# Protection of frequency tables – current work at Statistics Sweden

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**Abstract**. Statistics Sweden is currently working on finding a method for disclosure control of frequency tables, and in particular for the large number of frequency tables generated from the registers kept by the agency. These tables are often large in size, and large numbers of tables are generated for the same population. Relationships between tables are complex, including hierarchies or common cells or margins that appear in several tables. In addition, since tables are based on totally enumerated populations, they are exposed to higher disclosure risks than tables based on sample surveys. Current work focuses on testing the protection method developed by ABS (Australian Bureau of Statistics). Tables are confidentialised as they are requested by adding noise to cell values (perturbation). An attractive characteristic of the method is that it ensures that each cell is perturbed in the same way every time it is requested, that is there will be consistency between tables. The method is also tailored to protect against differencing. We will present the method, the tests and measures that have been carried out in order to evaluate the risk reduction and the utility of the protected tables, and the current plans for implementation of the method. It is clear that implementation of the method would not only generate sufficiently protected tables, but it would also lead to substantial changes in the production process and the current way of working.

## 1 Introduction

For Statistics Sweden, as well as all NSIs, it is important to protect data properly before it is disseminated. The premise of the Swedish legislation is that all official statistics are confidential without exception and may only be published if it can be guaranteed that a disclosure would not cause harm or damage to an individual, household, or business. Further, for the credibility of the agency, it is important for Statistics Sweden to show that we take all necessary measures in order to prevent violation of the privacy of individuals.

Decisions on dissemination of macro- and micro data are taken locally within the agency, but like many other producers of official statistics, Statistics Sweden is constantly moving towards a more standardized production. This applies to the entire production process, where protection of disseminated data is one activity among many others. The statistical methods used for identifying risks and protecting data have traditionally been decided locally within the agency, each survey specifying procedures and tools more or less on their own. A common approach is important for several reasons. There must be no uncertainty introduced among users because of inconsistencies due to different approaches locally within Statistics Sweden. The staff working in production must have proper support to make it possible for them to do their work. A common practice should also ensure that protection is actually being carried out where necessary, and that it is being done in a manner that follows best practices.

In 2008, Statistics Sweden started to work towards the (now accomplished) certification according to the international ISO 20252 standard on market, opinion, and social research. Statistical disclosure control (SDC) was identified as an area where a number of surveys did not fulfil the requirements set by the standard. As a consequence, the work to improve SDC at Statistics Sweden has in recent years been given some priority. However, it should be noted that the ISO-standard does not require procedures and tools to be standardized across surveys, but merely requires that proper procedures are in place for all surveys. We view the ISO-standard as a minimum and our goal is to have a set of common methods and tools that will suit the main part of the statistical products.

As a result of the previous work on common methodology and tools for disclosure control of tables and other published materials (not including micro data), Statistics Sweden now has a common solution for risk assessment and protection of magnitude tables. However, frequency tables require a different solution, and in particular tables generated from the large registers kept by the agency. Statistics Sweden keep three base registers that are widely used for table production within the agency; the Total Population Register (TPR), the Business Register (BR), and the Real Property Register. Tables produced from the TPR and the BR need to go through SDC before publication. These tables are often large in size, and large numbers of tables are often generated for the same population. Relationships between tables are complex, including hierarchies or common cells or margins that appear in several tables. In addition, since they are based on totally enumerated populations, these tables are exposed to higher disclosure risks than tables based on sample surveys.

Common methods, like cell suppression, that are used to protect tables after they have been produced are not good solutions for these kinds of tables (Hundepool et al., 2012). In addition to the need for a better common methodology, the large number of tables and their size and complex structure call for highly automated procedures of SDC. This is necessary in order to guarantee a reasonable workload and to assure the quality of the results. The same registers are used for many outputs, and without a common solution, the same table may look different when published at several occasions, depending on local production procedures. The current proposal is to focus on a methodology developed by ABS, the Australian Bureau of Statistics for their Census TableBuilder online tool, and adapt it to the needs of Statistics Sweden.

In this paper we describe the ABS method and compare it with two other perturbation methods: deterministic rounding to two significant digits and deterministic roundning to base 5. The methods are evaluated on measures of risk and utility. At the end of the paper we discuss implications of implementing the ABS method.

# 2 The ABS Census TableBuilder protection method

## 2.1 Introduction

The protection method used by ABS has been developed in order to protect Australian census frequency data as they are made available to the public through the Census TableBuilder, an online system to which users submit table queries. The confidentiality method was originally proposed in Fraser and Wooton (2005). The methodology is further described by Marley and Leaver (2011) and Thompson et al. (2013). Leaver (2009) describes the implementation of the method in the Census TableBuilder. Tables are confidentialised as they are requested by adding noise to cell values (perturbation). The same cell will always receive the same noise. In contrast to methods that randomly apply noise, the risk that somebody will be able to disclose information by repeatedly requiring the same table is thus avoided. The method was originally tailored to protect against differencing, that is the possibility to take the difference between tables for similar sub-populations and thereby disclose information for individuals.

To begin with, each object in the register is randomly assigned a permanent numeric value called a record key. When a frequency table is requested, the units defined to belong to a table cell are counted, and the record keys of the units within the cell are combined to create a cell key. The cell key is then reduced to a row index. A specific cell will always have the same row index. Via a perturbation look-up table (a fixed, two-dimensional array of numeric values), the row index and the original cell value are used to determine the amount of perturbation that is to be applied to a specific cell. Thus the cell will always receive the same amount of perturbation, independent of which table it belongs to or when it is requested. The method is further described in Sections 2.2 and 2.3.

An attractive characteristic of the method is that there will be consistency between tables. A drawback is that all cells are protected separately, so that sums of protected cell values might not equal the corresponding protected margins. Equality between margins and sums of cell values can be restored, but at the cost of consistency.

## 2.2 Constructing distributions for the perturbations

The amount of perturbation applied to an original (unconfidentialised) cell count $i = 1, \ldots, M$ is determined through the design of the look-up table. Given sets $\Pi_i = \{\pi_{Li}, \ldots, \pi_{Ui}\}$ with allowed perturbations for each $i$, a distribution for the perturbations $P(B_i = \pi)$ for all $\pi \in \Pi_i$, where $B_i$ is a stochastic variable, is calculated by maximising the entropy.

$$
\begin{aligned}
\text{Maximize} \qquad & -\sum_{\pi\in\Pi_i} P(B_i=\pi)\log(P(B_i=\pi)), \quad i=1,\ldots M \\
\text{subject to } \forall\pi\in\Pi_i \qquad & P(B_i=\pi)\geq 0, \\
& \sum_{\pi\in\Pi_i} P(B_i=\pi)=1, \\
& \pi+i\in\{0,l,l+1,\ldots\},\ l\geq 0 \\
& E(B_i=\pi)=0, \\
& V(B_i=\pi)\leq v_i.
\end{aligned}
\tag{1}
$$

The parameter values $l$ and $v_i$ are chosen by the NSI. For example, $l = 3$ implies that the cell values 1 and 2 are not allowed in the confidentialized table, while $l = 0$ implies that all cell values are allowed.

