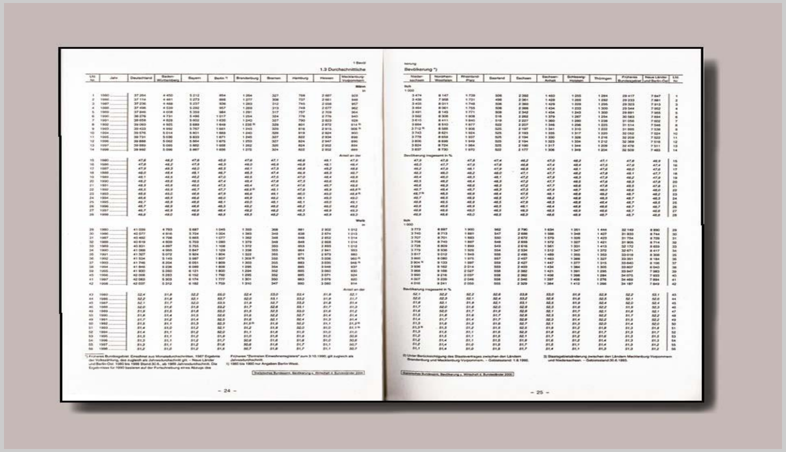
Testing CTA as Additivity Module



for Perturbed Census 2021 EU Hypercube Data

Tobias ENDERLE, Sarah GIESSING
Federal Statistical Office of Germany
Division Mathematical Statistical Methods

Background

EU-Project „Harmonized Protection of Census Data in the ESS“

- **recommended to test / use as SDC method for EU Census hypercubes and Grid Data**
  - **Targeted Record Swapping and/or**
  - **Random noise based on the Cell key method (Fraser and Wooton, ABS, 2005)**

Background

Properties of Cell key method noise:

- Statistical properties determined by noise design
  - Noise Variance V (constant, identical for all counts)
  - Noise Mean = 0 for each count („unbiased“)
  - Fixed Maximum Perturbation D
  - No small counts (1‘s, 2‘s) in noisy data
- Noise exactly consistent between hypercubes / grids
- Noise „drawn“ independently, based on Cell Keys

Background

Hypercube Data after CKM noise not additive

- **Viz.: Males + Females ≠ Total**

Naive approach

- **(hierarchical) aggregation of „interior“ cells**

Problem

- **Naive approach damages design properties of the noise:**
  - **Increase of maximum perturbation (like: from 3 to > 300)**
  - **Increase of noise variance (like: from 0,5 to 130)**
- **Large (absolute) inconsistencies between hypercubes**

„Calibrating“ noise might do better?

- **Iterative Methods (f.i. Iterative Proportional Fitting), or**
- **Optimization based method**
  - **Controlled Tabular Adjustment (CTA)**

Controlled Tabular Adjustment (CTA, Castro et al.)

Find vector of adjusted counts x_i subject to sets of constraints:

- (1) „Adjusted hypercube is additive“
- (2) Box constraints on deviations between „original“ (y_i) and adjusted (x_i) entries of the vector of counts
- (3) Protection levels for „sensitive“ cells ($i \in P$)
  - Upper protecion $x_i \geq y_i + upl_i$ **or**
  - Lower protection $x_i \leq y_i - lpl_i$

CTA Optimization problem in term of deviations $z_i := x_i - y_i$:

$$\min_z \sum_{i=1}^{n} w_i \, | z_i |$$

$$s.t. \quad (1)\ Az = 0$$

$$(2)\ l_i \leq z_i \leq u_i \quad , i = 1, \ldots, n$$

$$(3)\ z_i \leq lpl_i \text{ or } z_i \geq upl_i, \ i \in P$$

Scenario I: Test CTA to restore additivity to EU-Census hypercube 9.2 after perturbation by CKM noise

EU-Census hypercube 9.2 (one of ca. 150 EU-Census hypercubes)

- 4-dimensional, hierarchical hypercube presenting frequency counts by
  - geo_m 42 categories („synthetic" part of NW Germany)
  - sex 2 categories
  - age_m 21 categories
  - yae_h 24 categories
- Number of cells: 133,560
- Number of „additivity constraints": 129,822

Test hypercube 9.2: GEO.M x AGE.M x YAE.H x SEX

GEO.M

Format Ansicht ?

