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Assessing risk in statistical disclosure limitation

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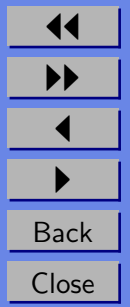

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- Public variables that allow identification are called key variables (KV).
- **Disclosure risk** is specific to a **cell** in the **contingency table** built by **cross-tabulating** the key variables.

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[Fienberg and Makov (1998); Omori (1998); Takemura (1998); Forster (2004); Benedetti and Franconi (1998)]

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EB: use Empirical Bayes approach (Efron and Morris) to estimate model parameters using the observed data distribution; then substitute these estimates into $[F_k|\underline{f}]$ to obtain an estimate of risk.

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Model V a Dirichlet-multinomial-multinomial model.

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- BF use $\hat{p}_k = f_k/\hat{F}_k^D$, where $\hat{F}_k^D$ is an estimate of F_k using the sampling design weights. This can sometimes be problematic.

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- A drawback: gamma hyperprior strongly concentrated on a small mean by the constraints on $[\pi_k]$ (K usually large): low variation across cells. Model I is less constrained.

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- The mean of each beta distribution in $(*)$ is $\hat{p}_k$. Here we make use of the sampling design weights through $\hat{p}_k$.

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- Finally, we estimate the risk using mean or mode of $[1/F_k|f_k]$.

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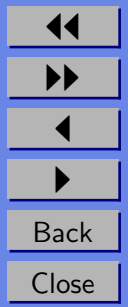

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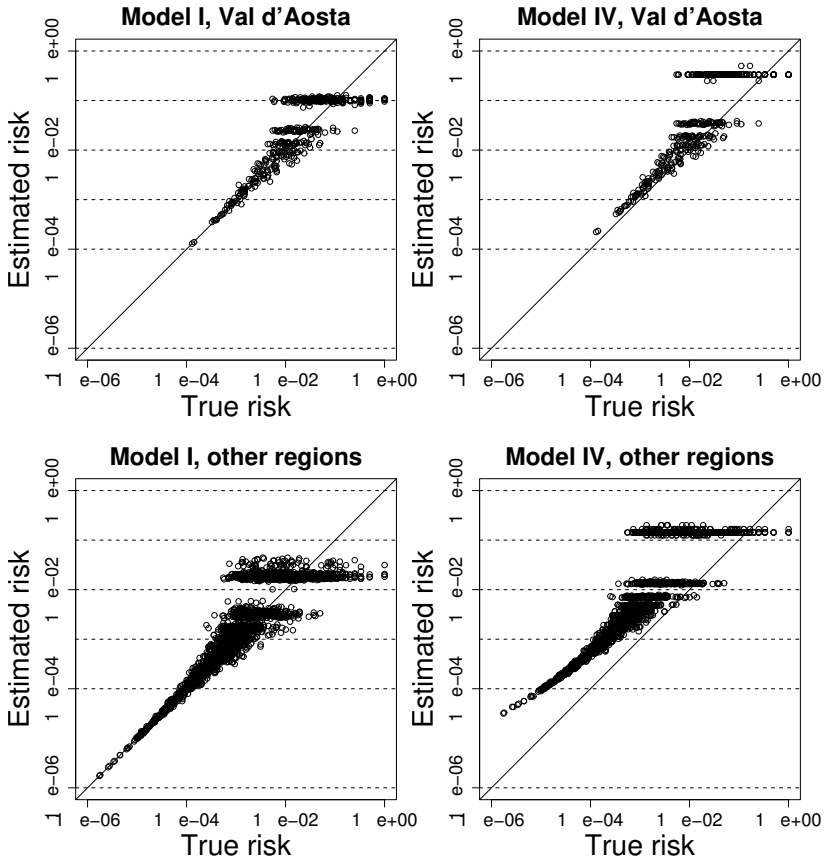
- $K = 20384$ but the number of nonempty cells is 12526 in the population and 2966 in the sample.

Models I and IV give similar patterns:



Model IV performs better with risky cells

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- Here is our model:

$$\begin{aligned} \underline{\pi} &\sim \text{Dirichlet}(\alpha_1, \ldots, \alpha_K) \\ \underline{F}|\underline{\pi} &\sim \text{multinomial}(N; \pi_1, \ldots, \pi_K), \\ \underline{f}|\underline{F} &\sim \text{multinomial}(n; F_1/N, \ldots, F_K/N), \end{aligned}$$

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- We perform inference using Markov chain Monte Carlo methods, implemented using `WinBUGS` and our own code.

