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ABSTRACT

Millions of goods and services are now unavailable in many countries due to the current coronavirus pandemic, dramatically impacting on the construction of key economic statistics used for informing policy. This situation is unprecedented; hence methods to address it have not previously been developed. Current advice to national statistical offices from the IMF, Eurostat and the UN is shown to result in downward bias in the CPI and upward bias in real consumption. We conclude that the only way to produce a meaningful CPI within the lockdown period is through establishing a continuous consumer expenditure survey.

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"The CPI is a critical input to economic policy making,
particularly during periods of economic uncertainty."
– IMF (2020)

1. Introduction

The coronavirus pandemic has led to widespread lockdowns[1] of non-essential retail outlets and many service industries such as regular restaurant services, tourism activities, sporting events, gyms, international air travel and most types of personal services. There are significant consequences of the sudden unavailability of many consumer goods and services for the measurement of inflation and real consumption.

A lockdown of economic activity means that there is a massive *disappearing products problem*. Countries base the construction of their Consumer Price Index (CPI), the main index of inflation, on a fixed basket of goods that people typically buy. But with lockdowns, the fixed basket becomes almost totally irrelevant as many of those typical items are no longer available; see Carvalho et al. (2020) and Dunn, Hood, and Driessen (2020) for evidence of dramatically changing consumer expenditure patterns arising from the pandemic. If consumer prices cannot be measured accurately, then real consumption cannot be measured accurately either. Policy makers and the public will eventually lose confidence in such indexes. This creates a crisis for policy and business decisions that rely heavily on these statistics.

Advice to national statistical offices (NSOs) from the International Monetary Fund, Eurostat and the United Nations for dealing with this problem is to simply implement a carry-forward methodology; the price for a commodity for the period prior to the lockdown is used (with some adjustment for inflation) when an item is missing; see IMF (2020), Eurostat (2020), UNECE (2020) and the summary in Appendix A. With the missing prices restored, index number construction can continue as before with the fixed consumption basket. We show that following this advice leads to a downward bias in

[1] The term "lockdown" is used to indicate a spectrum of restrictions which have different effects on the availability of products and the associated impacts on consumer behaviour.

estimating changes in the cost of living and an upward bias in estimating changes in real consumption.

With a lockdown, we have the opposite of the *new products problem*: a product is available for purchase in the current period but was not available in the previous period. We take the reservation price approach for the treatment of new products, which was developed by Hicks (1940; 114) and adapt it to cover the case of disappearing products following Hofsten (1952; 95-97).[2] *Reservation prices* are those that correspond to zero demand for the products. The unavailability of goods and services under a lockdown will lead to a substantial drop in welfare. Using the economic approach to index number theory in order to measure these declines in welfare, it is likely that the reservation prices for unavailable products will have to be much greater than the corresponding prices in the previous period. This approach allows us to identify biases from the approaches currently being recommended by international agencies.

We also provide a broader review of the price and quantity indexes that statistical agencies are likely to produce during lockdown conditions. The options open to an NSO will depend on its access to current household expenditure data.

Our conclusions for constructing the most informative prices indexes in this context can be summarised as follows:

1. In the short run, collect whatever prices are available and supplement these from scanner data and web scraped prices to make up for missing prices due to changes in price collection methodology.[3] For prices which are still missing, use inflation adjusted carry forward prices, consistent with current advice from the international agencies.
2. At the same time, put in motion some method for getting *current expenditure weights* for the consumption basket. This would require either a continuous

[2] For related materials on modeling new and disappearing goods bias, see Hausman (1996) (1999) (2003), de Haan and Krsinich (2012) (2014) and Diewert, Fox and Schreyer (2017).

[3] Some NSOs already make extensive use of scanner data from retail chains and are relatively well placed in this regard; see e.g. Australian Bureau of Statistics (2017). Problems remain, however, for broader categories of goods and services.

consumer expenditure survey or the use of new sources of data. These new sources could include credit card companies and companies that produce household expenditure data from households scanning barcodes of purchased items ("Homescan" data).

3. Once the new consumer expenditure information becomes available, produce a new analytic CPI. This would be revisable while the new methodology was developed further. This would supplement the existing CPI, which would likely be heavily compromised due to the treatment of missing prices and use of out-of-date expenditure weights.

For what follows, it is important to explain why the Fisher price index is preferred to the Laspeyres or Paasche price indexes. The Laspeyres price index prices out the basket of goods and services that is consumed in the *base period* at base period prices and at current period prices. The Laspeyres price index P_L is the ratio of these two costs using the prices of the current period in the numerator and the prices of the base period in the denominator. The Paasche price index prices out the basket of goods and services that is consumed in the *current period* at base period prices and at current period prices. The Paasche price index P_P is the ratio of these two costs using the prices of the current period in the numerator and the prices of the base period in the denominator. Both indexes are equally plausible and intuitively easy to understand. But the Laspeyres index will tend to show a greater increase in prices than the corresponding Paasche index.

Good statistical practice suggests that when one has two equally plausible measures of the same phenomenon, it is better to take an average of the two measures to give a single measure of the phenomenon. The geometric average of the Laspeyres and Paasche price indexes is a useful average of P_L and P_P which is equal to the Fisher price index. The Fisher index satisfies the important time reversal test (and many other tests) so that inflation measured going forward is the reciprocal of inflation going backwards. The good test performance of the Fisher index[4] explains why taking the geometric average of P_L and P_P is preferred rather than taking some other form of average of P_L and P_P. Thus

[4] See Diewert (1992) for a listing of the tests that the Fisher price index satisfies.

the paper will focus on how to measure *lockdown bias* which we define to be the difference between a Laspeyres or Lowe (or fixed basket) CPI and the Fisher price index.

The paper is organised as follows. In section 2, we study how the sudden unavailability of many goods and services affects the measurement of real consumption; that is, *quantity indexes* for household consumption. It may seem odd that in a paper that is focussed on potential bias in a CPI that we first look at the problems associated with the measurement of real consumption (and by extension, with the measurement of real GDP). However, construction of a consumer price index goes hand in hand with the construction of a corresponding measure of real consumption. In the end, economists and governments are concerned with the *welfare implications* of the pandemic on real consumption. It proves to be convenient to look at the problems associated with the measurement of real consumption during lockdown conditions before we look at the associated CPI measurement problems. In sections 3 and 4, our focus shifts to consumer price indexes. Section 3 looks at comparisons between the Laspeyres, Paasche and Fisher price indexes under pandemic conditions while section 4 examines the properties of a fixed basket or Lowe price index, which is used in construction of the CPI in many countries. Section 5 looks at the advantages and disadvantages of using various "practical" price and quantity indexes that statistical agencies are likely to produce during lockdown conditions. We note that the way forward will depend on what types of data are available to the NSO.

Section 6 looks at the problem of a lack of matching product prices at the elementary index level. Seven possible methods for dealing with this problem are discussed, depending on the availability of data. Section 7 takes a brief look at other practical measurement problems that an NSO may encounter when it attempts to produce a meaningful CPI under pandemic conditions. Section 8 concludes.

Appendix A provides a brief survey of the methods suggested by various statistical agencies. Appendix B specializes the algebra explained for the many commodity case in the main text to the case of only two commodities. The advantage of this simplification is that the various concepts can be illustrated in a simple diagram. Appendix C provides some theoretical guidance on how to construct generalized reservation prices when some market consumer goods and services are provided by the government or businesses to the household at zero cost (or at a highly subsidized prices) as responses to the pandemic. For example, how should a temporary rent holiday be treated in a CPI? The reservation price for the household in this situation is equal to its willingness to pay for this good or service and the Appendix indicates how in theory this price could be estimated using econometric techniques.