Note that the constraint on the expected value implies that table cell counts of 0 are not perturbed. Since $i = 1, \ldots, M$ this optimization problem is actually $M$ constrained non-linear optimization problems since the entropy is a nonlinear function of $P(B_i = \pi)$. These problems are solved numerically by an optimization module in SAS (SAS Institute Inc.), resulting in distributions from which the perturbations for each table cell value $i$ are generated and stored in the look up table.

### 2.3 The look-up table

An arbitrary frequency table may be viewed as a vector of cell frequencies, here denoted by $\mathbf{t} \in \mathbb{N}^K$, where $K$ is the number of cells in the table. The perturbations $\mathbf{p} = (p_1, \ldots, p_K) \in \mathbb{N}^K$ for unperturbed cell values $\mathbf{t} = (t_1, \ldots, t_K)$ are stored in the look-up table. Each column in the look-up table represents a cell count.

For each cell $k$ the perturbation $p_k$ is taken from column $t_k$ in the look up-table. In order to find the corresponding row in the look-up table, all records in the register are assigned a permanent random number, a record key. When the frequency in a specific table cell is calculated, the record keys of the objects contributing to the cell are summed to a cell key. The cell key is taken modulo a big prime number and written as a 32-bit binary number. From these numbers we get the row index of the look-up table specifying from which row in the look up table the perturbation for a specific cell will be taken. Once the perturbations $\mathbf{p}$ for table $\mathbf{t}$ are constructed, the confidentialised table is given by $\mathbf{c} = \mathbf{t} + \mathbf{p}$. This construction assures that the same perturbation is added to a specific cell, independent of which frequency table it appears in. For more details of the construction of the look-up table, see Fraser and Wooton (2005) and Thompson et al. (2013).

### 2.4 Risk versus utility

When determining the look-up table, the constraints for maximising the entropy will determine the amount of protection applied to a table, but protection will at the same time cause a reduction in utility. For example, allowing the maximum amount added to the cells as well as the variance of the perturbations to be large is likely to provide better protection but also to cause larger information loss, compared to constraints implying a small maximum amount of perturbation and low variance. It is thus important that the risk reduction and information loss is evaluated before implementation of a look-up table. An R-U graph (Duncan et al., 2001) provides a summary where risk and utility are plotted in order to find acceptable levels for both. In Section 3, we present results for four test cases, using such plots.

In order to describe the disclosure risk and the utility loss for a specific table when it is protected by perturbations constructed as in Section 2.2, an indicator set describing the structure of a table $\mathbf{t}$ is introduced according to

$$I_i(\mathbf{t}) = \{k | t_k = i\},\ k = 1, \ldots, K. \tag{2}$$

For example, for a table consisting of the cells

$$\mathbf{t}_{\text{ex}} = (2, 3, 1, 3, 1, 5)$$

the indicator sets are $I_1 = \{3, 5\}$, $I_2 = \{1\}$, $I_3 = \{2, 4\}$, $I_4 = \{\}$ and $I_5 = \{6\}$.

The noise that is added to the cells in $\mathbf{t}$ is random, so the variance of the noise can be considered for a measure of the risk (Marley and Leaver, 2011). The larger the variance, the lower is the risk of disclosing the original cell value. Let

$$\mathbf{v}(\mathbf{t}) = (\frac{1}{v_1(\mathbf{t})}, \frac{1}{v_2(\mathbf{t})}, \ldots, \frac{1}{v_M(\mathbf{t})}),$$

be a vector containing the inverted variances where

$$v_i(\mathbf{t}) = V(p_k | k \in I_i),\ i = 1, \ldots, M, \tag{3}$$

is the variance of the perturbations for all cells with unconfidentialized cell count $i$. The Euclidean norm of $\mathbf{v}(\mathbf{t})$

$$\mathcal{R}_1(\mathbf{t}) = \|\mathbf{v}(\mathbf{t})\|_2 \tag{4}$$

is used as a measure of disclosure risk. This risk measure apply to the whole table $\mathbf{t}$ and will decrease as the variances of the perturbations increase. Further, the Euclidean norm allow small variances to have large influence on the risk measure for the whole table, since they become large when inverted.

As a second measure of risk, the percentage of cells in a table that are unchanged is used. Let $p_{\text{zero}} = \{p_k | p_k = 0\}$. Then

$$\mathcal{R}_2(\mathbf{t}) = \frac{|p_{\text{zero}}|}{K}. \tag{5}$$

This measure applies to the whole table. A large value indicates low level of protection and low value indicates a higher level of protection.

One could argue that as long as any cell in a table is under risk, the whole table is unsafe to publish, and thus that table measures should be supplemented with cell measures. However, with perturbation methods, further uncertainty is introduced for all cells since it is unknown which cells have been protected and which have not. We regard table measures as sufficient for the evaluation in this paper.

To measure the utility loss that is introduced by the protection of the table, we chose to use the Hellinger distance (Shlomo, 2007). For tables $\mathbf{t}$ and $\mathbf{c}$, $\mathcal{U}(\mathbf{t}, \mathbf{c}) \in \mathbb{R}$ is defined as

$$\mathcal{U}(\mathbf{t}, \mathbf{c}) = \text{HD}(\mathbf{t}, \mathbf{c}) = \frac{1}{\sqrt{2K}} \sqrt{\sum_{i=1}^{M} (\sqrt{|I_i(\mathbf{t})|} - \sqrt{|I_i(\mathbf{c})|})^2}. \tag{6}$$

Note that numeric measures of utility will only capture quantitative aspects of the quality of protected tables, for qualitative aspects of their usefulness, discussion with users is necessary.

# 3 Test cases

## 3.1 Data and methods

The test cases are based on data from the Total Population Register (TPR) at Statistics Sweden. The register consists of all persons permanently residing in Sweden and contain approximately 9,6 million observations. Four tables regarding the total Swedish population with different levels of sensitivity, due to varying number of cells and cells with small values were used as test cases. The domains in the tables were region divided by county (21) and parish (1371), age (6), civil status (7) and country of birth (216). The figure in brackets refers to the number of categories in the domain.

- Test case 1: County, age and civil status.
- Test case 2: Parish, age and civil status.
- Test case 3: County, age and country of birth.
- Test case 4: Parish, age and country of birth.

In order to compare protection methods on risk level and information loss, the measures in Section 2.4 will be computed for the entropy based ABS method, deterministic rounding to two significant digits, and deterministic rounding to base 5.

