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The Well-being Index: the weights of different components

How to limit side-effects of acceptable perceptions

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Agenda

- Presentation of the Well-being Index (WBI) for Portugal: 2005-2014
- The weighting problem and the WBI
  - Equal and differential weighting
  - Towards a mixed solution: a simulation with a rescaling function
- Conclusions

WBI conceptualization: 2 perspectives

Material living conditions

and

Quality of life

Material living conditions

Dimensions

Current and future prospects of consumption

Achievement of material well-being

Inequalities in the distribution of wealth

Self-assessment of material living conditions

Monetary poverty

Material deprivation

Indebtedness

Housing vulnerability

Participation and social inclusion

Work and gender vulnerabilities

Work quality

Material living conditions – Dimensions by Domains

Economic well-being

- Current and future prospects of consumption
- Achievement of material well-being
- Self-assessment of material living conditions

Labour and earning

- Participation and social inclusion
- Work and gender vulnerabilities
- Work quality

Economic vulnerability

- Monetary poverty
- Material deprivation
- Indebtedness
- Housing vulnerability

Quality of life

Dimensions

- Health outcome indicators
- Health care provision assessment
- Risk factors to health
- Reconciliation between time affected to family and work
- Work-life balance self-assessment
- Formal education
- Life-long learning
- Culture
- Education quality and acquired capabilities level
- Research and development and Innovation
- Social subjective well-being
- Individual subjective well-being
- Civic and political participation
- Trust in institutions
- Criminality
- Personal security self-assessment
- Water quality
- Air quality
- Noise pollution
- Biodiversity
- Waste

Quality of life – Dimensions by Domains (1)

Health

- Health outcome indicators
- Health care provision assessment
- Risk factors to health

Work-life balance

- Reconciliation between time affected to family and work
- Work-life balance self-assessment

Education

- Formal education
- Life-long learning
- Culture
- Education quality and acquired capabilities level
- Research and development and Innovation

»

Quality of life - Dimensions by Domains (2)

Social relations and well-being

- Social subjective well-being
- Individual subjective well-being

Civic participation and governance

- Civic and political participation
- Trust in institutions

Personal security

- Criminality
- Personal security self-assessment

Environment

- Water quality
- Air quality
- Noise pollution
- Biodiversity
- Waste

WBI by Perspective and Domain

» Number of indicators by Perspective and Domain

Material living conditions	#Ind	Quality of life	#Ind
1. Economic well-being	*9*	*4. Health*	*9*
2. Economic vulnerability	*7*	*5. Work-life balance*	*5*
3. Labour and earning	*13*	*6. Education*	*11*
		7. Social relations and well-being	*5*
		8. Civic participation and governance	*6*
		9. Personal security	*6*
		10. Environment	*8*

Number of indicators: 79

Subjective indicators: 14/79

Construction of the WBI (1)

The index of variable x for the year t is:

$$i_{x_t} = \frac{x_t}{x_0} \times 100 \text{ (for "positive" indicators)}$$

$$i_{x_t} = \frac{1/x_t}{1/x_0} \times 100 \text{ (for "negative" indicators)}$$

x_t is the value of variable x in year t
x_0 is the value of variable x in the base year

Construction of the WBI (2)

The index of domain d for the year t is:

$$I_{d.t} = \frac{1}{k_d} \sum_{j=1}^{k_d} i_{j.t}$$

$i_{j.t}$ is the value of the index associated to indicator j for the year t

k_d is the number of indicators of domain d

Within each domain, all indicators also have the same weight

Construction of the WBI (3)