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AGE.M

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YAE.H

Format Ansicht ?

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SEX

Format Ansicht ?

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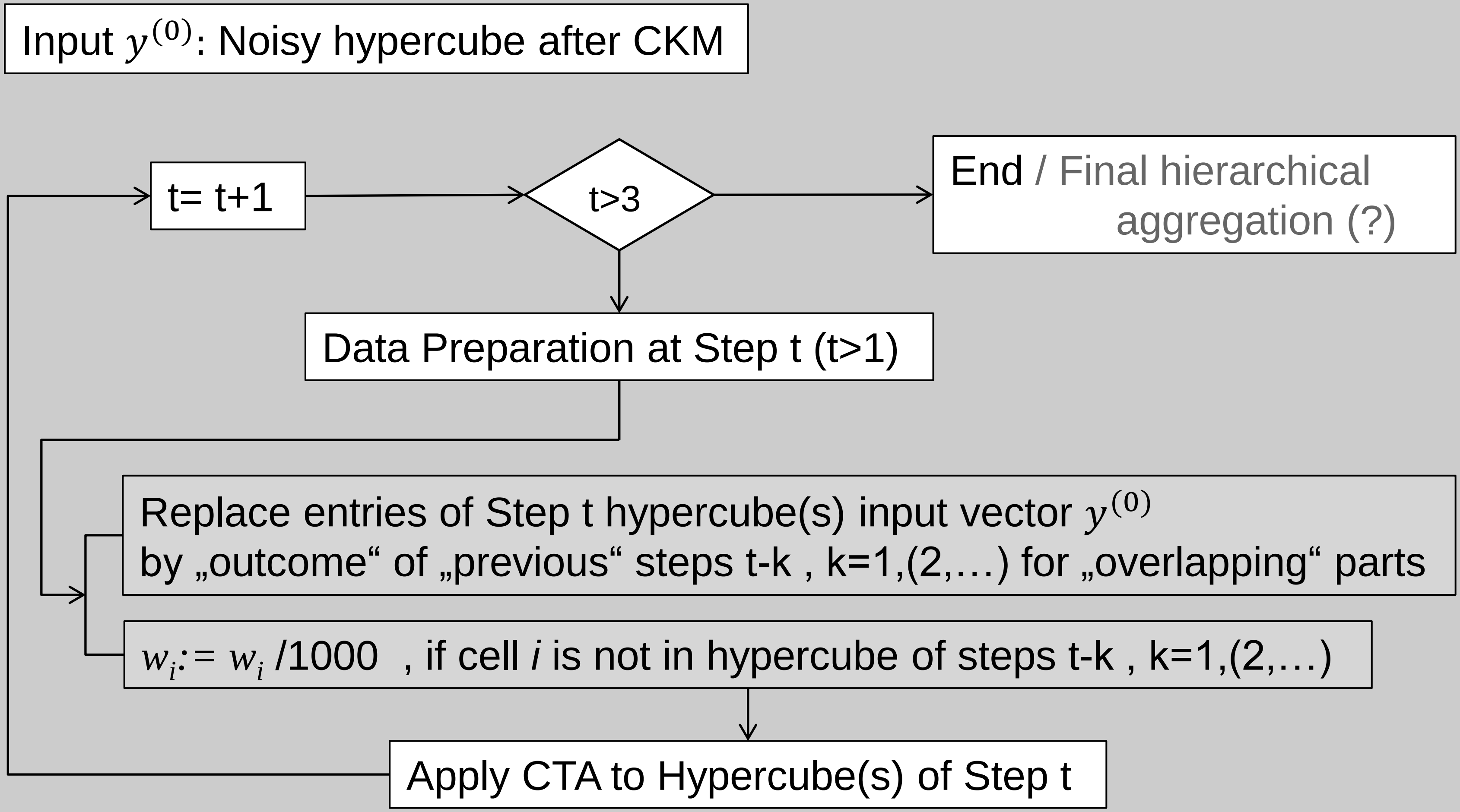
Settings of scenario I