### 3.2 Parameters for the ABS method

In order to compare the ABS method with the other methods, we need to choose the best set of parameters for the optimization in (1). The following constraints were chosen: The sets of allowed perturbations are $\Pi_A = \{-1, \dots, 1\}$, $\Pi_B = \{-3, \dots, 3\}$, and $\Pi_C = \{-5, \dots, 5\}$. The maximum allowed variance is varied over the set $\{1, 5, 10, 15, 30\}$, for all $i$, and $l$ either 0 or 3. For the case $l = 3$, only $\Pi_B$ and $\Pi_C$ are used because using $\Pi_A$ puts too hard constraints on the optimization problem. The results for the test cases in Section 3 are presented as R-U graphs in Figures 1 - 8 in the Appendix with $\mathcal{R}_1$ as the risk measure. The different combinations of parameters are listed in Tables 1 and 2. Figures 1 - 4 show the results when the variance and the set of allowed perturbations are varied, keeping $l = 0$ (that is, all cell values are allowed in the perturbed table). From these plots we choose the parameter combination 14 to be the best choice combining very low risk with reasonable utility loss for all tested tables. The parameters in this case are $v = 15$ and $\Pi_C = \{-5, \dots, 5\}$.

Figures 5 - 8 show the result when the variance and the set of allowed perturbations are varied but $l = 3$, that is the cell values 1 and 2 are not allowed in the perturbed tables. The different combinations are listed in Table 2. From these plots we choose the parameter combination 8 to be the best choice combining low risk with reasonable utility loss for all tested tables. The parameters in this case are $v = 10$ and $\Pi_C = \{-5, \dots, 5\}$.

When selecting parameter values, we have valued small risk higher than small utility loss. Looking at Figures 1 - 4, parameter combinations 6, 11, and 12 could also be possible candidates. Together with combination 14, they make up an imaginary line, representing varying degree of low risk and low utility loss. The parameter combinations above this line perform worse. The choice among the combinations on the line must be guided by the policy of the agency, weighing utility loss against the level of protection.

| Number | $\Pi_i$ | $v$ |
|---|---|---|
| 1 | $\Pi_A$ | 1 |
| 2 | $\Pi_A$ | 5 |
| 3 | $\Pi_A$ | 10 |
| 4 | $\Pi_A$ | 15 |
| 5 | $\Pi_A$ | 30 |
| 6 | $\Pi_B$ | 1 |
| 7 | $\Pi_B$ | 5 |
| 8 | $\Pi_B$ | 10 |
| 9 | $\Pi_B$ | 15 |
| 10 | $\Pi_B$ | 30 |
| 11 | $\Pi_C$ | 1 |
| 12 | $\Pi_C$ | 5 |
| 13 | $\Pi_C$ | 10 |
| 14 | $\Pi_C$ | 15 |
| 15 | $\Pi_C$ | 30 |

Table 1: Combinations of parameters for the test cases in Figures 1 - 4, the ABS-method with $l = 0$.

| Number | $\Pi_i$ | $v$ |
|---|---|---|
| 1 | $\Pi_B$ | 1 |
| 2 | $\Pi_B$ | 5 |
| 3 | $\Pi_B$ | 10 |
| 4 | $\Pi_B$ | 15 |
| 5 | $\Pi_B$ | 30 |
| 6 | $\Pi_C$ | 1 |
| 7 | $\Pi_C$ | 5 |
| 8 | $\Pi_C$ | 10 |
| 9 | $\Pi_C$ | 15 |
| 10 | $\Pi_C$ | 30 |

Table 2: Combinations of parameters for the test cases in Figures 5 - 8, the ABS-method with $l = 3$.

In Section 3.3, the chosen combinations of parameters are used when the ABS method is compared to the different rounding methods.

### 3.3 Comparing disclosure protection methods

Table 3 shows the results when the ABS method with two different combinations of parameters is compared to deterministic rounding to two significant digits and deterministic rounding to base 5, for Test case 1-4. The performance of the methods is evaluated on utility loss as measured by the Hellinger distance in (6), and the two risk measures $\mathcal{R}_1$, the norm of the vector of inverted variances of the perturbations as in (4), and $\mathcal{R}_2$, the percentage of unchanged cell values, as in (5).

The utility loss is lowest for the ABS method with parameters ($\Pi_C$,15,0), but the loss for the same method with combination($\Pi_C$,10,3) is not that much higher. Both rounding methods have much higher utility loss. In particular, rounding to two digits causes a high utility loss.

Risk as measured by the percentage of unchanged cell values is low for rounding to two digits for all four test cases, and is also fairly low for the ABS method with ($\Pi_C$,10,3).

Risk measured using the variance of the perturbations does not differ much between the two parameter combinations used for the ABS method.

The results are not clearly in favor of one method, Even though the risk is low when rounding to two digits, the utility loss is very high compared to the other methods. No measure favors deterministic rounding to base 5. From these results we conclude that the choice of method again depends on how the agency choses to regard risk versus utility.

| Test case | Measure | Entropy $\Pi_C$ $v = 15$ $l = 0$ | Entropy $\Pi_C$ $v = 10$ $l = 3$ | Det roundning to mod(5) | Det roundning two digits |
|---|---|---|---|---|---|
| 1 | $\mathcal{U}$ | 0.662 | 0.703 | 0.871 | 0.958 |
| | $\mathcal{R}_1$ | 3.24 | 2.95 | - | - |
| | $\mathcal{R}_2$ | 13.1 | 8.66 | 19.2 | 4.09 |
| 2 | $\mathcal{U}$ | 0.156 | 0.220 | 0.779 | 0.867 |
| | $\mathcal{R}_1$ | 19.5 | 20.5 | - | - |
| | $\mathcal{R}_2$ | 10.6 | 8.76 | 18.1 | 7.72 |
| 3 | $\mathcal{U}$ | 0.172 | 0.310 | 0.818 | 0.901 |
| | $\mathcal{R}_1$ | 11.1 | 13.5 | - | - |
| | $\mathcal{R}_2$ | 12.6 | 7.89 | 15.2 | 5.82 |
| 4 | $\mathcal{U}$ | 0.065 | 0.187 | 0.901 | 0.958 |
| | $\mathcal{R}_1$ | 20.7 | 20.5 | - | - |
| | $\mathcal{R}_2$ | 18.0 | 5.32 | 8.06 | 2.36 |

Table 3: Utility loss and risk for Test cases 1-4.

# 4 Implementation at Statistics Sweden

## 4.1 Defining risk

Before implementing a protection method, the agency needs to define sensitive cells. Small values are generally thought of as being particularly sensitive and at risk for disclosure. A single object in a cell might be identified, and new information might be disclosed if the identified object can also be linked to attributes. Two objects in a cell can be a risk since one of them might identify the other. Further, publishing small values might give the impression that the agency is less concerned with protecting individuals and there is a risk that the credibility of the agency is negatively affected. Larger cell values might also require protection if a variable is particularly sensitive, or if all or almost all objects belong to the same category (group disclosure). Another concern is the risk of differencing, i. e. comparing two outputs on the same population with only minor differences and being able to disclose information from the difference between the two tables. In order to design the look-up tables, it is necessary to agree on risk scenarios and define risk accordingly.

