_1 = (1 - 2n + 4n^2)d_1$$

We could then write

$$S_{i3} = S_e(1 - 2n(1 - 2n + 4n^2))^2$$
$$S_3 = S_e(1 - 2n + 4n^2 - 8n^3)$$

It must be noted that the expression within brackets in last equation is a geometric series with a reason -2n that is smaller than 1. So, the addition of the infinite terms tends to 1/(1+2n). So

$$d_a = \frac{d_1}{1 + 2n}$$
$$S_a = S_e\left(\frac{1}{1 + 2n}\right)$$

And the converged K value (K_a) will be

$$K_a = \frac{Q}{S_a \Delta T} = \frac{\lambda}{d_a} = \frac{\lambda}{d_1}(1 + 2n) = K_e + 2\frac{\lambda}{b}$$

It obviously shows a dependence with λ which is symmetric to the former case for vans.

It is worth noting the dependence of the error of the iteration on the assumed conductivity λ. Let us call K_0 the actual value, which is unknown in reality, but we can define it and manage it in our analysis. It holds

$$K_0 = \frac{Q}{S_0 \Delta T}$$

We first need a geometric specification of the actual case, which can go as follows:

$$S_e = 6b^2$$
$$S_{i0} = 6(b - 2d_0)^2$$
$$S_0 = S_e\left(1 - 2\frac{d_0}{b}\right) = S_e(1 - 2n_0)$$

And we will include it in the general law for the converged K

$$K_{a0} = K_e + 2\frac{\lambda_0}{b} = \frac{Q}{S_e \Delta T} + 2\frac{K_o d_0}{b}$$

Where we have used the relation

$$K_0 = \frac{\lambda_0}{d_0}$$

And finally

$$K_{a0} = \frac{Q(1 - 2n_0)}{S_0 \Delta T} + 2K_0 n_0 = K_0 - 2K_0 n_0 + 2K_0 n_0 = K_0$$

For cubic box is easy to demonstrate that the asymptotic value of the iteration process coincides with the actual value of K if we used as assumed conductivity the real. If the assumed conductivity is lower, the converged K is also lower than the actual K; and it is higher if the assumed conductivity is higher.

Although the simplest tank (a cubic box) has been useful to understand the qualitative topology of the iterative process (going inwards) it is much better to study an example that could actually represent commercial tanks. We are going to consider a cylindrical body 10 m long and 1 m radius, of external dimensions, So, the outer surface will be 69.08 m^2. The thickness of the walls is taken 0.1 m with a conductivity of 0.04 W/m·ºC. Using the thin wall approximation, we obtain a K value of 0.4 $W/m^{2o}C$ (=0.04/0.1). The internal surface 60.5 m^2, and therefore the ratio between surfaces is 1.1416.

We apply the former description of the iteration for tanks, starting by choosing a value of conductivity, and we find the following set of calculations for a number of assumptions on the conductivity value.

conductivity	d1	S int1	S1	d2	Sint2	S2	d3

0,02	0,05342647	64,4693551	66,7348713	0,05161275	64,6253078	66,8155391	0,05167514
0,025	0,06678309	63,3247077	66,1397823	0,06394063	63,5677397	66,2665787	0,06406321
0,03	0,08013971	62,1867858	65,5428346	0,07603624	62,5356663	65,7264317	0,07624923
0,035	0,09349633	61,0555893	64,9439767	0,08789843	61,5288671	65,1952003	0,08823844
0,04	**0,10685294**	**59,9311184**	**64,3431555**	**0,09952599**	**60,5471292**	**64,6729904**	**0,10003618**
0,045	0,12020956	58,813373	63,7403154	0,11091771	59,5902475	64,1599119	0,11164787
0,05	0,13356618	57,7023531	63,1353986	0,12207229	58,6580252	63,6560789	0,12307903

It is seen that the method converges in two steps, plus the starting point. Next table shows that the error is relatively small, and it actually vanishes for the cases with an assumed conductivity close to the real one.

conductivity	K	ratio S	K/0,4-1
0,025	0,39023955	1,08671475	0,02440113
0,03	0,39344658	1,10464962	0,01638355
0,035	0,39665251	1,12272504	0,00836873
0,04	**0,39985533**	**1,1409294**	**0,00036168**
0,045	0,40305292	1,15925009	0,00763229
0,05	0,40624305	1,17767347	0,01560763

As the actual conductivity is not known, it would be wise to complement this method with any other giving valuable information. That additional method can rely on the conductivity value as such, or in some statistical study of the state of art. Figure 6 shows the distribution of the ratio between the external and the internal surfaces, for more than 200 cases of tanks analyzed in our test station.