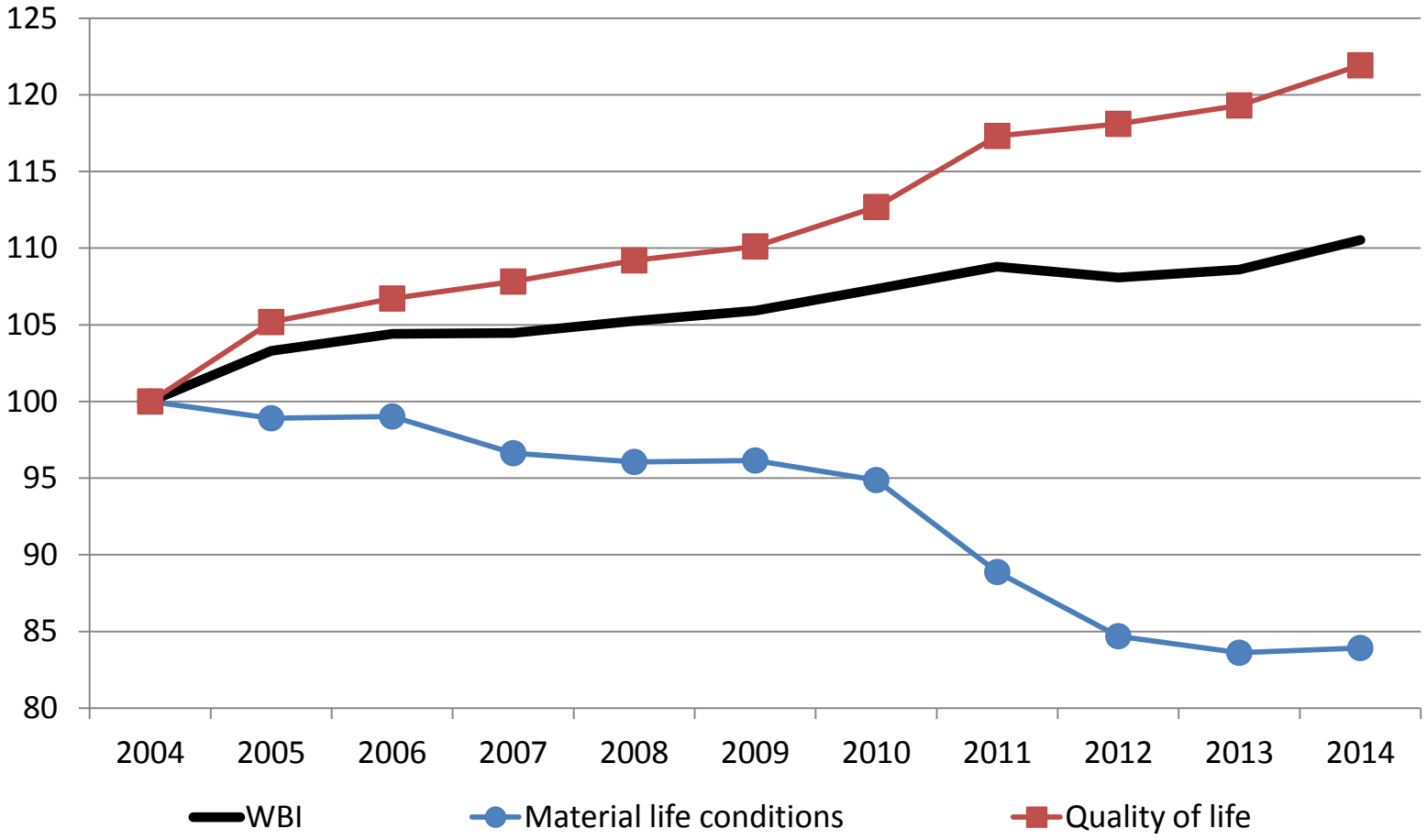
The global index (WBI) for the year t is:

$$WBI_t = \frac{1}{n} \sum_{d=1}^{n} I_{d.t}$$

$I_{d.t}$ is the value of the index of domain d in year t
n is the number of domains (10)

All domains have the same weight

WBI and Perspectives (2004-2014)



Equal versus differential weighting

- "*If surveys of citizens' weights are not available, then equal weighting minimizes the worst disagreements*" (Hagerty and Land, 2012)
- "*Equal weighting does not mean 'no weights', but implicitly imply that the weights are equal*" (OECD, 2008)
- "*A whole set of weights able to express in a perfect way the contribution of each indicator does not exist*" (Maggino)

Differential weighting methods

- Based on PCA/FA
- Data envelopment analysis
- Benefit of the doubt
- Unobserved component model
- Budget allocation process
- Public opinion
- Analytic hierarchy process
- Conjoint analysis

- Statistical methods
- Multiatribute models
- Subjective methods
- Participatory methods

Equal and differential weighting: a mixed solution?