- Cell key method noise
  - D=3 and V=1

- No sensitive cells ($P = \emptyset$)
- Weights: $w_i = (y_i)^{-\frac{1}{2}}$
- Box constraints: $l_i = -10$ und $u_i = +10$
- Commercial LP/MILP Solver

- Two approaches:
  - Solve in a single step
    - Not successfull (?)
    - Execution stopped (manually) after 1 week
  - Stepwise procedure
    - Blocking Strategy

Stepwise blocking strategy – 3 steps



Blocking strategy – Step 1

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YAE.H

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Step 1
„CTA for hypercube of 3-way margins“

Blocking strategy – Step 1

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Format Ansicht ?

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Step 1
„CTA for hypercube of 3-way margins“

X Skip variable

Blocking strategy – Step 2

GEO.M

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SEX

Format Ansicht ?

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Step 2
„CTA for hypercube of 4-way GEO.L margins“

Blocking strategy – Step 2

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SEX

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Step 2
„CTA for hypercube of 4-way GEO.L margins“

- GEO.L margins

Blocking strategy – Step 3

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Step 3
„CTA for six 4-way hypercubes“

Blocking strategy – Step 3

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Format Ansicht ?

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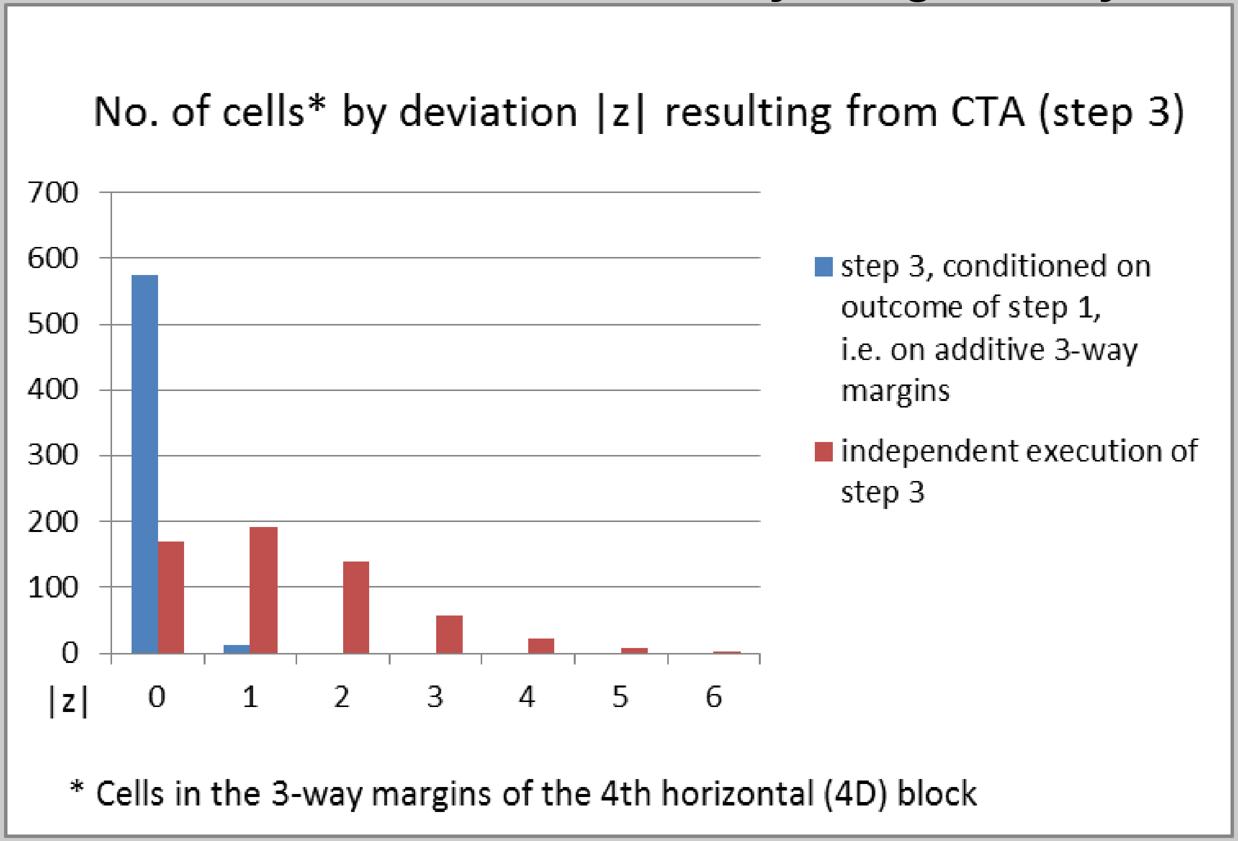
Step 3
„CTA for six
4-way hypercubes“