So far, the discussion at Statistics Sweden has concentrated on small values, but the risk for differencing has also been investigated. It can be discussed whether or not small values pose a risk, but in this context it is important to consider what generally constitutes a good table and good statistics. Conclusions based on a frequency table are probably not dependent on the 1's and 2's, and the small values should be of little relevance for most users. In order to circumvent the discussion, a decision not to publish small unprotected values is helpful.

### 4.2 Production at Statistics Sweden

Statistics Sweden does not have a dissemination system as flexible as the ABS Census TableBuilder. Tables are produced on appropriation and disseminated on the website through the Statistical Database (SSD), but also on commission, i. e. on a commercial basis. The SSD does not produce tables from micro data but requires that tables are protected before uploading. Implementing a common SDC method for the tables produced from the base registers implies that the method must be made available in the production systems at the units producing tables from the registers, both for the SSD and on commission. The fact that the TPR and the BR are used by several units and for several statistical products means that the method must be implemented simultaneously for all products, or consistency between tables will be lost.

The units producing tables from the base registers publish a lot of tables, but they are of similar types and, at least for the SSD, the same tables are published in regular intervals. It is likely that a few look-up tables will be sufficient for a large number of tables, which simplifies the implementation. The look-up tables have to be reconsidered regularly in order to catch changing demands.

An important aspect is to decide what is most important: consistency between tables or margins equal to sums of protected cell values (additivity). This is mainly related to what the users find most important, and how the method is described and explained to the users. Additivity can be restored at the cost of some consistency, but we have not yet considered this option.

### 4.3 Practical issues

The tables produced for SSD are tailored so as to avoid too much detail; i. e. some disclosure risk is avoided by aggregation. The smallest geographical level displayed is municipality. Implementing the ABS method for these tables will require some effort but it is estimated that this is manageable since once the system is set up, the production is regularly reoccurring. Some of this regular table production has not been reviewed for some time, and an implementation of a common SDC method might also be an opportunity for a complete table make-over, including both content and production process.

Tables produced on commission pose a somewhat different challenge. They represent a wide variety of usage, magnitude, and possible impact on society. Customers range from government agencies responsible for official statistics or for analyses or allocation of state funds, to businesses using data for marketing purposes. The demand for statistics at a detailed level is high. The different users and uses imply that consequences of adding noise will differ. With detailed calculations sensitive to small changes, noise may have an unwanted impact. The main challenge for the statistical agency, irrespetive of the protection method applied, is to explain to users why disclosure control is necessary, and the implications of the method applied.

## 5 Future work

The work to find a method for disclosure control of frequency tables based on totally enumerated populations is not yet completed at Statistics Sweden. The ABS method is a good candidate, but much work still remains and more evaluation is necessary. The choice of parameters for the optimization needs to be further investigated, and the method needs to be compared to other rounding options, for example random rounding of small values. We plan to apply the methods to other test cases, including data from the business register and data with a higher level of geographic detail. Many of the issues concerning implementation of (any) method also need to be solved.

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## Appendix

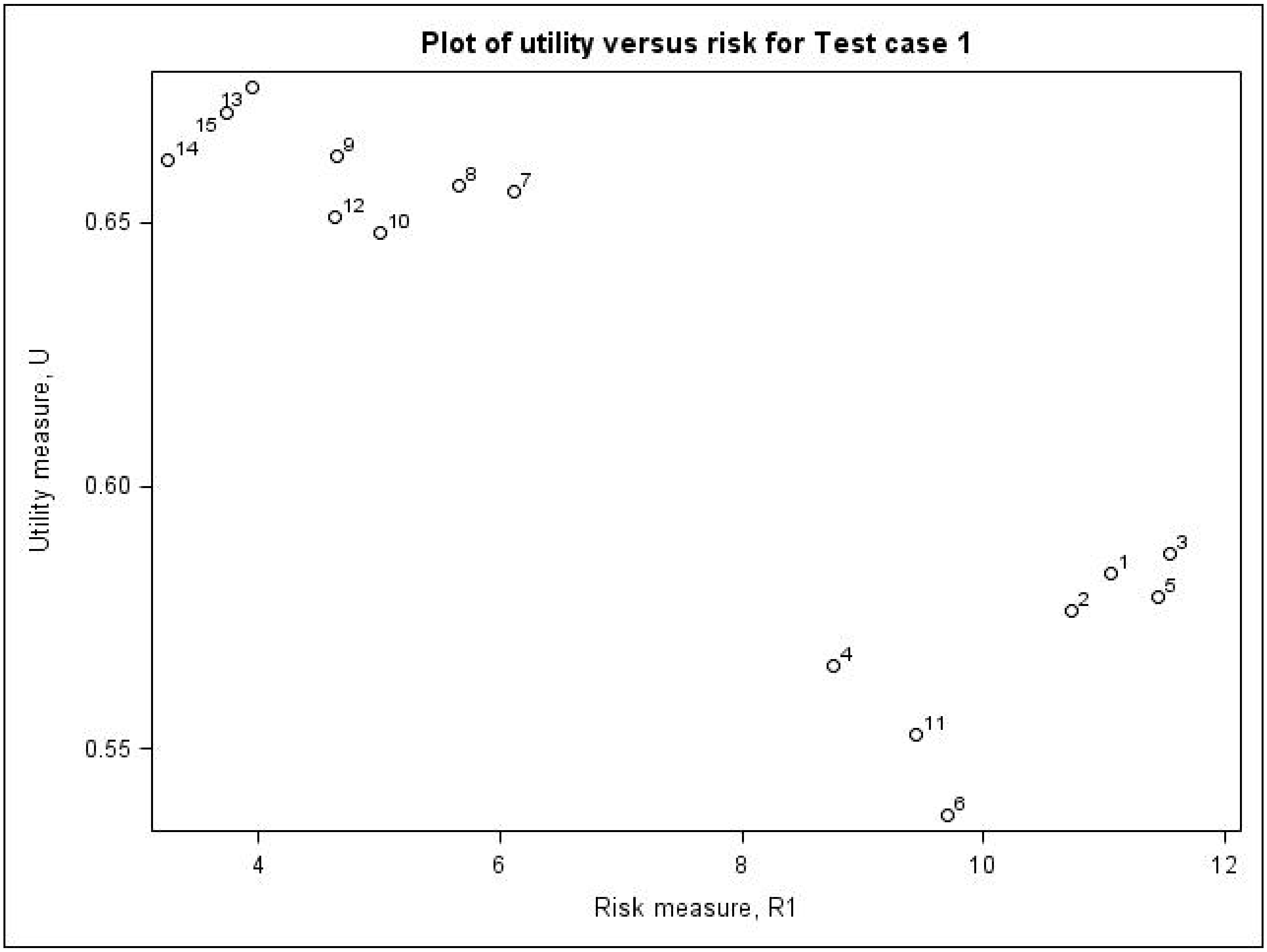


Figure 1: An $RU$-plot for Test case 1 perturbed by the ABS-method, $l = 0$.