2. Measuring Real Consumption when Transitioning to a Lockdown Economy

Suppose that the period prior to the lockdown is period 0 and the first lockdown period is the subsequent period 1. The period of time could be a month or a quarter. We initially assume that data on prices and quantities that are in scope for the CPI are available to the statistical agency. We divide the CPI goods and services into two groups: Group 1 prices and quantities are available in periods 0 and 1 and Group 2 prices and quantities are only available in period 0.[5] Denote the period t price and quantity vectors for Group 1 products by $p^t \equiv [p_1{}^t,...,p_M{}^t] >> 0_M$ and $q^t \equiv [q_1{}^t,...,q_M{}^t] >> 0_M$ for t = 0,1.[6] The Group 2 price and quantity vectors for period 0 are $P^0 \equiv [P_1{}^0,...,P_N{}^0] >> 0_N$ and $Q^0 \equiv [Q_1{}^0,...,Q_N{}^0] >> 0_N$. We take the Group 2 quantity vector for period 1 to be a vector of zero components; i.e., we assume that:

$$(1)\ Q^1 \equiv 0_N.$$

It is not clear how to define the period 1 price vector P^1 for the products that are not available in period 1. In these notes, we will apply the *economic approach to index number theory* and assume that the appropriate definition for the missing prices are the

[5] The methodology to be developed below can be applied to a subset of the CPI; i.e., to an elementary category. However, the category has to be broad enough so that it contains some continuing commodities or products and some commodities that have disappeared due to the lockdown. The algebra that follows assumes that information on (unit value) prices and the corresponding quantities are available for the commodities in scope during period 0.

[6] Notation: $p^t >> 0_M$ ($\geq 0_M$) means that all components of the M dimensional vector p^t are positive (nonnegative). The inner product of the vectors p^t and q^t is defined as $p^t \cdot q^t \equiv \Sigma_{m=1}{}^M\, p_m{}^t q_m{}^t$.

relevant *Hicksian reservation prices*; i.e., $P^{1*} \equiv [P_1^{1*},...,P_N^{1*}] >> 0_N$ where $P_n^{1*} > 0$ is the price for commodity n that will cause the corresponding period 1 consumer demand Q_n^1 to equal 0 for that commodity for n = 1,...,N.

In most cases, it is safe to assume that the reservation price for product n in period 1, P_n^{1*}, will be considerably larger than the actual price for product n in period 0, P_n^0; i.e., it is reasonable to assume that:[7]

(2) $P^{1*} >> P^0$.

It is important to understand why the reservation price for a product that is suddenly unavailable is likely to be greater than the prevailing price in the prior period when it was available. In the most extreme form of lockdown, individuals are confined to their houses or apartments. If they live in a small apartment, the resulting confinement is very similar to being in prison. Individuals are typically willing to pay large sums of money to avoid going to prison (think of high lawyer fees). Going to jail or being subject to a strict lockdown decreases welfare to a very substantial degree. A way of measuring the decrease in welfare is to use the reservation price concept. In order to get a substantial drop in welfare due to a lockdown, the reservation prices for unavailable goods and services will have to be much greater than the corresponding prices in the previous period.

Before we look at the implications of our assumptions for the construction of a Consumer Price index going from period 0 to 1, we look at the implications for the measurement of real consumption. The *"true" overall Laspeyres quantity index*, Q_L, is defined as follows:[8]

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[7] In Appendix B, we assume M = 1 and N = 1. Using the economic approach to index number theory, it is possible to show that it must be the case that $P^{1*} \geq (p^1/p^0)P^0$ where p^0 and P^0 are the observed period 0 prices for the two commodities, p^1 is the period 1 price for the continuing commodity and P^{1*} is the period 1 reservation price vector for the unavailable commodity. P^{1*} will equal $(p^1/p^0)P^0$ only if the two commodities are perfect substitutes. Note that $(p^1/p^0)P^0$ is the inflation adjusted carry forward price vector for the unavailable commodity. See Appendix C for information on how to define reservation prices.

[8] Note that the reservation prices P^{1*} do not enter the definition for Q_L.

$$(3)\ Q_L \equiv [p^0\cdot q^1 + P^0\cdot Q^1]/[p^0\cdot q^0 + P^0\cdot Q^0]$$
$$= p^0\cdot q^1/[p^0\cdot q^0 + P^0\cdot Q^0] \qquad \text{using assumption (1)}$$
$$= [p^0\cdot q^1/p^0\cdot q^0]\{p^0\cdot q^0/[p^0\cdot q^0 + P^0\cdot Q^0]\}$$
$$= Q_{Lq}s_q^0$$

where the *Laspeyres quantity index for always present commodities* Q_{Lq} is defined as

$$(4)\ Q_{Lq} \equiv p^0\cdot q^1/p^0\cdot q^0.$$

The *period 0 expenditure share of always present commodities* is s_q^0 defined as

$$(5)\ s_q^0 \equiv p^0\cdot q^0/[p^0\cdot q^0 + P^0\cdot Q^0].$$

Thus overall Laspeyres real consumption growth, Q_L, is equal to Laspeyres consumption growth of always available commodities Q_{Lq} times the share of always available commodities in period 0, s_q^0. Note that Q_L, Q_{Lq} and s_q^0 depend on observable data and, in principle, can be constructed by the statistical agency. This is not the case for the Paasche quantity index.

The *"true" overall Paasche quantity index*, Q_P^*, is defined as follows:[9]

$$(6)\ Q_P^* \equiv [p^1\cdot q^1 + P^{1*}\cdot Q^1]/[p^1\cdot q^0 + P^{1*}\cdot Q^0]$$
$$= p^1\cdot q^1/[p^1\cdot q^0 + P^{1*}\cdot Q^0] \qquad \text{using assumption (1)}$$
$$= [p^1\cdot q^1/p^0\cdot q^0]p^0\cdot q^0/[(p^1\cdot q^0/p^0\cdot q^0)p^0\cdot q^0 + (P^{1*}\cdot Q^0/P^0\cdot Q^0)P^0\cdot Q^0]$$
$$= [p^1\cdot q^1/p^0\cdot q^0]s_q^0/[P_{Lq}s_q^0 + P_{LQ}^*s_Q^0]$$
$$= P_{Lq}Q_{Pq}s_q^0/[P_{Lq}s_q^0 + P_{LQ}^*s_Q^0] \qquad \text{since } p^1\cdot q^1/p^0\cdot q^0 = P_{Lq}Q_{Pq}$$

[9] Note that the reservation prices P^{1*} do enter the definition for Q_P: the bigger is the gap between $P^{1*}\cdot Q^0$ and $P^0\cdot Q^0$, the smaller Q_P^* becomes. Thus in order to compute real consumption (using either the test or economic approaches to index number theory), it is absolutely necessary to compute the Paasche quantity index (or an approximation to it) in addition to the Laspeyres quantity index which misses the effect of higher reservation prices in period 1. The Fisher quantity index will usually provide an adequate approximation to the change in welfare due to the lockdown; see Diewert (1976) (2020a).

$= Q_{Pq}s_q{}^0P_{Lq}/[P_{Lq}s_q{}^0 + P_{LQ}{}^*s_Q{}^0]$

where the *Paasche quantity index for always available commodities* Q_{Pq} is defined as

(7) $Q_{Pq} \equiv p^1\cdot q^1/p^1\cdot q^0$.

The *Laspeyres price indexes for the always available and unavailable commodities,* P_{Lq} and $P_{LQ}{}^*$ respectively, are defined as follows:

(8) $P_{Lq} \equiv p^1\cdot q^0/p^0\cdot q^0$;
(9) $P_{LQ}{}^* \equiv P^{1*}\cdot Q^0/P^0\cdot Q^0$.

The *period 0 expenditure shares of the available and unavailable commodities* are $s_q{}^0$ and $s_Q{}^0$ defined as

(10) $s_q{}^0 \equiv p^0\cdot q^0/[p^0\cdot q^0 + P^0\cdot Q^0]$;
(11) $s_Q{}^0 \equiv P^0\cdot Q^0/[p^0\cdot q^0 + P^0\cdot Q^0]$.