6 separate GEO blocks

Results for blocking approch in test scenario I

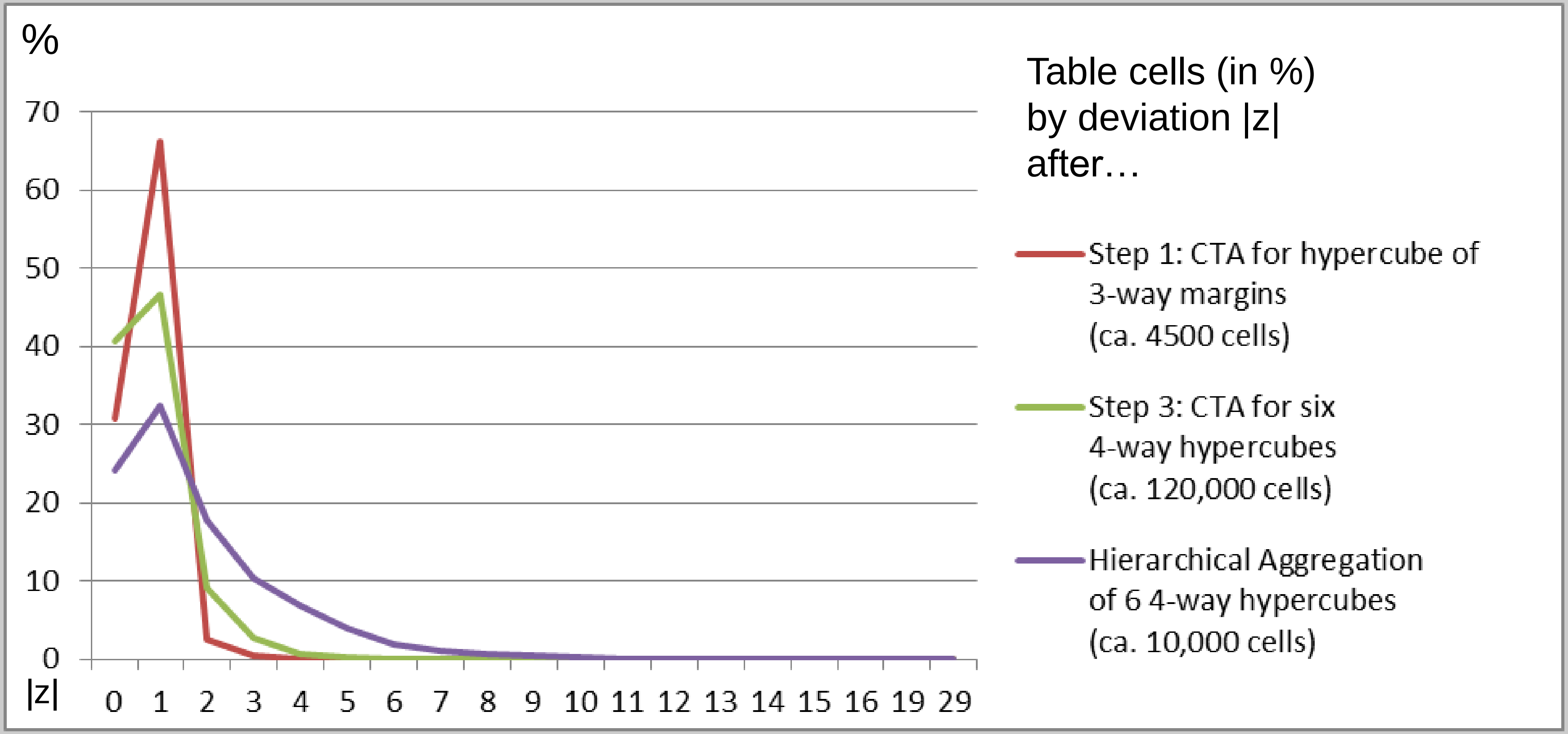
Step 1 (CTA on 3-way margin hypercube) preserves between-hypercube consistency very well