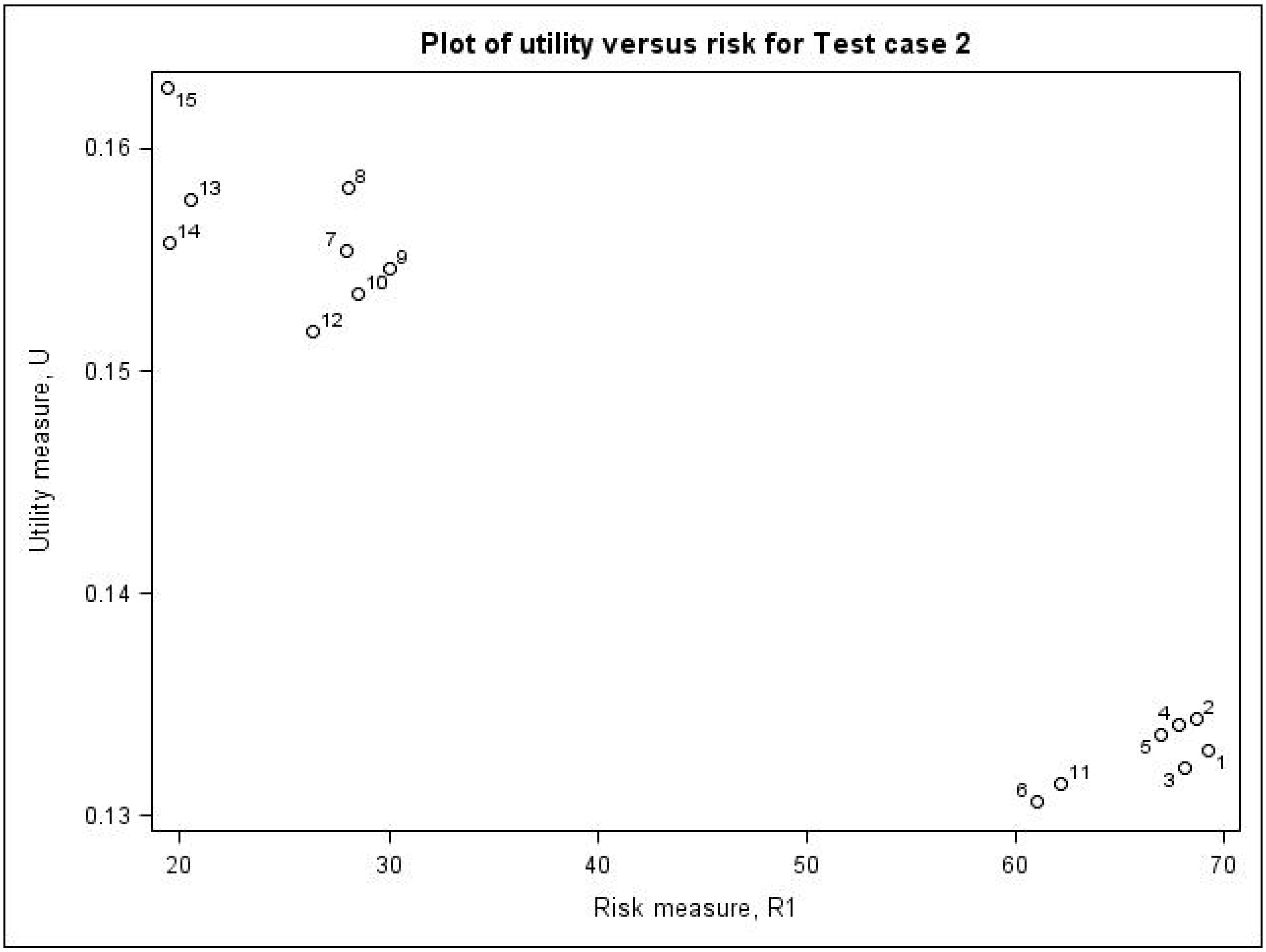


Figure 2: An $RU$-plot for Test case 2 perturbed by the ABS-method, $l = 0$.

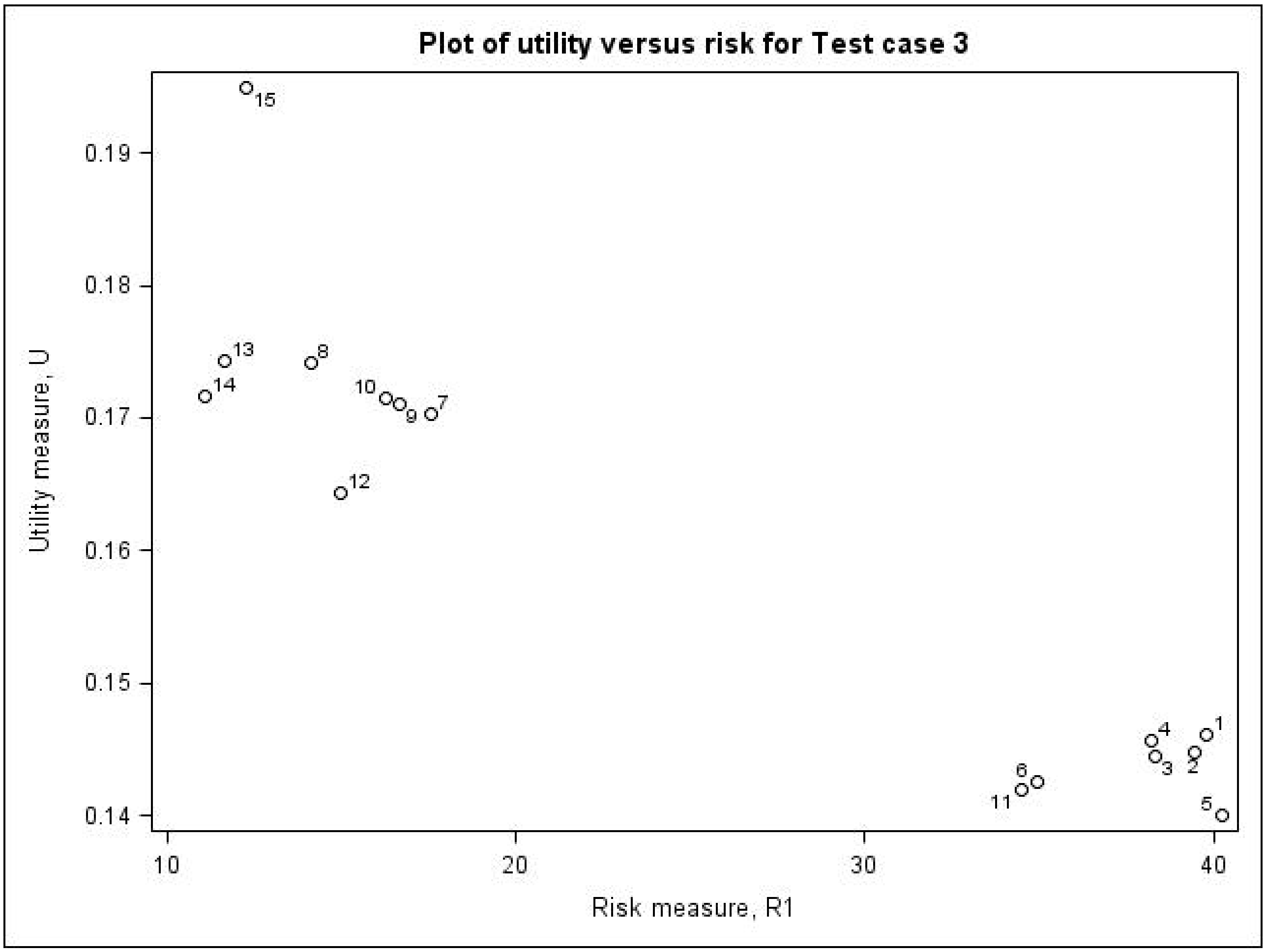


Figure 3: An $RU$-plot for Test case 3 perturbed by the ABS-method, $l = 0$.