Note that definition (9) for the Laspeyres index for the unavailable commodities, $P_{LQ}{}^* \equiv P^{1*}\cdot Q^0/P^0\cdot Q^0$, depends on the vector of period 1 reservation prices P^{1*} for these products.[10] Thus formula (6) for the true overall Paasche quantity index, $Q_P{}^* = Q_{Pq}s_q{}^0P_{Lq}/[P_{Lq}s_q{}^0 + P_{LQ}{}^*s_Q{}^0]$ would seem to be of only theoretical interest. However, it is possible to provide some very rough estimates for P^{1*}.

Suppose that the Laspeyres inflation for unavailable products was equal to Laspeyres inflation for the continuing products; i.e., suppose that $P^{1*}\cdot Q^0/P^0\cdot Q^0 = p^1\cdot q^0/p^0\cdot q^0$. This means that the reservation prices P^{1*} are such that:

[10] The fact that the true Laspeyres price index $P_{LQ}{}^*$ for unavailable commodities has an asterisk superscript is meant to alert the reader that the index depends on the reservation prices for the unavailable products which are not directly observable. Since the true overall Paasche quantity index, $Q_P{}^*$, depends on $P_{LQ}{}^*$, we have added an asterisk superscript to Q_P as well.

(12) $P_{LQ}^{*} = P_{Lq}$.

Assumption (12) sets Laspeyres type inflation P_{LQ}^{*} for the unavailable goods and services equal to the actual Laspeyres inflation for the always available products P_{Lq}. This is consistent with a form of *inflation adjusted carry forward pricing* for unavailable products; i.e., if we set $P^{1*} = P_{Lq}P^{0} = (p^{1}\cdot q^{0}/p^{0}\cdot q^{0})P^{0}$, then assumption (12) will be satisfied.[11] Under this assumption, define the corresponding overall Approximate Paasche quantity index, Q_{P}^{A}, as follows:[12]

$$
\begin{aligned}
(13)\ Q_{P}^{A} &\equiv Q_{Pq}s_{q}^{0}P_{Lq}/[P_{Lq}s_{q}^{0} + P_{Lq}s_{Q}^{0}] && \text{using definition (6) and assumption (12)} \\
&= Q_{Pq}s_{q}^{0} \\
&> Q_{Pq}s_{q}^{0}P_{Lq}/[P_{Lq}s_{q}^{0} + P_{LQ}^{*}s_{Q}^{0}] && \text{if } P_{LQ}^{*} > P_{Lq} \\
&= Q_{P}^{*}.
\end{aligned}
$$

since the period 0 expenditure shares sum to 1; i.e., we have $s_{q}^{0} + s_{Q}^{0} = 1$.

The relationships in (13) show that the practical approximation to the true Paasche quantity index defined by Q_{P}^{A} is equal to the Paasche quantity index for continuing commodities, Q_{Pq}, times the expenditure share of continuing commodities in the base period s_{q}^{0}. The implication of the equality in (13) is that the practical Paasche index Q_{P}^{A} has an *upward bias* relative to the true Paasche quantity index Q_{P}^{*} provided that the true Laspeyres price index for unavailable commodities P_{LQ}^{*} is greater than the Laspeyres price index for continuing commodities P_{Lq}.[13] For many countries experiencing massive shutdowns of various industries, it is almost certain that P_{LQ}^{*} is very much bigger than P_{Lq} so that the upward bias in Q_{P}^{A} relative to Q_{P} is likely to be very large indeed.

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[11] From Appendix B, it can be seen that in general, these inflation adjusted carry forward reservation prices, $P_{Lq}P^{0}$, will be too low from the perspective of the economic approach to index number theory.

[12] The superscript A was added to Q_{P} to indicate that the resulting overall Paasche quantity index is only an approximation to the true Paasche quantity index, Q_{P}. However, Q_{P}^{A} does not depend on the unobservable reservation prices P^{1*} and so Q_{P}^{A} is a practical approximation to Q_{P}.

[13] Alternatively, $Q_{P}^{A} > Q_{P}^{*}$ if the true reservation prices $P^{1*} > P_{Lq}P^{0}$ where $P_{Lq}P^{0} = (p^{1}\cdot q^{0}/p^{0}\cdot q^{0})P^{0}$ are the inflation adjusted carry forward prices for unavailable commodities in period 1.

Note the similarity of expressions (3) and (13) for the Laspeyres and approximate Paasche quantity indexes. Under assumption (12), we obtain the following simple formula for the *Approximate Fisher quantity index*, Q_F^A:

$$(14)\ Q_F^A \equiv [Q_L Q_P^A]^{1/2}$$
$$= [Q_{Lq} s_q^0 Q_{Pq} s_q^0]^{1/2} \qquad \text{using (13)}$$
$$= s_q^0 [Q_{Lq} Q_{Pq}]^{1/2}$$
$$= s_q^0 Q_{Fq}$$

where $Q_{Fq} \equiv [Q_{Lq} Q_{Pq}]^{1/2}$ is the Fisher quantity index defined over only available commodities where Q_{Lq} is defined by (4) and Q_{Pq} is defined by (7). Thus under assumption (12), real consumption going from period 0 to 1 can be measured by the Fisher quantity index Q_{Fq} defined over only always available commodities times the expenditure share on always available commodities in period 0.[14]

However, as indicated above, it is very likely that the reservation prices P^{1*} are very much higher than their period 0 counterpart prices P^0 and hence the implicit inflation for the unavailable commodities will likely be much greater than the observed inflation in the always available commodities. As was the case for the inequality in (13), we make the following assumption:

$$(15)\ P_{LQ}^* > P_{Lq}.$$

Using assumption (15) and the inequality in (13) ($Q_P^A > Q_P^*$), we see that the true overall Paasche quantity index, Q_P^*, (which may be difficult to accurately calculate due to a lack of information on the reservation prices of the unavailable products P^{1*}) will be less than the share adjusted Paasche quantity index calculated using only the always available commodities, $Q_{Pq} s_q^0$ (which can be calculated). Thus under the reasonable assumption (15), we have the following relationships between the true overall Laspeyres, Paasche

[14] This result is similar to the inflation adjusted carry forward price methodology explained in Triplett (2004), de Haan and Krsinich (2012) (2014) and Diewert, Fox and Schreyer (2017).

and Fisher measures of real consumption, Q_L, Q_P^* and $Q_F^* \equiv [Q_L Q_P^*]^{1/2}$, and their counterpart subindexes that use only information on the prices and quantities of available products, Q_{Lq}, Q_{Pq} and Q_{Fq}:

(16) $Q_L = Q_{Lq} s_q^0$;
(17) $Q_P^* < Q_{Pq} s_q^0$;
(18) $Q_F^* < Q_{Fq} s_q^0$.

Thus if assumption (15) holds, the true overall Paasche and Fisher quantity indexes, Q_P^* and Q_F^*, will be *less* than their share adjusted counterpart indexes, $Q_{Pq} s_q^0$ and $Q_{Fq} s_q^0$, which do not require information on reservation prices. The size of the gaps in the inequalities (17) and (18) will grow as the size of the gap in the inequality (15) grows.[15] It is very likely that the gaps in the inequalities (15), (17) and (18) are substantial for many countries that have implemented significant lockdowns of economic activity.[16]

At the time of writing, it has become clear that the COVID-19 virus has changed the preferences of many households. Thus households are reluctant to shop for goods and services in person and they are reluctant to offer labour services at unsafe workplaces. This fact affects the above analysis in at least two ways:

- Government mandated shutdowns of many industries is only part of the lockdown story; many business establishments will shut down due to safety concerns for both their customers and workers. Thus the lockdown effects on output and consumption are bigger than just government mandated shutdowns of business and household activities. Thus the products that correspond to the vector Q include not only products that have disappeared due to government orders but they also include products that households no longer wish to purchase due to safety concerns.

[15] Of course, the size of the gap in (17) will always be larger than the size of the gap in (18).
[16] See Appendix B for some rough and ready methodology which will enable the reader to form an approximation to the gap in (18).