- **Example: CTA deviations after step 3, for block 4**
  - **Results for 3-way margins only**



Results for blocking approch in test scenario I

- „Smaller hypercube“ (Step 1) => „Smaller perturbation“
- Changes after CTA (Step 3) much smaller than after (final) hierarchical aggregation
  - Including Step 2 (CTA of Geo.L 4-way hypercube, not yet tried) might help



Scenario II - complex case: regard ones and twoes as sensitive cells

Find vector of adjusted counts x_i subject to sets of constraints:

- (1) „Adjusted hypercube is additive“
- (2) Box constraints on deviations between „original“ (y_i) and adjusted (x_i) entries of the vector of counts
- **(3) Protection levels for „sensitive“ cells ($i \in P$)**
  - **Upper protecion** $x_i \geq y_i + upl_i$ **or**
  - **Lower protection** $x_i \leq y_i - lpl_i$

CTA Optimization problem in term of deviations $z_i := x_i - y_i$:

$$\min_z \sum_{i=1}^{n} w_i \, | z_i |$$

$$s.t. \quad Az = 0$$

$$l_i \leq z_i \leq u_i \quad , i = 1, \ldots, n$$

$$z_i \leq -lpl_i \text{ or } z_i \geq upl_i, \quad i \in P$$

Scenario II: Restore additivity and don‘t allow for sensitive cells

EU-Census sub-hypercube 9.2

- 4-dimensional, hierarchical hypercube presenting frequency counts by
  - geo_m 42 categories („synthetic“ part of NW Germany)
  - sex 2 categories
  - age_m 21 categories
  - yae **3** categories
- Number of cells: **16,800**
- Number of „additivity constraints“: **16,352**
- Number of bottom level cells: 5,292

Iterative strategy for the complex case

Feasible solution: entries need to be zero or above a given threshold N (i.e. N=2)

Iterative strategy consists of three phases:

- Initial Phase
  - Starting with a suitable non-additive random noise solution
  - Run CTA instance as applied in the „simple case“ (scenario I)
- Second Phase (iteratively)
  - Define sensitive cells $P = \{i_1, i_2, \dots, i_p\}$ and protection levels
  - Run CTA instance using protection levels, modified boundaries (especially for non-sensitive bottom level cells) and weights
- Final Phase
  - Stop second phase if …
    - … feasible solution is available OR
    - … no further reduction of sensistive cells is possible
  - In case of remaining sensitive cells:
    deterministic rounding and/or aggegation of bottom level cells