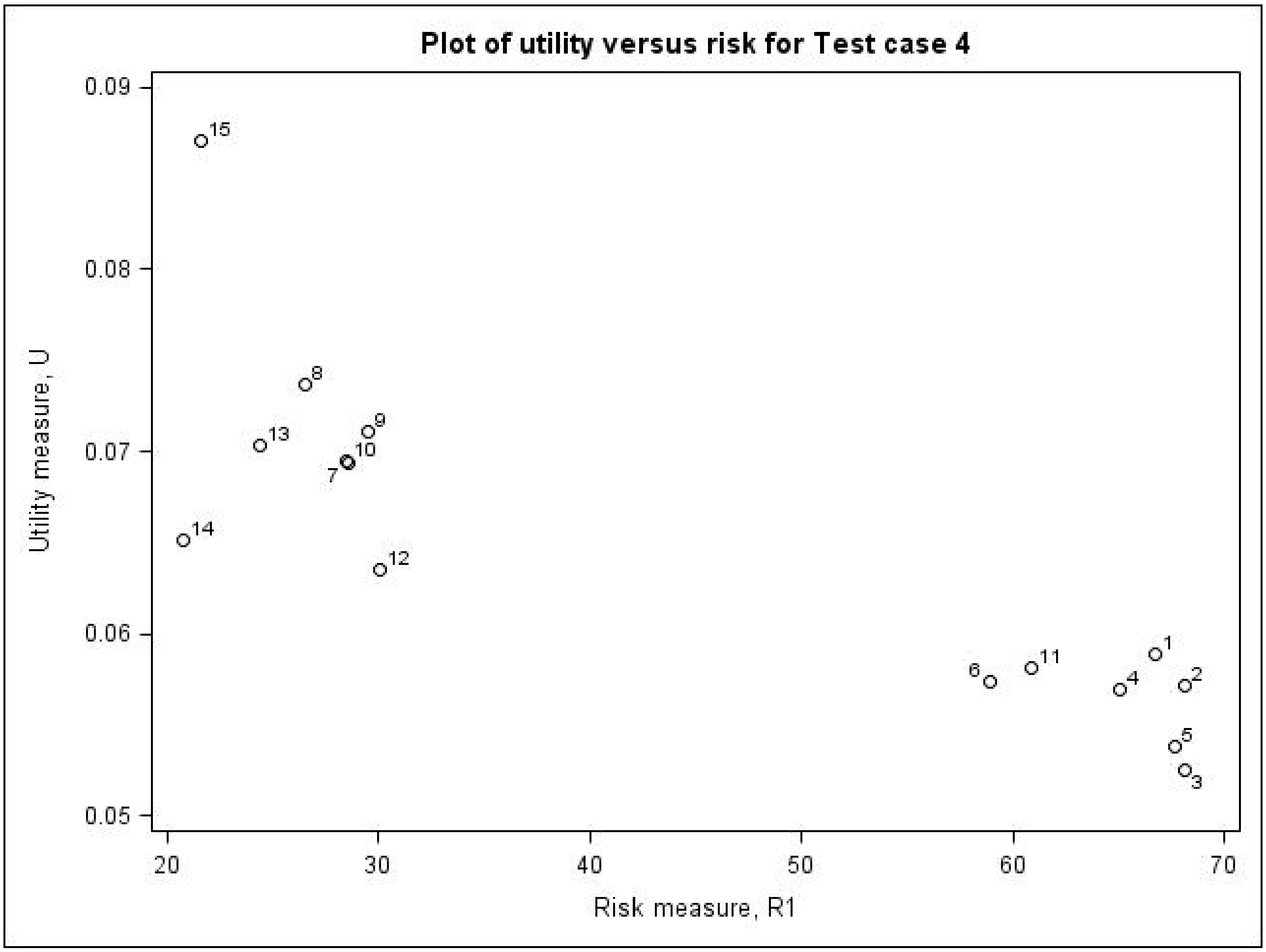


Figure 4: An $RU$-plot for Test case 4 perturbed by the ABS-method, $l = 0$.