- The reservation prices which appear in the above algebra (and in Appendix B) are reservation prices which are based on the preferences that prevailed before the lockdowns took place. For the post lockdown preferences, the reservation prices for the products in the Q vector are essentially infinite. Basically, changing preferences mean that a new CPI series along with a new real consumption series needs to be constructed for the lockdown period.[17]

Once the economy has stabilized under lockdown conditions, it is appropriate to use chained Fisher indexes using only commodities that are available under shutdown conditions.[18] If the degree of shutdown increases from month to month, the analysis in the present section can again be applied to the smaller set of available commodities as we go from month to month. However, when the lockdown ends in period $\tau > 1$, instead of linking period τ to period $\tau - 1$, linking period τ to period 0 will minimize any potential chain drift problems.[19]

3. Constructing a Cost of Living Index when Transitioning to a Lockdown Economy

The *true overall Paasche price index*, P_P, is defined as follows:[20]

$$(19)\ P_P \equiv [p^1 \cdot q^1 + P^{1*} \cdot Q^1]/[p^0 \cdot q^1 + P^0 \cdot Q^1]$$

$$= p^1 \cdot q^1/p^0 \cdot q^1 \qquad \text{using assumption (1), } Q^1 = 0_N$$

$$\equiv P_{Pq}$$

[17] Suppose household preferences in the pre-lockdown period can be represented by the utility function, $F(f^1(q),f^2(Q))$ where all three functions are increasing, linearly homogeneous and concave functions. Then using pre-lockdown preferences, real consumption growth going from period 0 to 1 is equal to $F(f^1(q^1),f^2(0_N))/F(f^1(q^0),f^2(Q^0))$. Post lockdown preferences set $Q = 0_N$ and using these preferences, real consumption growth going from period 0 to 1 can be set equal to $F(f^1(q^1),f^2(0_N))/F(f^1(q^0),f^2(0_N))$ or more simply to $f^1(q^1)/f^1(q^0)$. Thus these two real consumption series going from period 0 to 1 are not really comparable. If period t is a lockdown period, then real consumption growth relative to period 0 can be represented by $f^1(q^t)/f^1(q^0)$. If a subsequent period τ is a "back to normal" period, then we can measure consumption growth relative to period 0 using period 0 preferences as $F(f^1(q^\tau),f^2(Q^\tau))/F(f^1(q^0),f^2(Q^0))$.

[18] We will discuss this recommendation in more detail in section 6 below.

[19] The basic idea here is that the quantity vector $[q^\tau,Q^\tau]$ is likely to be more similar to $[q^0,Q^0]$ than to the quantity vectors that pertain to the intervening lockdown periods.

[20] Note that the true Paasche index P_P does not depend on the vector of period 1 reservation prices P^{1*}. Thus P_P is likely to be very much lower than the true Laspeyres index P_L^* to be defined shortly.

where P_{Pq} is the Paasche price index that uses only the price and quantity data pertaining to the always available products in the two periods being compared. The above equality tells us that the restricted domain Paasche price index, P_{Pq}, is equal to the true overall Paasche price index, P_P. Thus the overall Paasche price index can be constructed without the use of imputed price data for period 1 prices, P^{1*}.

The *overall true Laspeyres price index* P_L^* is defined as follows:[21]

$$(20)\ P_L^* \equiv [p^1\cdot q^0 + P^{1*}\cdot Q^0]/[p^0\cdot q^0 + P^0\cdot Q^0]$$
$$= [(p^1\cdot q^0/p^0\cdot q^0)p^0\cdot q^0 + (P^{1*}\cdot Q^0/P^0\cdot Q^0)P^0\cdot Q^0]/[p^0\cdot q^0 + P^0\cdot Q^0]$$
$$= P_{Lq}s_q^0 + P_{LQ}^*s_Q^0$$
$$= P_{Lq}\{s_q^0 + [(P^{1*}\cdot Q^0/P^0\cdot Q^0)/(p^1\cdot q^0/p^0\cdot q^0)]s_Q^0\}$$

where the period 0 expenditure shares s_q^0 and s_Q^0 were defined by (10) and (11), the Laspeyres price index for continuing products P_{Lq} was defined by (8) and the Laspeyres price index for unavailable products P_{LQ}^* was defined by (9).[22] The problem with definition (20) is that the vector of period 1 reservation prices P^{1*} is not directly observable and hence the overall Laspeyres index P_L^* and the "true" Laspeyres index for unavailable commodities in period 1 P_{LQ}^* cannot be readily calculated.

When a commodity is temporarily out of stock in a retail outlet, many statistical agencies simply *carry forward* the observed (unit value) price for the product from the previous period and use this carry forward price in place of the Hicksian reservation prices that were used above in our index number calculations. Thus define the overall *Laspeyres price index* P_L^C *using* (*inflation unadjusted*) *carry forward prices* P^0 in place of the reservation prices P^{1*} as follows:

$$(21)\ P_L^C \equiv [p^1\cdot q^0 + P^0\cdot Q^0]/[p^0\cdot q^0 + P^0\cdot Q^0]$$

[21] Again we add a superscript asterisk to P_L to indicate that the true overall Laspeyres price index requires a knowledge of the period 1 reservation prices for unavailable commodities, P^{1*}.

[22] Compare (20) with the simpler expression defined by (B35) in Appendix B.

$= [(p^1 \cdot q^0/p^0 \cdot q^0)p^0 \cdot q^0 + P^0 \cdot Q^0]/[p^0 \cdot q^0 + P^0 \cdot Q^0]$

$= P_{Lq}s_q{}^0 + s_Q{}^0$

where the period 0 expenditure shares $s_q{}^0$ and $s_Q{}^0$ were defined by (10) and (11) and the Laspeyres price index for continuing products P_{Lq} was defined by (8).

Using expressions (20) and (21), it can be seen that the following inequality holds between the simple unadjusted carry forward Laspeyres index, $P_L{}^C$, and the true Laspeyres index, $P_L{}^*$:

(22) $P_L{}^* > P_L{}^C$ if and only if $P_{LQ}{}^* > 1$.

As in the previous section, it is reasonable to assume that reservation prices P^{1*} are considerably larger than prices P^0 that prevailed prior to lockdown conditions. Hence we can safely assume that the true Laspeyres subindex for unavailable commodities, $P_{LQ}{}^* \equiv P^{1*} \cdot Q^0/P^0 \cdot Q^0$, is greater than 1 and thus $P_L{}^C$ will have a *substantial downward bias* compared to the true Laspeyres price index $P_L{}^*$ that uses reservation prices.

Following Triplett (2004) and many other authors,[23] statistical offices often multiply the prior period prices that are not available in the present period by the price inflation of related commodities. We will use either the Laspeyres or Paasche price index for continuing commodities to do this indexation and compare the resulting overall Laspeyres price indexes to $P_L{}^C$.

Define the *inflation adjusted carry forward prices* P^{1L} using the Laspeyres price index for continuing commodities, P_{Lq}, as the inflation adjusting index as follows:

(23) $P^{1L} \equiv P_{Lq}P^0 = (p^1 \cdot q^0/p^0 \cdot q^0)P^0$.

[23] See for example de Haan and Krsinich (2012) (2014), Diewert, Fox and Schreyer (2017) or Diewert (2020c). This approach is implicit in the use of equation (12) in the previous section; see equation (12).

Define the Laspeyres index using the inflation adjusted carry forward prices P^{1L} in place of the true prices P^{1*} as P_L^{CL}:

$$
\begin{aligned}
(24)\ P_L^{CL} &\equiv [p^1\cdot q^0 + P^{1L}\cdot Q^0]/[p^0\cdot q^0 + P^0\cdot Q^0] \\
&= [p^1\cdot q^0 + (p^1\cdot q^0/p^0\cdot q^0)P^0\cdot Q^0]/[p^0\cdot q^0 + P^0\cdot Q^0] && \text{using (23)} \\
&= [(p^1\cdot q^0/p^0\cdot q^0)][s_q^0 + s_Q^0] && \text{using definitions (10) and (11)} \\
&= P_{Lq}[s_q^0 + s_Q^0] && \text{using definitions (8) and (9)} \\
&= P_{Lq}.
\end{aligned}
$$

Thus the use of P_{Lq} as the indexing index in our inflation adjusted carry forward prices leads to a Laspeyres type index, P_L^{CL}, that turns out to equal P_{Lq}, the Laspeyres price index for continuing goods and services. This is a useful result since P_{Lq} is a “real” index with no imputations whereas P_L^{CL} is constructed using a great many imputations if N is large. Note that both P_L^{C} and P_L^{CL} can be constructed using only knowledge of p^0, p^1, P^0, q^0, q^1 and Q^0.