Results of the iterative strategy

Cells	Before CTA	Initial Phase	Second Phase – Iteration …						Final Phase*
			1	2	3	4	5	5*	
0	1,474	1,488	1,552	1,564	1,570	1,570	1,570	1,075	1,076
1	0	61	16	10	4	5	4	1	0
2	0	106	31	14	6	2	2	1	0
> 2	15,326	15,145	15,201	15,212	15,220	15,223	15,224	4,215	4,216
All	16,800	16,800	16,800	16,800	16,800	16,800	16,800	5,292	5,292
CPU time		56 min	7 min	6 s	6 s	3 s	3 s		

* bottom level cells

Initial/Second Phase:

- Modified lower boundaries for non-sensitive cells: $\min\left(10; \max(y_i - 3; 0)\right)$
  - in order to prevent these cells to be changed to sensitive cells
  - ”Optimal CTA table found (optimal within tolerances)”

Results of the iterative strategy

Cells	Before CTA	Initial Phase	Second Phase – Iteration …						Final Phase*
			1	2	3	4	5	5*	
0	1,474	1,488	1,552	1,564	1,570	1,570	1,570	1,075	1,076
1	0	61	16	10	4	5	4	1	0
2	0	106	31	14	6	2	2	1	0
> 2	15,326	15,145	15,201	15,212	15,220	15,223	15,224	4,215	4,216
All	16,800	16,800	16,800	16,800	16,800	16,800	16,800	5,292	5,292
CPU time		56 min	7 min	6 s	6 s	3 s	3 s		