- Differential weighting may result in indexes too distant from those based on equal weights
- Legitimacy: citizens may have perceptions and should have a voice about the relative importance of indicators. Giving them the opportunity to express their opinion strengthens the index legitimacy
- “But even surveys of citizens’ weights may be not correct” (Hagerty and Land, 2012): dynamically changing weights

- Is it possible to conciliate freedom of choice of weights by the users with the eventual virtues of equal weighting?

Weighting: rescaling function

The rescaling function *f* maps original weights (user defined weights) into smoothed weights

» $$f : w_d \mapsto w'_d$$

$$w'_d = \alpha + \beta w_d, \quad \alpha > 0 \quad \wedge \quad 0 < \beta \leq 1$$

w_d original weights of the domain W
w'_d smoothed weights of the domain W

Weighting: an example

» With $\alpha = 1$ and $\beta = 0.1$
Any weight $w_d \in \{1, 2, 3, 4, 5, 6\}$
is mapped into
$w'_d \in \{1.1, 1.2, 1.3, 1.4, 1.5, 1.6\}$

Strengths:

1. **Rankings** between weights are maintained;
2. **Distances** between weights are homothetic to the original distances;
3. ***Ratios*** *between weights are smaller.*

Procedure: generation of WBI series with random weights

1. Generate all possible combinations of domain weights (in fact we used a random sample (5000) of those combinations (6^{10}))

2. Rescale the generated domain weights using a linear transformation:

$$\alpha + \beta w_i \text{ where } \alpha=1 \text{ and } 0<\beta\leq 1$$

3. For each year of the series:
   - Compute each index (WBI) using all of these sets of weights
   - Compute Percentiles 5 and 95 of the distribution of the indexes (WBI)

Algorithm 1 Simulation algorithm

procedure GENERATION OF WBI SERIES WITH RANDOM WEIGHTS

Require: $1 \leq d \leq 10$: $d \in \mathbb{N}$ ▷ Domains

$\mathbf{W} = (w_1, \ldots, w_{10})$ ▷ Vector of original weights by domain

Require: Matrix $\mathbf{I}(10 \times 10)$ of indexes i_d ▷ 10 domains by 10 years

$k \leftarrow 5000$ ▷ Number of samples

Require: $\alpha \leftarrow 1$

Require: $w_d \in \mathbf{S} = \{1, 2, 3, 4, 5, 6\}$

for all *year* such that $2004 \leq year \leq 2013$ **do**

for $\beta = 0$ to 1 **do**

for $j = 1$ to k **do**

Choose randomly a vector of weights of dimension 10 from the 6^{10} possible combinations (w_d)