Comparing (21) with (24) leads to the following inequality:

$$(25)\ P_L^{CL} > P_L^{C} \text{ if and only if } P_{Lq} > 1.$$

Instead of using the Laspeyres price index for continuing commodities, we could use the Paasche index for continuing commodities as the indexing index in the definition of the inflation adjusted carry forward prices for the unavailable commodities. Define the *inflation adjusted carry forward prices* P^{1P} using the Paasche price index for continuing commodities, P_{Pq}, as the inflation adjusting index as follows:

$$(26)\ P^{1P} \equiv P_{Pq}P^0 = (p^1\cdot q^1/p^0\cdot q^1)P^0.$$

Define the Laspeyres index using the inflation adjusted carry forward prices P^{1P} in place of the true prices P^{1*} as P_L^{CP}:

$$(27)\ P_L^{CP} \equiv [p^1 \cdot q^0 + P^{1P} \cdot Q^0]/[p^0 \cdot q^0 + P^0 \cdot Q^0]$$
$$= [p^1 \cdot q^0 + (p^1 \cdot q^1/p^0 \cdot q^1)P^0 \cdot Q^0]/[p^0 \cdot q^0 + P^0 \cdot Q^0] \quad \text{using (26)}$$
$$= [(p^1 \cdot q^0/p^0 \cdot q^0)p^0 \cdot q^0 + (p^1 \cdot q^1/p^0 \cdot q^1)P^0 \cdot Q^0]/[p^0 \cdot q^0 + P^0 \cdot Q^0]$$
$$= (p^1 \cdot q^0/p^0 \cdot q^0)s_q^0 + (p^1 \cdot q^1/p^0 \cdot q^1)s_Q^0 \quad \text{using definitions (10) and (11)}$$
$$= P_{Lq}s_q^0 + P_{Pq}s_Q^0 \quad \text{using definitions (8)}$$
$$< P_L^{CL} \quad \text{if and only if } P_{Lq} > P_{Pq}.$$

Thus the use of P_{Pq} as the inflation index in our inflation adjusted carry forward prices leads to a price index, P_L^{CP}, that turns out to equal a share weighted average of P_{Lq} and P_{Pq}, the Laspeyres and Paasche price indexes for continuing goods and services. Since P_{Pq} is likely to be less than its Laspeyres counterpart, P_{Lq}, it is likely that P_L^{CP} will be less than P_L^{CL}. Note that the scope of P_L^{CL} and P_L^{CP} is the set of all M+N commodities and N imputations are required to construct these indexes, whereas P_{Lq} and P_{Pq} are “real” indexes with no imputations. A knowledge of q^1 is required to construct P_L^{CP} whereas no knowledge of q^1 was required to construct P_L^{CL}.

Using (20) and (24), it is easy to show that $P_L^* > P_L^{CL}$ if and only if $P_{LQ}^* > P_{Lq}$. Using (20) and (27), it is easy to show that $P_L^* > P_L^{CP}$ if and only if $P_{LQ}^* > P_{Pq}$. The bottom line is that all three practical indexes that use some form of carry forward pricing for missing products, P_L^C, P_L^{CL} and P_L^{CP}, *will have substantial downward biases* relative to the true Laspeyres index, P_L^*.

It is too early to say whether a given country will have rising or falling prices for continuing products as lockdowns take place. Demand for available commodities may fall due to declining household incomes as industries shut down and this may lead to lower prices for continuing products. On the other hand, there may be supply shortages for some highly demanded consumer products like toilet paper, face masks and hand sanitizers which will lead to higher prices for these products. Workers in supermarkets that are open may demand higher wages to compensate them for increased risk of infection which will lead to higher grocery prices in general. Truckers will also be subject to increased risk of infection and may get higher wages a result and this will lead to higher prices for available products. Meat processing plants may shut down due to virus spread which again could lead to higher prices. Also, there will be rapid growth of home deliveries for available products which will lead to higher prices for products ordered online since transport costs must be included for the home delivered goods. A shortage of delivery trucks and drivers may drive up transportation costs. Border shutdowns may restrict imports of food and medical products, again leading to higher prices for continuing products. Thus it will be important for statistical agencies to produce inflation estimates for continuing commodities and this can be accomplished if the agency produces the Laspeyres index for continuing products, P_{Lq}. If the agency is able to obtain approximate expenditure data for current period values for continuing products $p^1 \cdot q^1$, then it would be desirable to produce the Paasche index for continuing product, P_{Pq}, as well so that the Fisher index for continuing products, $P_{Fq} = [P_{Lq}P_{Pq}]^{1/2}$, could also be produced.

4. Constructing a Lowe Index when Transitioning to a Lockdown Economy

The analysis in the previous section can be adapted to study the behavior of *fixed basket Lowe price indexes* when a country closes down industries. This type of index is of interest because it is used in the construction of the CPI in most countries.

Let $q^b \equiv [q_1{}^b,...,q_M{}^b] >> 0_M$ and $Q^b \equiv [Q_1{}^b,...,Q_N{}^b] >> 0_N$ be a vector of "representative" commodities that households are purchasing in period 0 and prior periods. The *fixed basket Lowe price index* going from period 0 to 1, P_B, is defined as follows:

$$(28)\ P_B{}^* \equiv [p^1 \cdot q^b + P^{1*} \cdot Q^b]/[p^0 \cdot q^b + P^0 \cdot Q^b]$$
$$= [(p^1 \cdot q^b/p^0 \cdot q^b)p^0 \cdot q^b + (P^{1*} \cdot Q^b/P^0 \cdot Q^b)P^0 \cdot Q^b]/[p^0 \cdot q^b + P^0 \cdot Q^b]$$
$$= P_{Bq}s_q{}^b + P_{BQ}{}^*s_Q{}^b$$

where the *fixed basket subindexes* for continuing commodities and unavailable commodities, P_{Bq} and P_{BQ} are defined as follows:

(29) $P_{Bq} \equiv p^1 \cdot q^b / p^0 \cdot q^b$;

(30) $P_{BQ}^* \equiv P^{1*} \cdot Q^b / P^0 \cdot Q^b$.

The *base period hybrid shares* (prices of period 0 but quantities for a prior year b) for the continuing and disappearing commodity groups, s_q^b and s_Q^b, are defined as follows:

(31) $s_q^b \equiv p^0 \cdot q^b / [p^0 \cdot q^b + P^0 \cdot Q^b]$;

(32) $s_Q^b \equiv P^0 \cdot Q^b / [p^0 \cdot q^b + P^0 \cdot Q^b]$.

In definitions (28) and (30), we have used reservation prices to value goods and services that are no longer available. It makes sense to use reservation prices to value unavailable products in the context of the economic approach to index number theory but it is not clear that it is appropriate to use them to value the fixed basket Q^b of unavailable products in period 1. However, the logic of the fixed basket approach works as follows: there is a representative basket of commodities that households are consuming in periods prior to period 1, (q^b,Q^b). The fixed basket methodology simply prices out this basket of goods and services at the prevailing market prices of periods 0 and 1 and the ratio of these costs becomes the fixed basket price index. When goods and services are unavailable in period 1, it is reasonable to use reservation prices for these absent market prices since they are (imputed) *market clearing prices* that will ration demand down to zero for the absent commodities in period 1.