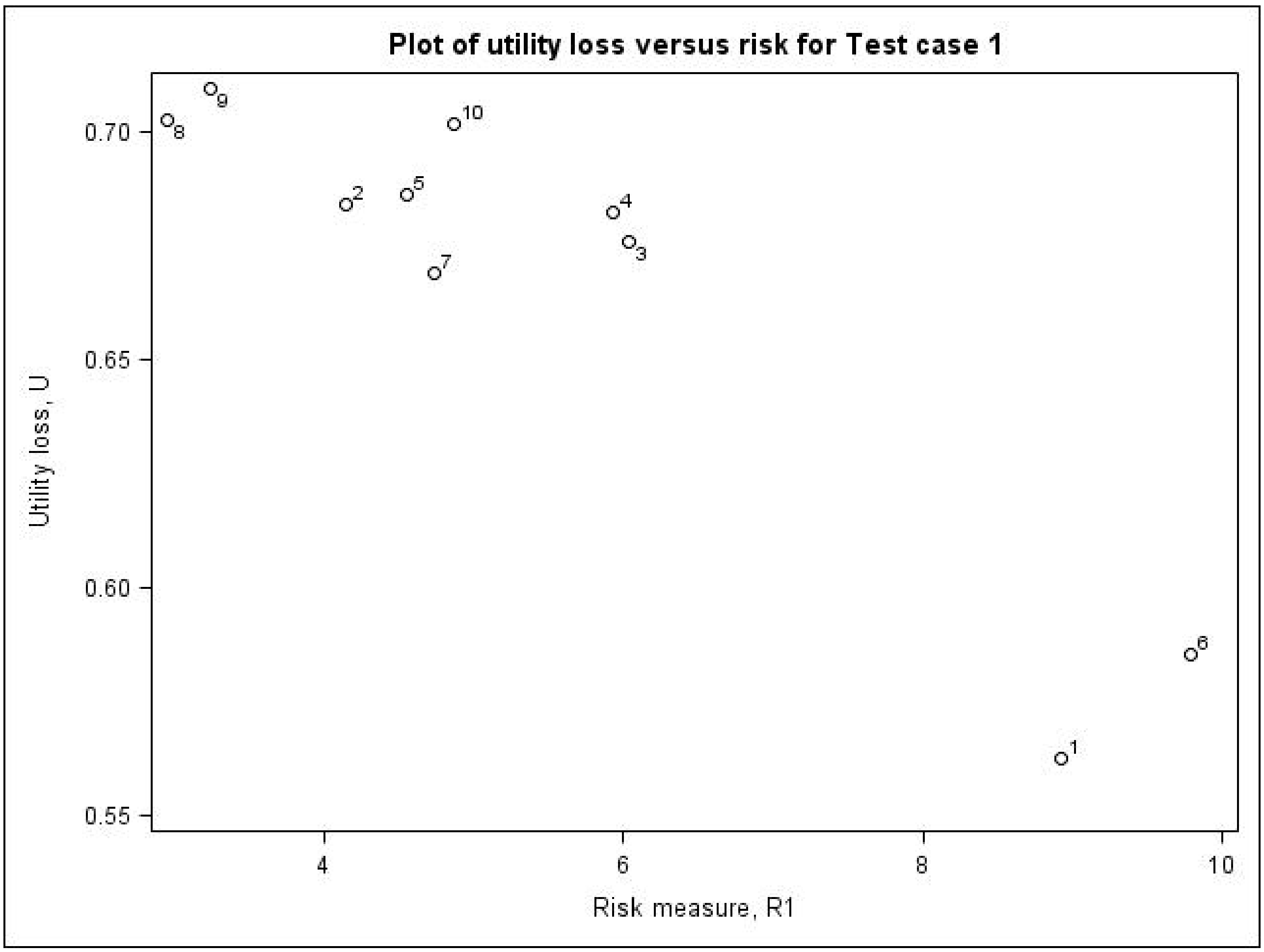


Figure 5: An $RU$-plot for Test case 1 perturbed by the ABS-method, $l = 3$.

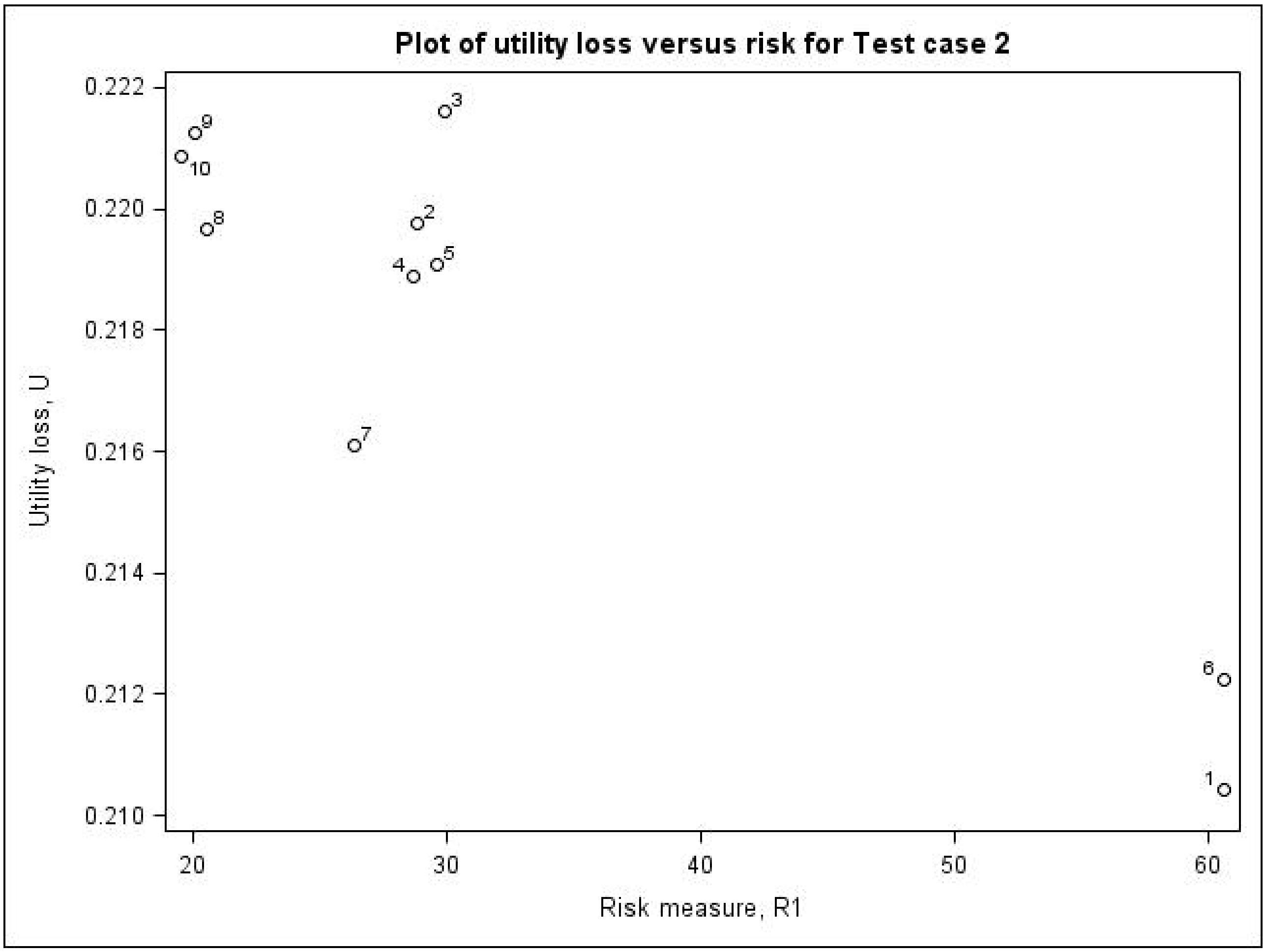


Figure 6: An $RU$-plot for Test case 2 perturbed by the ABS-method, $l = 3$.