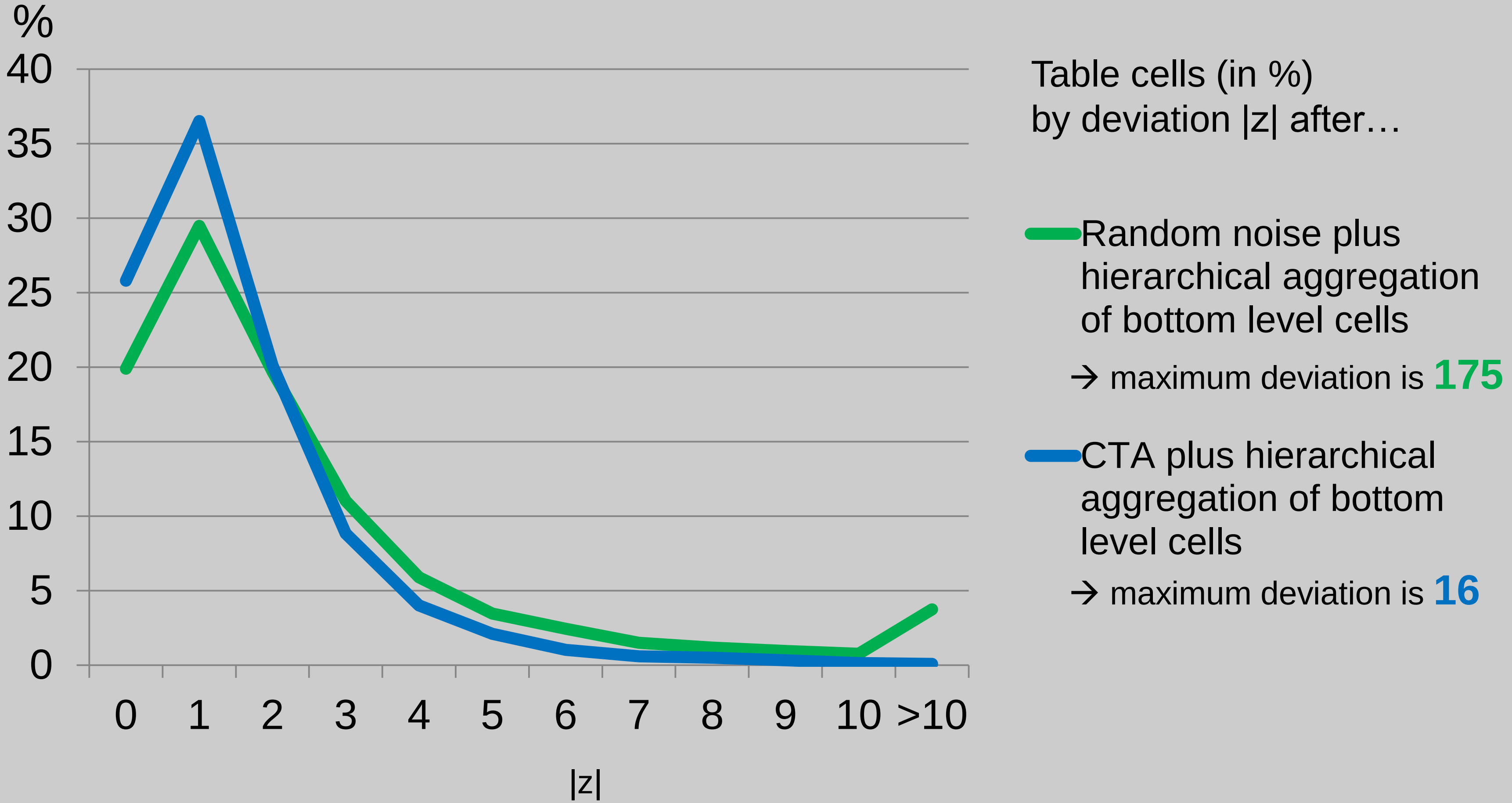
reduction of sensitive cells

* bottom level cells

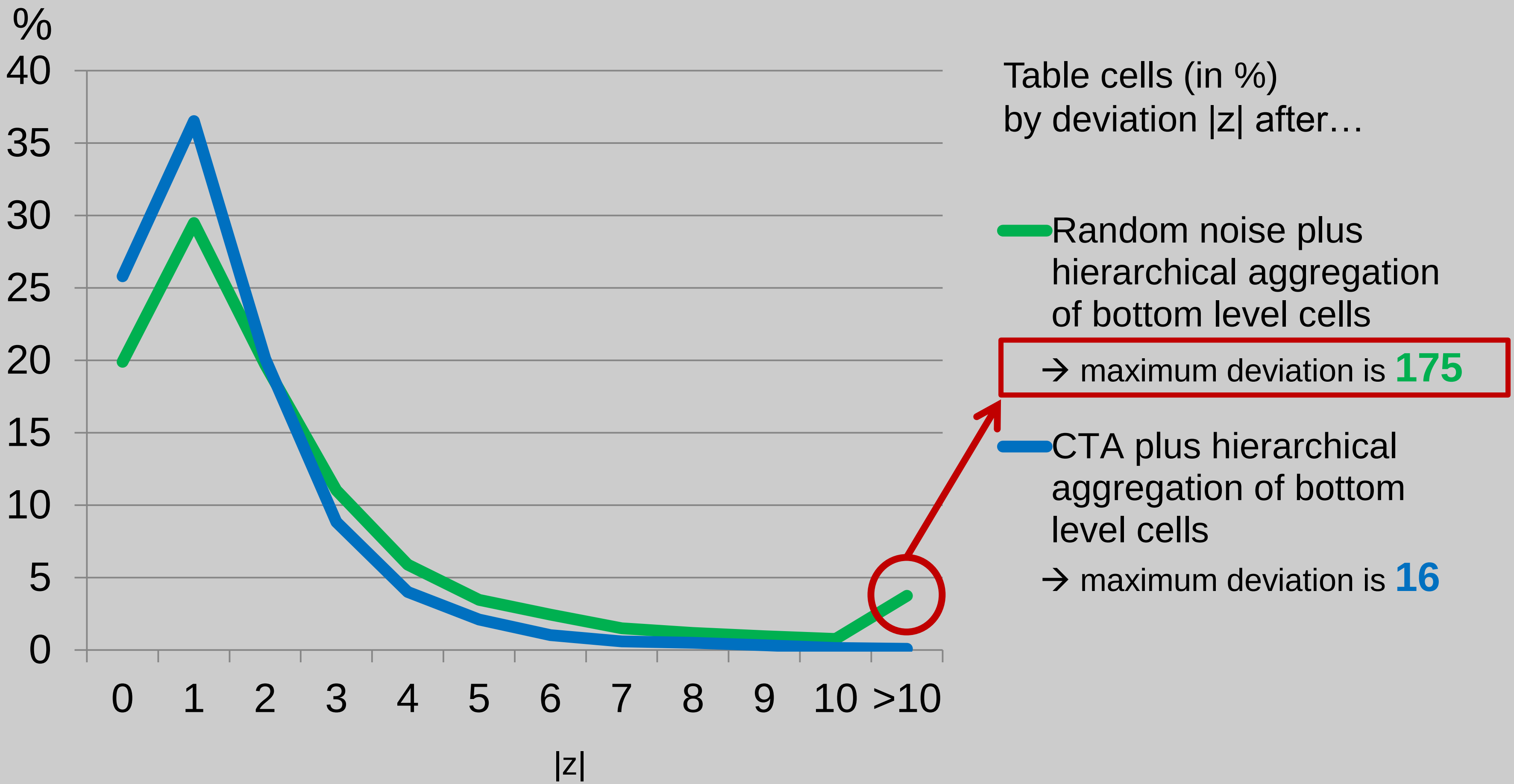
Initial/Second Phase:

- Modified lower boundaries for non-sensitive cells: $\min\left(10; \max(y_i - 3; 0)\right)$
  - in order to prevent these cells to be changed to sensitive cells
  - ”Optimal CTA table found (optimal within tolerances)”

Comparison: iterative CTA strategy vs. simple aggregation



Comparison: iterative CTA strategy vs. simple aggregation



Summary and Conclusion

Summary:

- Starting point: Non-additive but consistent hypercubes protected by random noise with Cell Key Method design properties
- Evaluations using synthetic EU hypercube data
- Restoring additivity at once too complex, therefore …
  - Blocking strategies and iterative approaches make CTA optimization problems feasible
  - CTA techniques lead to much less perturbation in hypercube margins than simple summation of the noisy lowest level counts

Conclusions:

- Recommendation: to preserve consistency and keep perturbation “low” the “goals” of additivity “corrections” should be revised:
  - either not restore additivity at all, or
  - restore additivity to sub-hypercubes rather than to full hypercubes (i.e. devise heuristical strategies)

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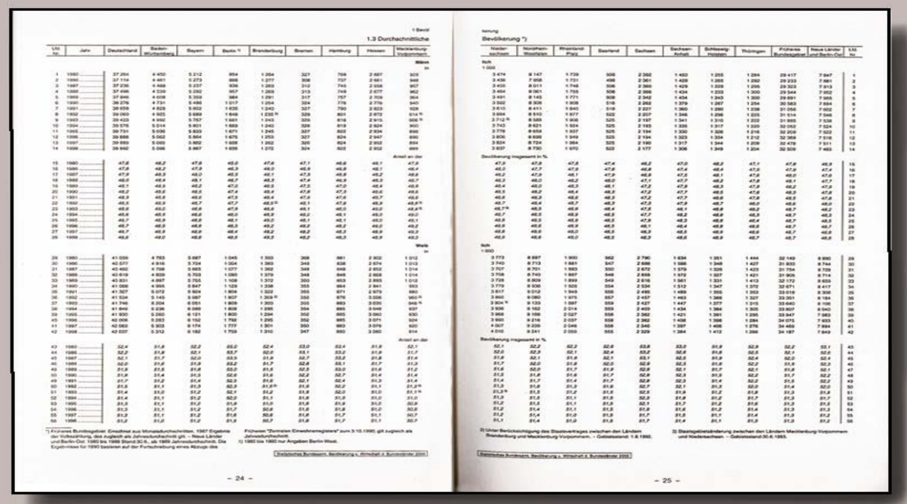
1.3 Durchschnittliche
Bevölkerung *)
– 24 –
– 25 –

Thanks for your attention