As in the previous section, instead of using reservation prices, the prices P^{1*} could be set equal to the base period prices, giving rise to the following *carry forward basket price index*, P_B^C, defined as follows:

(33) $P_B^C \equiv [p^1 \cdot q^b + P^0 \cdot Q^b]/[p^0 \cdot q^b + P^0 \cdot Q^b]$

$= [(p^1 \cdot q^b / p^0 \cdot q^b) p^0 \cdot q^b + (P^0 \cdot Q^b / P^0 \cdot Q^b) P^0 \cdot Q^b]/[p^0 \cdot q^b + P^0 \cdot Q^b]$

$= P_{Bq} s_q^b + s_Q^b$

$< P_{Bq} s_q^b + P_{BQ}^* s_Q^b$ assuming that $P_{BQ}^* \equiv P^{1*} \cdot Q^b / P^0 \cdot Q^b > 1$

$= P_B^*$ using definition (28).

Thus the carry forward fixed basket index will have a downward bias if $P^{1*}\cdot Q^b > P^0\cdot Q^b$, which is a reasonable assumption.

An alternative to using carry forward prices is to use the price index for continuing commodities, P_{Bq} defined by (29) to form the *inflation adjusted fixed basket carry forward price vector* $P^{1I} \equiv P_{Bq}P^0 = (p^1\cdot q^b/p^0\cdot q^b)P^0$ as an estimate for P^{1*}. This leads to the following *inflation adjusted fixed basket price index*, P_B^{CI}, defined as follows:

(34) $P_B^{CI} \equiv [p^1\cdot q^b + P^{1I}\cdot Q^b]/[p^0\cdot q^b + P^0\cdot Q^b]$

$= [p^1\cdot q^b + P_{Bq}P^0\cdot Q^b]/[p^0\cdot q^b + P^0\cdot Q^b]$

$= [(p^1\cdot q^b/p^0\cdot q^b)p^0\cdot q^b + P_{Bq}(P^0\cdot Q^b/P^0\cdot Q^b)P^0\cdot Q^b]/[p^0\cdot q^b + P^0\cdot Q^b]$

$= P_{Bq}s_q^b + P_{Bq}s_Q^b$

$= P_{Bq}$ since $s_q^b + s_Q^b = 1$

$< P_{Bq}s_q^b + P_{BQ}^* s_Q^b$ assuming that $P_{BQ}^* > P_{Bq}$

$= P_B^*$ using definition (28).

Again, it is reasonable to assume that $P_{BQ}^* > P_{Bq}$; i.e., that (imputed) inflation for unavailable commodities is greater than observed inflation for continuing commodities. Thus the version of the fixed basket index that makes use of inflation adjusted carry forward prices for the unavailable commodities, P_B^{CI}, will be less than the “true” fixed basket price index P_B^*.

As was the case in the previous section, we could try to determine which practical fixed basket price index, P_B^C or P_B^{CI}, is greater because bias will be minimized if we choose the maximum of these two indexes. We have:

(35) $P_B^C/P_B^{CI} = [P_{Bq}s_q^b + s_Q^b]/P_{Bq}$ using definitions (33) and (34)

$= s_q^b + [s_Q^b/P_{Bq}]$.

If $P_{Bq} > 1$, so that fixed basket inflation for continuing commodities is positive, then $P_B^{CI} > P_B^{C}$ and it is preferable (from the viewpoint of minimizing bias) to use the inflation adjusted carry forward fixed basket price index P_B^{CI}. On the other hand, if $P_{Bq} < 1$, so that there is deflation for continuing products, then it would be preferable to use the carry forward fixed basket price index P_B^{C}.

The above discussion of the fixed basket price indexes parallels our discussion of the Laspeyres price index. In fact, if $q^b = q^0$ and $Q^b = Q^0$, then the fixed basket Lowe index equals the Laspeyres index. However, in the previous section, the analysis of the Laspeyres index was not the end of the story: in the end, the Laspeyres index was combined with the Paasche index to form an approximation to a cost of living index. *In the present section, there is no Paasche counterpart to the fixed basket index*. A fixed basket index, like the Laspeyres index has a certain amount of *substitution bias.* Normally, if consumption patterns do not change much over time, a fixed basket index that uses a representative basket will not be subject to a great deal of substitution bias. However, in the present context, there are massive changes in the actual consumption vectors as we move from a pre lockdown period to a post lockdown period. And there are massive changes in the corresponding market prices.[24] Thus the amount of substitution bias in a Laspeyres (too high) or Paasche (too low) or fixed basket index (too high) will also be very large indeed under these conditions. When we take an average of the Paasche and Laspeyres indexes to form a Fisher index, we greatly reduce the amount of substitution bias.

In addition to being subject to substitution bias under normal conditions, there is another problem with using a fixed basket price index during a pandemic. The public will regard a fixed basket price index as a "reasonable" index, provided that the basket vectors, q^b and Q^b, are not too far removed from "normal" consumption patterns. A fixed basket index is very easy to explain and is perfectly reasonable under normal conditions. *But a fixed basket index is not intuitively plausible when a substantial fraction of the fixed basket commodities are simply not available*. In order to maintain public confidence in

[24] We interpret Hicksian reservation prices for unavailable commodities as market prices.

the CPI, it will be necessary for *statistical agencies to move to more representative baskets that are relevant for lockdown conditions*. This means that the national statistical agency will need to find ways to update their historical baskets more quickly—updated baskets in real time would be ideal.[25]

To explain how updated baskets could work in producing a reasonable CPI for continuing goods and services, suppose the price and quantity data for period 0 are (p^0,q^{0b}) and (P^0, Q^{0b}) but for period 1, the data are (p^1,q^{1b}) and $(P^{1*},0_N)$. It is unlikely that national statistical agencies will be able to produce the reservation prices P^{1*} and so we will concentrate on how to construct a price index for continuing commodities. Note that we are assuming that a new representative basket for continuing commodities in period 1, q^{1b}, is available to the agency.[26] Under these assumptions, the following *pseudo Laspeyres Paasche and Fisher price indexes*, P_{Bq0}, P_{Bq1} and P_{BqF}, for continuing commodities could be produced:

(36) $P_{Bq0} \equiv p^1 \cdot q^{0b}/p^0 \cdot q^{0b} = P_{Bq}$;

(37) $P_{Bq1} \equiv p^1 \cdot q^{1b}/p^0 \cdot q^{1b}$;

(38) $P_{BqF} \equiv [P_{Bq0}P_{Bq1}]^{1/2}$

where P_{Bq} was defined by (29) and we have used the fact that q^{0b} is equal to our old q^b. The logic for preferring the pseudo Fisher index over its Laspeyres and Paasche counterparts is the usual one: P_{Bq0} and P_{Bq1} are both fixed basket indexes that attempt to measure general inflation going from period 0 to 1. Both indexes are equally plausible so good statistical practice suggests that we take an average of the two to obtain a single point estimate of overall inflation between the periods. If q^{0b} is close to actual period 0 consumption of the continuing commodities, q^0, then P_{Bq0} will be a good approximation

[25] Quantity baskets of the form q^b can be replaced by expenditure share information (the vector of expenditure shares s^b) for the same period provided that price information for the same period, p^b, is also available since $q_m{}^b$ will be proportional to $s_m{}^b/p_m{}^b$ for m = 1,...,M.

[26] For some elementary index strata, the statistical agency may have scanner data available. In this case, we set $q^{0b} = q^0$ and $q^{1b} = q^1$; i.e., there is no need to use approximations to q^t in this case. The actual consumption vector for a period is the most representative consumption vector for that period. An elementary index is simply an index constructed at the lowest level of aggregation; see Diewert (2020b).

to the Laspeyres index P_{qL}. If q^{1b} is close to actual period 1 consumption of the continuing commodities, q^1, then P_{Bq1} will be a good approximation to the Paasche index P_{qP}. If both approximations are good, then P_{BqF} will be close to our preferred index, P_{qF}.