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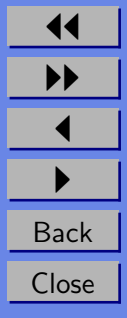

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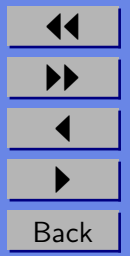

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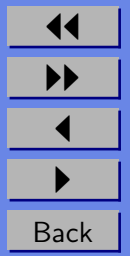
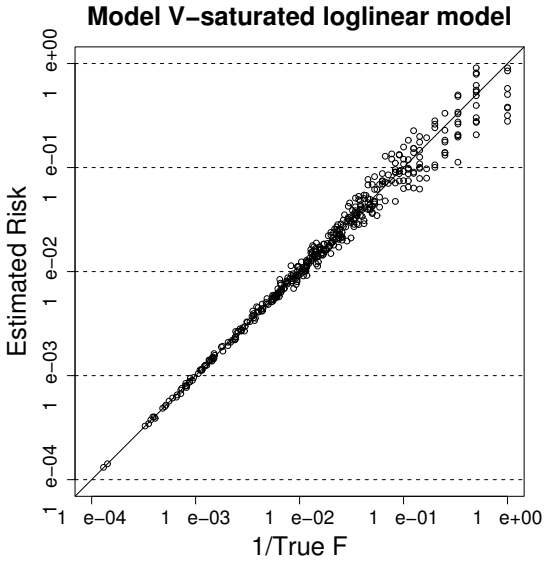
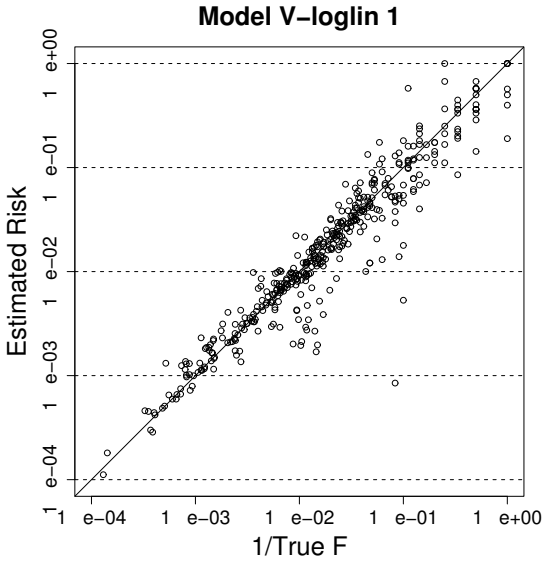
- Loglinear models used (by region):
  - Loglin 1: `sex+(rel+age+posprof)^3`
  - Loglin 2: `rel+(sex+age+posprof)^3`



Results for Val d'Aosta region (44% sample uniques; 77% of sample frequencies in 1-5.)





	$r_k \leq 0.05$	$r_k > 0.05$
$\hat{r}_k \leq 0.05$	303	7
$\hat{r}_k > 0.05$	9	102

Sensitivity is 0.94

Specificity is 0.97

	$r_k \leq 0.05$	$r_k > 0.05$
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Back
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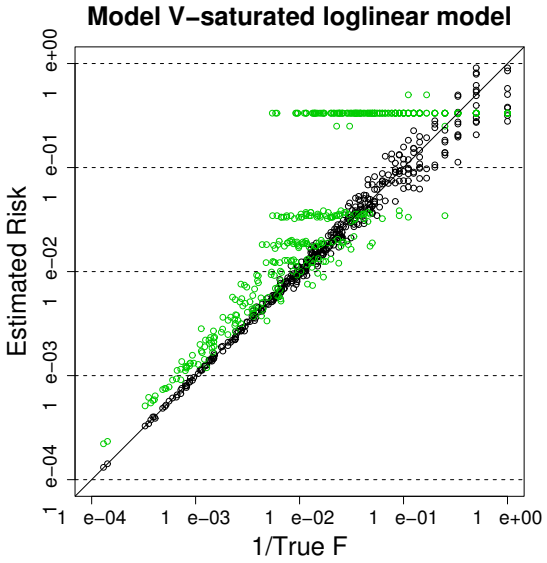