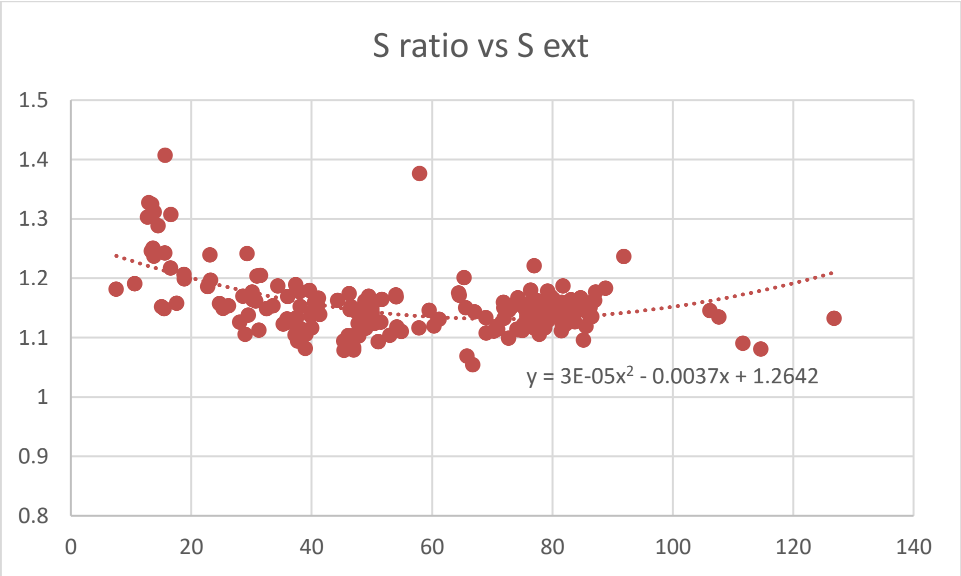

Figure 6. Statistical distribution of the ratio between surfaces as a function of the external surface

It is worth adding that the average value of the ratio between external and internal surfaces is 1.1466 and the standard deviation is 0.05. This means that 95% of the cases, in round numbers, will have a ratio between 1.0466 and 1.2466. We can also recall that the case-example formerly studied presented a ratio of 1,1409, so very close to the mean value.
From our statistical experience, the use of the assumption of a conductivity value compatible with this average value, can be qualified as best estimate. Nevertheless, the estimate can be improved if the statistical study is restricted to a domain of cases with clear analogies, for instance, depending on the size. Figure 7 shows the statistical distribution of what can be called main family of tanks (avoiding those of small sizes or very peculiar geometries).

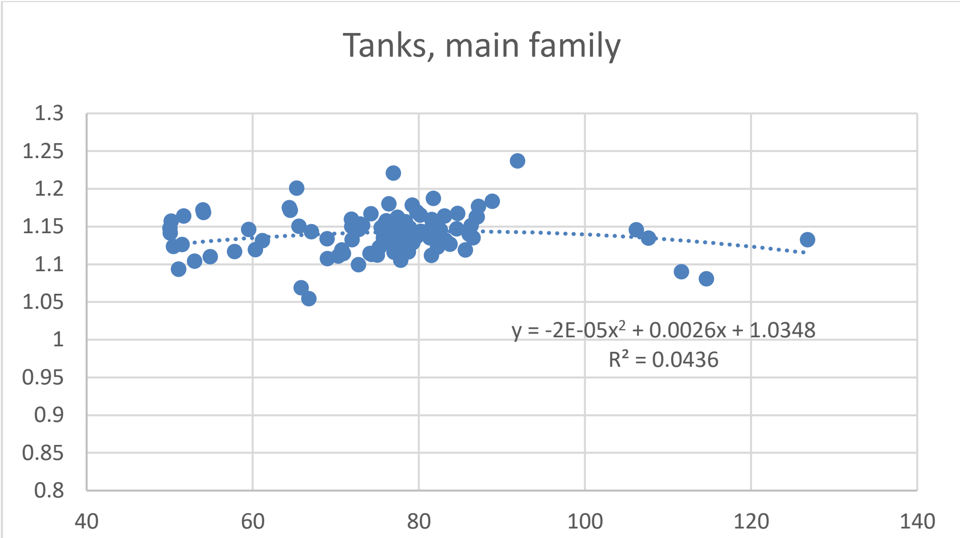

Figure 7. Surface ratio Se/Si for the main family of tanks, versus external surfaces.