If the scope of the lockdown does not change materially during the lockdown, then the statistical agency could go back to using a fixed basket (equal to q^{1b}) for the duration of the lockdown. However, it is unlikely that countries will keep the scope of their lockdowns constant. Thus the number of continuing commodities that are present in two consecutive periods is unlikely to be constant. In this case, constructing chained maximum overlap pseudo Fisher indexes over the lockdown periods is preferable.[27]

If the lockdown ends at the beginning of period τ, then it will be necessary to construct a new basket ($q^{\tau b}$,$Q^{\tau b}$) that is representative for period τ. Recall that the basket for period 0 was (q^{0b},Q^{0b}). In order to construct the period τ price level, instead of comparing period τ prices to the prices of period $\tau - 1$, it is best to compare period τ prices, (p^τ,P^τ), to the prices of period 0, (p^0,P^0), since the overlap of products will be much larger than the overlap of products available in period τ to products available during the lockdown periods.[28] Thus it is preferable that the following *pseudo Laspeyres Paasche and Fisher price indexes*, P_{B0}, $P_{B\tau}$ and $P_{B0\tau}$, be produced:

(39) $P_{B0} \equiv [p^\tau \cdot q^{0b} + P^\tau \cdot Q^{0b}]/[p^0 \cdot q^{0b} + P^0 \cdot Q^{0b}]$;

(40) $P_{B\tau} \equiv [p^\tau \cdot q^{\tau b} + P^\tau \cdot Q^{\tau b}]/[p^0 \cdot q^{\tau b} + P^0 \cdot Q^{\tau b}]$;

(41) $P_{B0\tau} \equiv [P_{B0}P_{B\tau}]^{1/2}$.

[27] There may be a chain drift problem at the lowest level of aggregation. In this case, it may be necessary to use a multilateral index number. See Ivancic, Diewert and Fox (2011), Diewert and Fox (2018) and Diewert (2020b) for discussions of the chain drift problem and the use of multilateral methods. At higher levels of aggregation, chain drift will typically not be a major problem.

[28] Other forms of linking based on the similarity of the structure of prices and quantities going from one period to another period. For an explanation of how to implement these more sophisticated methods of linking observations over time, see section 20 of Diewert (2020c).

Thus the price level in period τ will be set equal to the price level in period 0 times the pseudo Fisher index,[29] $P_{Bq0\tau}$. This index will approximate the true Fisher index going from period 0 to period τ. However, the sequence of price levels going from period 1 to period $\tau - 1$ will not approximate economic price indexes or cost of living indexes because the big increase in the cost of living due to the lockdown in period 1 will not be reflected by the movements in the above basket price indexes for periods 1 to $\tau - 1$. In order to accurately measure the cost of living for those periods, we need estimates for the period 1 reservation prices P^{1*}.

5. The Way Forward

Statistical agencies that use a fixed basket methodology for constructing their CPI are faced with the fact that the fixed basket has become almost totally irrelevant.[30] A fixed basket index is very easy to explain and is perfectly reasonable under "normal" conditions. *But a fixed basket index is not intuitively plausible when a substantial fraction of the fixed basket commodities are simply not available*. At the very least, this suggests that *NSOs move to more representative baskets that are relevant for lockdown conditions*.

However, many NSOs will not have the resources to estimate representative baskets in real time. We will list a number of possible strategies that an agency could use in order to construct a CPI under pandemic conditions, depending on what kind of data they are able to collect. We will start with the assumption that very little data are available and finish with the way forward if ample data are available. For each of these cases, we will look at possible ways of addressing the lack of matching problem at the elementary index level.

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[29] Pseudo Fisher price indexes have been computed retrospectively; e.g., see Diewert, Huwiler and Kohli (2009).

[30] One might argue that the fixed basket that is used for continuing commodities is still relevant but in fact, there are substantial changes in consumption patterns due to the government imposed lockdowns of economic activity. A further complicating factor is household accumulation of storable goods. Thus in terms of our model, q^1 may differ substantially from q^0. Another complicating factor is the fact that online prices can differ from in store prices for the available food and pharmacy items that are still sold during a lockdown period. Furthermore, many households need to pay additional fees for hiring someone to do their shopping for them and deliver it to their doorstep. Thus there is a new goods problem even in categories of consumption that have not been shut down.

Case 1: Very Little Data Availability

For this case, we suppose that the agency has only a fixed basket (q^b,Q^b) along with price data for period 0 which is the period before the lockdown, (p^0,P^0). For the pandemic periods, the agency has only price data for always available goods and services, p^t for $t = 1,2,...\tau-1$, the pandemic periods. When the pandemic is over in period τ, we assume that the agency can collect price data for always available goods, p^τ, and for commodities that were available in period 0 and become available again in period τ, P^τ. For the lockdown periods, the agency can calculate the fixed basket index for always available commodities, $p^t\cdot q^b/p^0\cdot q^b$ for $t =1,2,...\tau-1$. These indexes may be suitable for (partial) compensation purposes; i.e., if period 0 household expenditures on the basket q^b were equal to $p^0\cdot q^b$, then using the index $p^t\cdot q^b/p^0\cdot q^b$ to escalate the household's period 0 "income" (equal to $p^0\cdot q^b$) would allow the household to purchase the bundle of commodities q^b in period t for $t = 1,2,...,\tau-1$.[31] The statistical agency would need to note that the CPI for these periods is not comparable to the CPI for either period 0 or period τ.[32] The suggested CPI for these lockdown periods will not provide a satisfactory approximation to a cost of living index. Because it does not involve the unavailable goods, it cannot provide a credible index that will reflect the increase in the effective prices of unavailable goods and services. However, to provide a useful estimate for a cost of living index relative to the standard of living in period 0 for the lockdown periods, we require estimates for reservation prices, P^{t*} for $t = 1,2,...,\tau-1$. Very few NSOs will venture to estimate reservation prices. What NSOs can do is to provide a credible CPI for goods and services which are actually available during the lockdown periods. When the lockdown ends and conditions approach "normality" in period τ, then the under-resourced statistical office can use its pre-lockdown basket, (q^b,Q^b), to calculate the price level in period τ relative to period 0 as the fixed base index $[p^\tau\cdot q^b + P^\tau\cdot Q^b]/[p^0\cdot q^b + P^0\cdot Q^b]$.

[31] This index may be subject to some substitution bias.
[32] Equations (34) show that the indexes $p^t\cdot q^b/p^0\cdot q^b$ for $t =1,2,...\tau-1$ could be generated by using the inflation adjusted carry forward price vectors for unavailable commodities defined as $P^{tI} \equiv (p^t\cdot q^b/p^0\cdot q^b)P^0$. If this is done, users need to be informed that the resulting indexes are not "true" fixed basket indexes in that part of the overall fixed basket, (q^b,Q^b), is simply not available for purchase in period t.

Case 2: Some Data Availability

We assume that the data availability is at least as good as in the above case. In addition, we assume that by period σ (where $1 < \sigma < \tau$), the statistical agency is able to obtain an estimate for a representative quantity vector q^σ for the always available quantities during the lockdown period. For the lockdown periods prior to period σ, the agency can calculate the fixed basket index for always available commodities, $\pi^t \equiv p^t \cdot q^b / p^0 \cdot q^b$ for $t = 1,2,...\sigma-1$. In period σ, the new basket q^σ becomes available so we can calculate the period t price index value for period t as $\pi^t \equiv \pi^{t-1}[p^t \cdot q^\sigma / p^{t-1} \cdot q^\sigma]$ for $t = \sigma,\sigma+1,...,\tau-1$. However, the price levels π^1, π^2,..., $\pi^{\sigma-1}$ may very unreliable due to the fact that the pre-lockdown quantity vector q^{b0} will probably be rather far from the actual consumption vectors q^1, q^2,...,$q^{\sigma-1}$ over the lockdown period extending from period 1 to period $\sigma-1$. Thus it may be preferable to define ρ^σ as the pseudo Fisher index comparing period 0 with period σ; i.e., define $\pi^\sigma \equiv [p^\sigma \cdot q^b / p^0 \cdot q^b]^{1/2} [p^\sigma \cdot q^\sigma / p^0 \cdot q^\sigma]^{1/2}$. For lockdown periods following period σ but prior to period τ, define $\pi^t \equiv \pi^{t-1}[p^t \cdot q^\sigma / p^{t-1} \cdot q^\sigma]$ for $t = \sigma+1,\sigma+2,...,\tau-1$. Again, these indexes may be suitable for partial indexation purposes but they will be subject to substantial downward biases as approximations to cost of living indexes. When we get to period τ, the moderately-resourced statistical office can calculate the fixed base index relative to period 0; i.e., set $\pi^\tau = [p^\tau \cdot q^b + P^\tau \cdot Q^b]/[p^0 \cdot q^b + P^0 \cdot Q^b]$.[33]

[33] If the statistical office has set in motion a continuous consumer expenditure survey so that a new period τ comprehensive basket (q^τ, Q^τ) can be constructed, then the office can calculate the pseudo Fisher index defined by (42).