$w'_d \leftarrow \alpha + \beta w_d$ ▷ Smooth the weights

Ensure: $\sum_{d=1}^{10} w^*_d = 1$

$w^*_d \leftarrow \frac{w'_d}{\sum_{d=1}^{10} w'_d}$

$WBI_j \leftarrow \sum_{d=1}^{10} w^*_d i_d$ ▷ Computing the index

end for

Compute P05 and P95 of WBI **return** $P05(WBI)$ and $P95(WBI)$

$\beta \leftarrow \beta + 0.05$

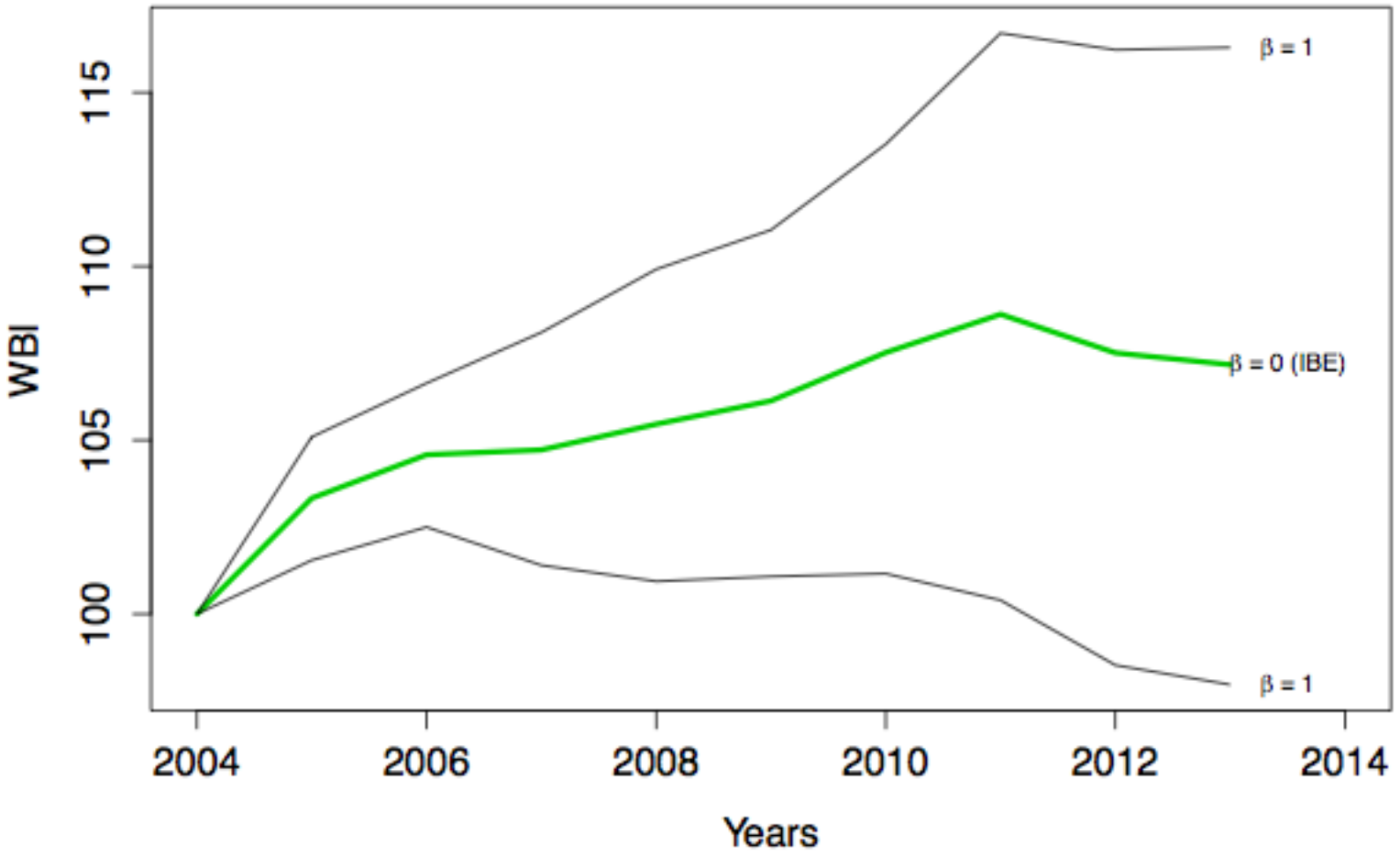
end for

end for

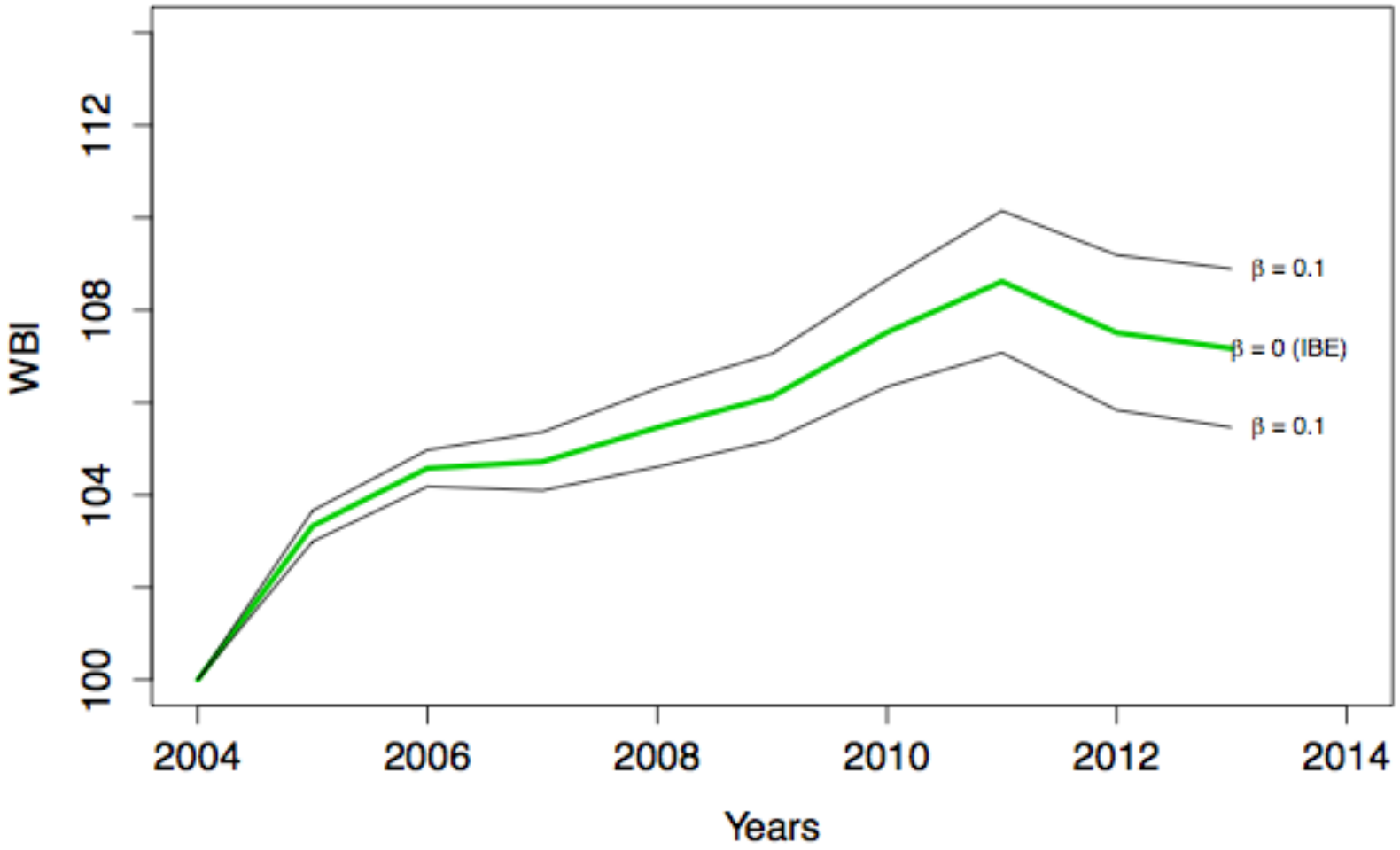
Plot a graph of all $P05$ and $P95$ for all β values from year 2004 to 2013