The average ratio is 1.14 and the standard deviation of the sample is 0.0135. This means that 95% of the cases will be between 1.113 and 1.167. This corresponds to an expanded uncertainty of 2,3%. So, if the outer surface can be measured with high accuracy, of the order or better than 1%, the iterative method could be applied, for a number of selected assumed conductivities, and after that the best estimate would be the one giving a surface ratio of 1.14. (For simple geometries, it could be done looking for the asymptotic expression, but in the general case, there is no better than making a few iteration cases, generating a map of results where the conductivity corresponding to a surface ratio of 1.14 would be the correct result).

General application of the "inwards" iterative method

It is worth underlining that the iterative method is a self-coherent way to determine both the internal surface of the tank (or box, or container) and the K value. To apply it, a test must be made with previous knowledge of S_e. The test gives the value of the heating power W and the difference of temperatures between the inner air and outer air, ΔT. An initial coefficient K_e is given by

$$K_e = \frac{W}{S_e \Delta T}$$

The iteration stars with the calculation of a thickness d_1 given by

$$d_1 = \lambda \Delta T S_e / W$$

A value of λ= 0.035 W/mºC is recommended for this method, unless the applicant has an accurate estimate of the conductivity of the insulator, either by physical measurement or by statistical studies of similar equipment.

Thickness d_1 is applied to the external surface, in order to define the internal surface, S_{i1} and this application will depend of the shape of the tank. Therefore, this method can only be applied if the person responsible for the test knows how to manage that geometric situation. The next step is to calculate S_1 as

$$S_1 = \sqrt[2]{S_{i1} S_e}$$

This new surface value gives a new estimate of K, which is

$$K_1 = \frac{W}{S_1 \Delta T}$$

And a new thickness d_2 is calculated by

$$d_2 = \lambda \Delta T S_1 / W$$

And the iteration continues until convergence on K and *d*..

As an example, in the case of a cylindrical tank, with an external radius R and length L

$$S_e = 2\pi R^2 + 2\pi R L$$

And

$$S_{i1} = 2\pi (R - d_1)^2 + 2\pi (R - d_1)(L - 2d_1)$$

And so forth.

In this method, a value λ=0.035 W/m·ºC has been selected, unless a better estimate could be obtained for a specific case. A justification of this value is given in the following.

Firstly, remember that for a thin wall (as anyone found in ATP tanks) the K value can be approximated by the conductivity divided by the thickness of the wall, but it must also be recalled that in an ATP test the total thermal resistance has three components (as explained in page 11). So, if the approximate expression is given for the main insulator of the tank, the K value would be

$$\frac{K}{j} = \frac{\lambda}{d}$$

Parameter *j* stands for the fraction of the thermal resistance that is contributed by the insulator, which is around 23,5/25 =0,94 because it is the ratio of the temperature jump between both faces of the insulator, inner and outer, divided by the total jump between inner and outer air.

Note that *d* stands for the thickness of the wall, which in turn is related to the ratio expressed in figure 9. In the corresponding statistical analysis, it was found that the ratio S_e/S_i must be around 1.14, which is the best value representing the state of the art. This value can be used for finding the λ value to be used in last equation.

A cylindrical tank will be used for this purpose. Of course, any type of geometry can be treated in the same way, but a cylinder leads to very manageable equations. The starting equations are

$$S_e = 2\pi R^2 + 2\pi RL = 2\pi R(R + L)$$

and

$$S_i = 2\pi(R-d)^2 + 2\pi(R-d)(L-2d) = 2\pi(R-d)\big((R-d) + (L-2d)\big)$$

An additional relation is needed between R and L, that can be identified from the statistical distribution. In fact, it does not have a strong influence on the result. In an real case, this relation will be known. In this study, the assumption is L=8R. So we can write

$$\frac{S_i}{S_e} = \frac{1}{1.14} = 0.877 = \frac{(R-d)}{R}\frac{(R-d) + (8R-2d)}{R(R+8R)}$$

If we use the interim variable x as

$$x = \frac{R-d}{R}$$

It can be rewritten

$$0.877 = x^2 + \frac{6}{9}x(1-x)$$

Which can be reorganized as follows

$$\frac{1}{3}x^2 + \frac{2}{3}x - 0.877 = 0$$

and it only has a positive root, x=0.9055, and therefore d=R(1-0.9055)= 0.0945R

So, we arrive at the equation

$$\frac{K}{0.94} = \frac{\lambda}{0{,}0945R}$$

As R will be in a short range around R=1m, we find λ=K/10 approximately. For a wider tank, with R= 1,2, it holds λ= K/8,3 (but for a wider tank, relation between L and R can change).

Note also that many tanks have an elliptical cross section, and the mean radius is smaller than the maximum width. Therefore, R values (equivalent) of 0.8 are also found, and λ= K/12.5

In summary, as good tanks have a K in the range 0.35 through 0.38, a value of 0.035 W/mºC has been chosen for λ as assumed parameter, if a better estimate is not available.

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