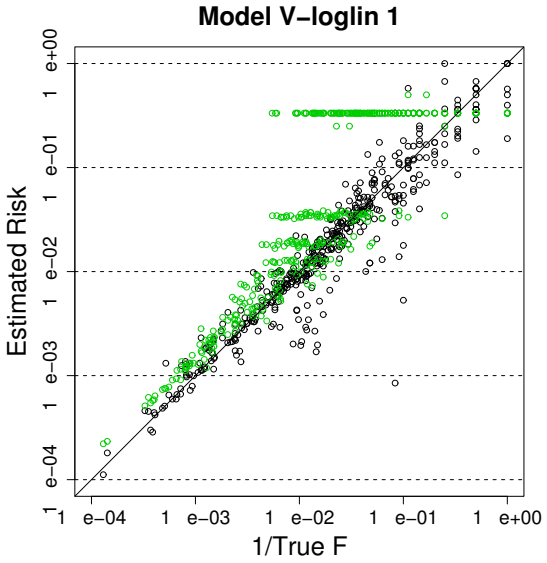

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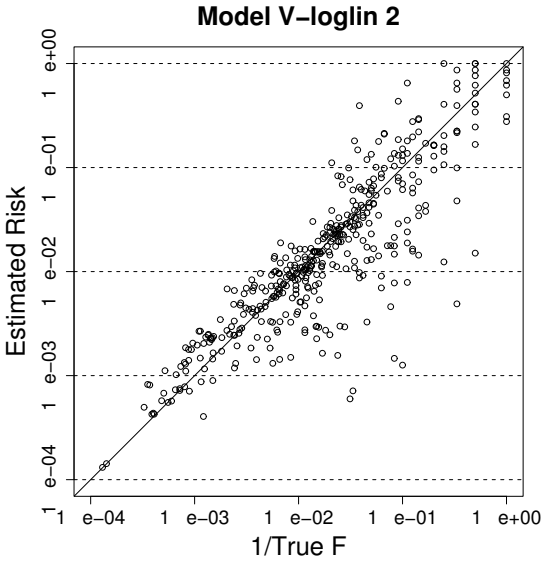

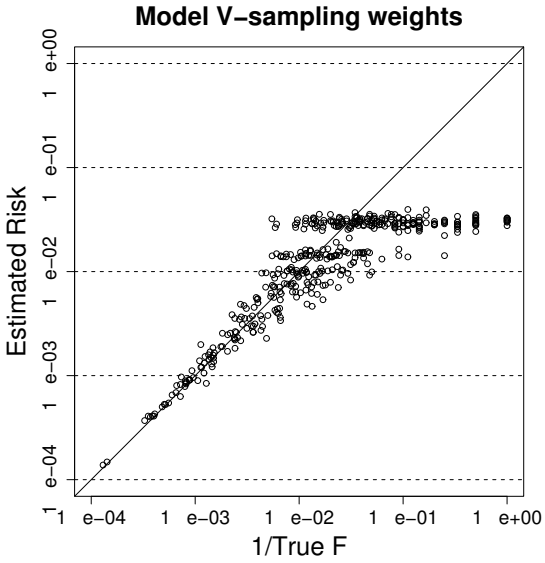

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Sensitivity is 0.71

Specificity is 0.94

	$r_k \leq 0.05$	$r_k > 0.05$
$\hat{r}_k \leq 0.05$	312	109
$\hat{r}_k > 0.05$	0	0

Sensitivity is 0

Specificity is 1

Back
Close

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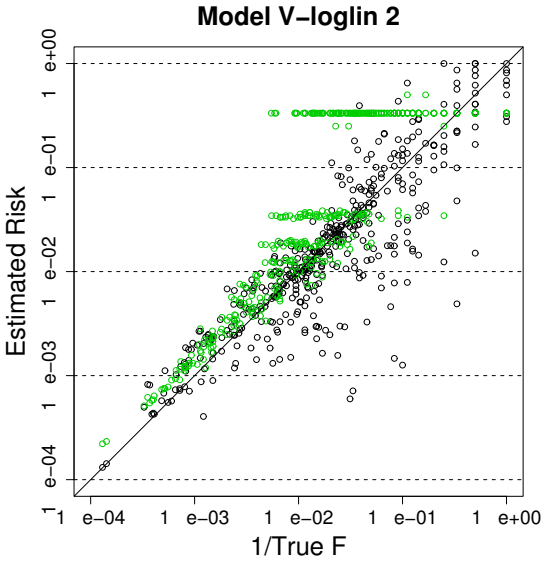

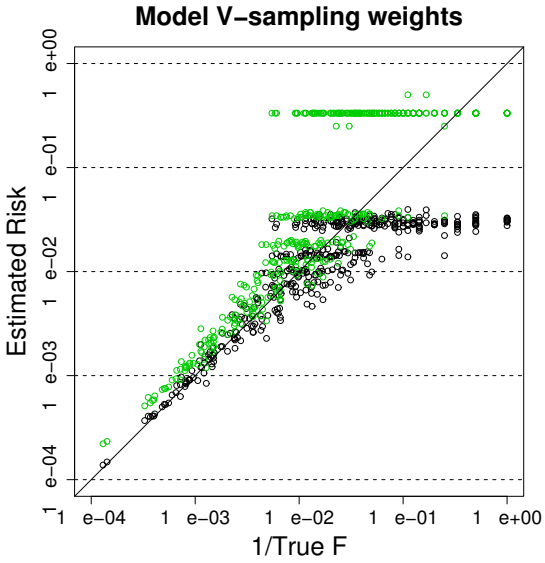

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Some References

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