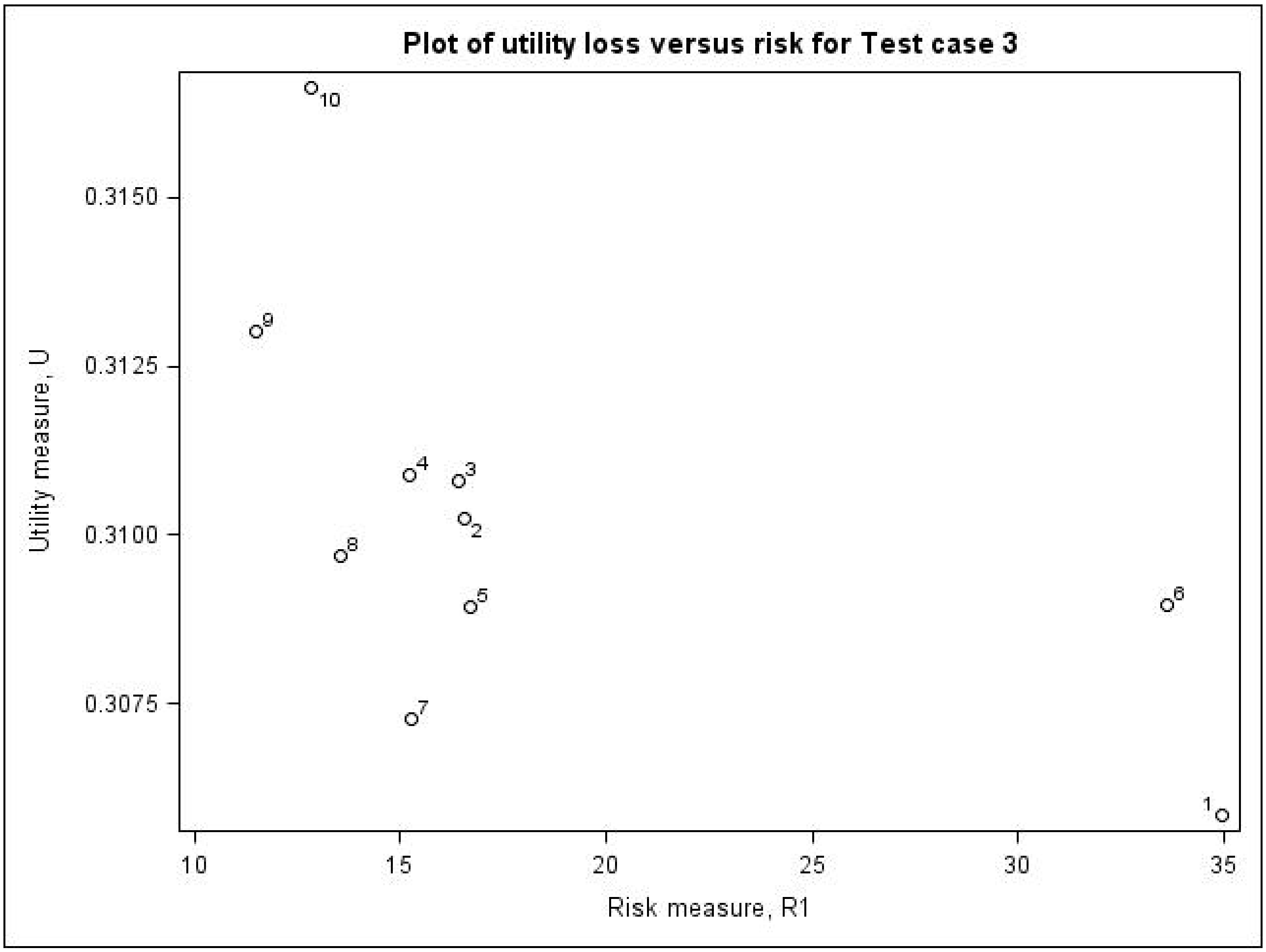


Figure 7: An $RU$-plot for Test case 3 perturbed by the ABS-method, $l = 3$.

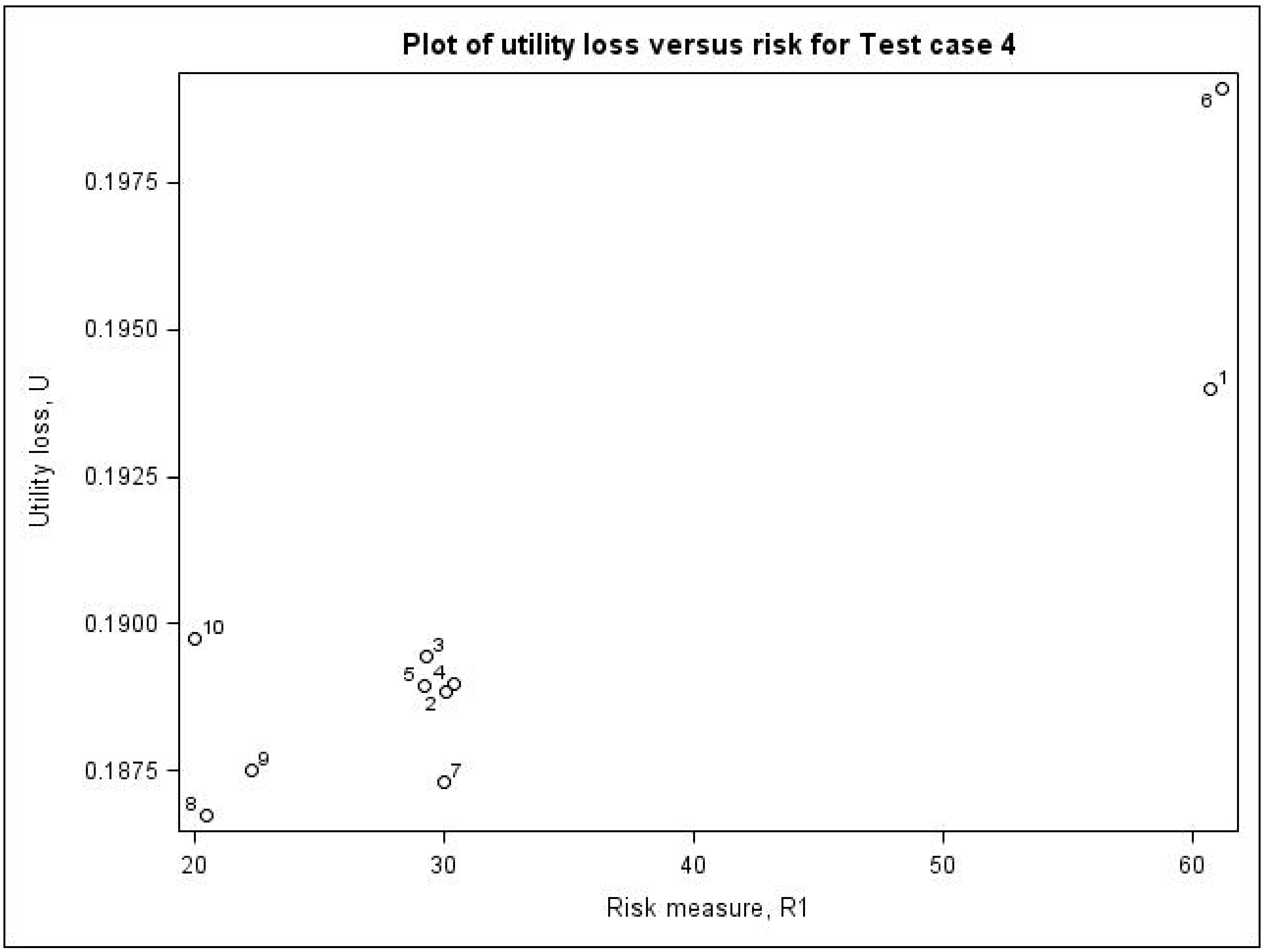


Figure 8: An $RU$-plot for Test case 4 perturbed by the ABS-method, $l = 3$.