end procedure

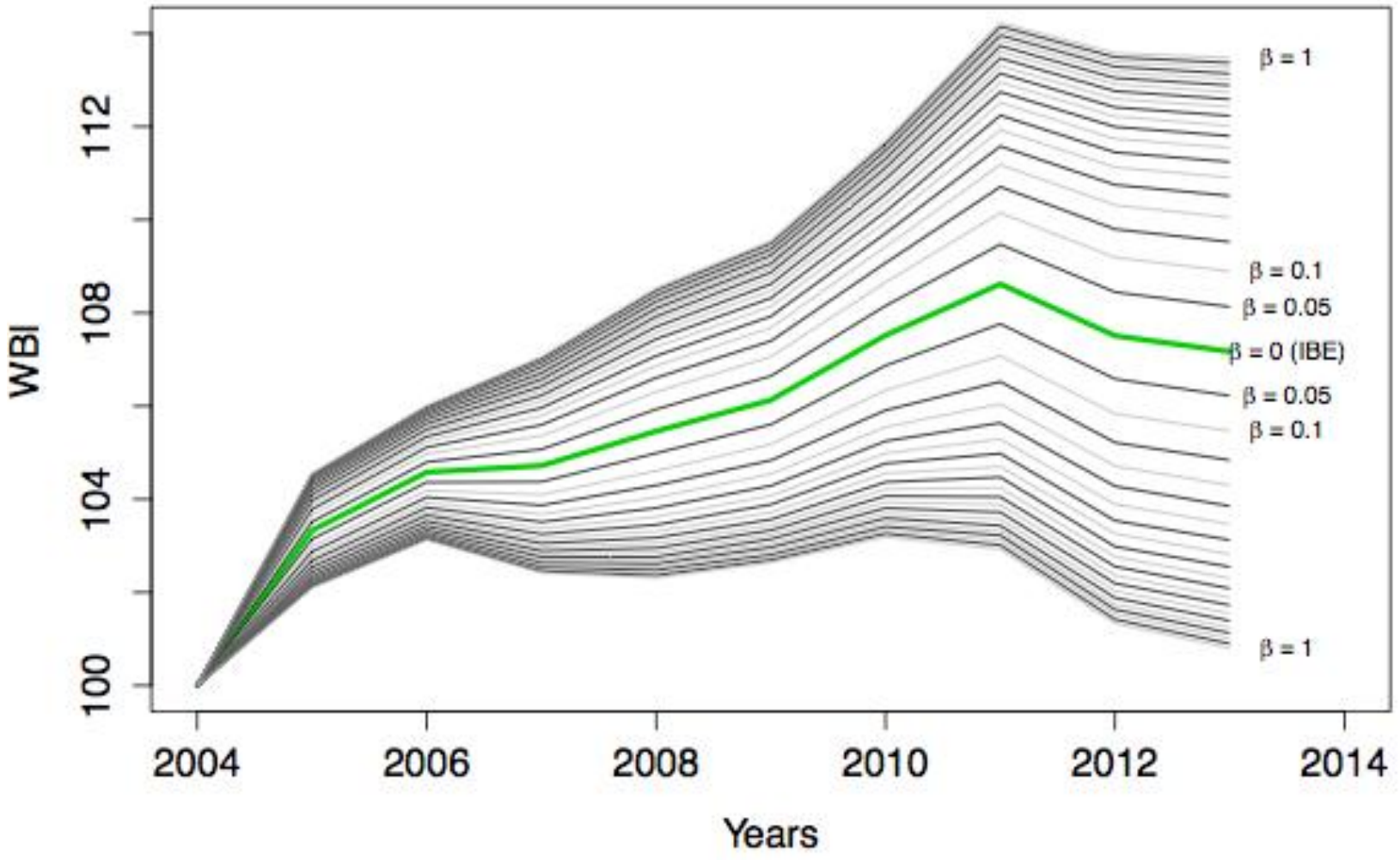
WBI (equal weights) vs. P05 and P95 of WBI (1 ≤ weights ≤ 6, weights ∈ ℕ) by year (α=0)



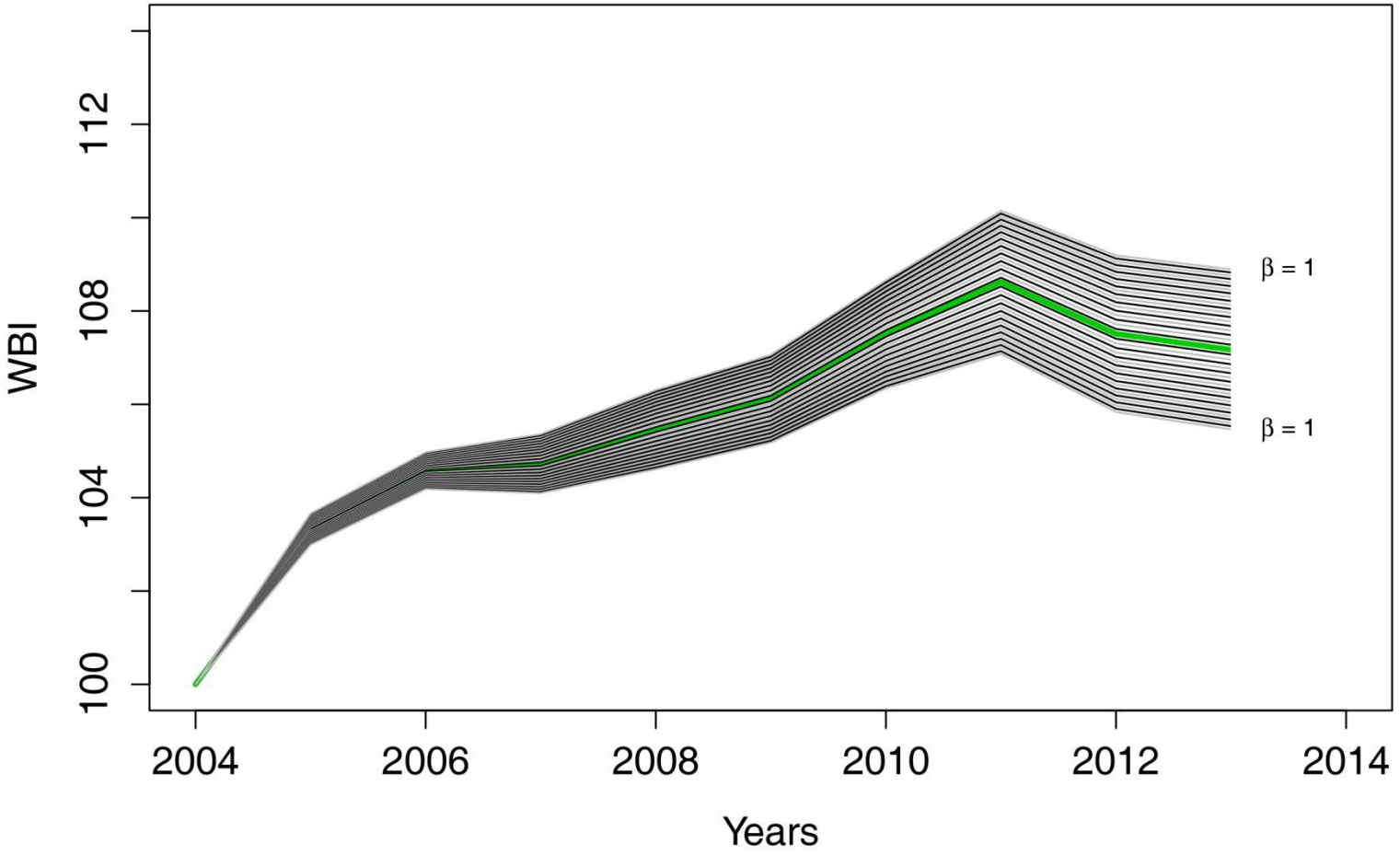
WBI (equal weights) vs. P05 and P95 of WBI (smoothed weights with β = 0.1) by year (α=1)



WBI (equal weights) vs. P05 and P95 of WBI (smoothed weights with $0<\beta\leq1$) by year ($\alpha=1$)



WBI (equal weights) vs. P05 and P95 of WBI (smoothed weights with $0<\beta\leq1$) by year ($\alpha=10$)



Conclusions (1)

When we assign different weights ($w_i < w_j < w_k$) to the index domains there are three implicit rationales:

1. ORDER - a domain (*j*) is more or equally important than the other (*i*)

2. INTERVALS - The difference between the importance of domain *j* and *i* is the same/greater/lesser than the difference between the importance of domain *j* and *k*

3. PROPORTIONS - The domain *j* becomes w_j/w_i times more important than domain *i*

However the third item is sometimes an *unintended consequence* of using a scale to assign the relative importance to each domain

Conclusions (2)

Our main contribution is:

➢A linear rescaling of the weights assigned by the user where we:

1. Respect ORDER and INTERVALS
2. Control for **side effects** of unintended assigned PROPORTIONS
3. Obtain a final result (composite index) very **similar** to what would be expected when equal weights are used (RESILIENCE)

Thank you for your attention