Case 3: Ample Data Availability

We assume the Case 1 data availability plus the availability of representative quantity vectors q^{bt} for all periods $t = 0,1,...,\tau$. We also assume that a representative quantity vector for the unavailable commodities is available for periods 0 and τ. Denote these vectors by Q^{b0} and $Q^{b\tau}$. The corresponding price vectors are P^0 and P^1. For period 0, define the price level as $\pi^0 \equiv 1$. For the lockdown periods, define the period 1 price index π^1 as the pseudo Fisher index $\pi^1 \equiv \{[p^1\cdot q^{b0}/p^0\cdot q^{b0}][p^1\cdot q^{b1}/p^0\cdot q^{b1}]\}^{1/2}$. For $t = 2,3,...\tau-1$ define the period t price index as $\pi^t \equiv \pi^{t-1}\{[p^t\cdot q^{bt-1}/p^{t-1}\cdot q^{bt-1}][p^t\cdot q^{bt}/p^{t-1}\cdot q^{bt}]\}^{1/2}$. Thus the period to period pseudo Fisher indexes are chained together to form the period t price level. For period τ, define the price level π^τ as the comprehensive pseudo Fisher price index connecting period 0 to period τ; i.e., define π^τ as follows:

$$(42)\quad \pi^\tau \equiv \{[p^\tau\cdot q^{b0} + P^\tau\cdot Q^{b0}]/[p^0\cdot q^{b0} + P^0\cdot Q^{b0}]\}^{1/2}\{[p^\tau\cdot q^{b\tau} + P^\tau\cdot Q^{b\tau}]/[p^0\cdot q^{b\tau} + P^0\cdot Q^{b\tau}]\}^{1/2}.$$

The reason for using chained pseudo Fisher price indexes for the available products during the lockdown period instead of fixed base pseudo Fisher price indexes is the likelihood that consumer purchases of available products over the lockdown periods may not be well approximated by a constant vector q^b. Initially, households will stock up on storable goods and cut back on purchases of consumer durables. If the lockdown period is long and the degree of lockdown varies, then it is quite likely that the vector of actual purchases of available commodities in period t, q^t, will be quite variable and hence a constant q^b will not provide a representative vector of household purchases over all of the lockdown periods.[34] Of course, the gold standard for the quantity vectors q^{bt} would be the actual period t consumption vectors, q^t, in which case, the pseudo Fisher indexes would become actual Fisher indexes.

[34] In reality, the set of available products will also vary over the lockdown periods. At the time of writing, food processers in many cities are not producing their full line of products; instead they are concentrating on increasing the volume of their best selling products. Thus for some elementary index categories, it may be necessary to use fixed base pseudo Fisher indexes in place of the chained indexes for the lockdown periods in order to eliminate chain drift.

6. The Lack of Matching Problem at the Elementary Index Level

A problem which has appeared as a result of country wide lockdowns is the *problem of missing products* in retail outlets. As a result of household stockpiling activities, many products are missing in grocery shops. In some cases, the missing products may reappear in a later period; in some cases, they may be gone for the duration of the lockdown.[35] If the products are gone for the duration of the lockdown and the remaining products are present during the current and prior lockdown periods, then we are in position to apply the theory above to the particular elementary aggregate under consideration; i.e., we need to switch from pricing out the pre-lockout basket of products to the new smaller set of products. However, real life will be more complicated than having a clear division between products present and products that have been discontinued for all lockout periods: products will be drifting in and out of scope in any particular retail outlet. This may lead to a massive lack of matching problem. We will briefly suggest possible solutions to this problem under two scenarios: (i) only web scraped data are available and (ii) scanner data are available. The analysis in this section differs from the analysis that was presented in the previous section where it was known that some commodities would be unavailable for the duration of the lockdown. In the present situation, we are assuming that products are present in some periods and not present in other periods; i.e., we are assuming that the full array of pre-lockdown products is not available in the lockdown periods.

Case 1: Only Price Data are Available

Method 1: Adapt the Section 4 Carry Forward Methodology

The adaptation here is to assume that q^0 and q^1 are equal to the vector of ones, 1_M, and Q^0 equals the vector of ones, 1_N. $Q^1 = 0_N$ as in section 4. Thus the q group of products are the *maximum overlap products* that are present in both periods and the Q products are present

[35] Many food manufacturers are focusing on their most popular items and producing them at scale to satisfy the stockpiling demands.

in the base period 0 but not in the current period 1. The given price vectors are p^0, p^1 and P^1. Applying the section 4 methodology using the above assumptions on prices and quantities leads to the following inflation adjusted carry forward price index using equation (34) adapted to the present situation:

$$(43)\ P_B^{CI} = P_{Bq} = p^1 \cdot 1_M / p^0 \cdot 1_M = \Sigma_{m=1}^M\, p_m^1 / \Sigma_{m=1}^M\, p_m^0.$$

The above index is the Dutot (1738) elementary index, defined over products that are present in both periods. It has an undesirable property: it is not invariant to changes in the units of measurement of the products. It will also give a higher weight to products that are more expensive which may not be a desirable property. Nevertheless, it does approximate the theoretically more desirable Jevons index under certain conditions.[36]

Method 2: Use Maximum Overlap Jevons Indexes

This method simply sets the price index equal to the Jevons (1865) index for the overlapping products in the two periods under consideration. Thus using the same notation as was used to describe Method 1 above, the maximum overlap Jevons index, P_{JMO}, is equal to the geometric mean of the price ratios for the overlapping products:

$$(44)\ P_{JMO} \equiv [\Pi_{m=1}^M\, (p_m^1/p_m^0)]^{1/M}.$$

The Jevons index has the best axiomatic properties for indexes (with no missing prices) that depend only on prices. Note in particular that the maximum overlap Jevons index is invariant to changes in the units of measurement for the products.[37]

[36] See Diewert (2020b) for a discussion of this index and its relationship to other elementary indexes.
[37] See Diewert (2020b) for the axiomatic properties of the Jevons index.

Method 3: Use the Multilateral Time Product Dummy Method

A problem with the above two methods is that they make use of price data covering only two periods. In the situation where closely related products are moving in and out of scope, constructing maximum overlap bilateral index numbers does not make use of all of the data and hence is inefficient from a statistical point of view. For example, suppose a product is present in periods 1 and 3 and another product is present in periods 2 and 4. In a bilateral index setup, the information pertaining to these two products would not be used which is inefficient since price comparisons for product 1 between periods 1 and 3 and for product 2 between periods 2 and 4 are perfectly valid comparisons and should be used somehow in constructing the sequence of price indexes. The way forward here is to use a *multilateral index* which utilizes the price information for all periods.

Our preferred multilateral method is the *Time Product Dummy Method* with missing observations.[38] We introduce some new notation in order to describe this method. We now assume that there are N products and T time periods but not all products are purchased (or sold) in all time periods. The price and quantity vectors for period t are denoted by $p^t \equiv [p_{t1},...,p_{tN}]$ and $q^t \equiv [q_{t1},...,q_{tN}]$. If product n in period t is missing, we set the corresponding price and quantity, p_{tn} and q_{tn}, equal to 0. For each period t, define the set of products n that are present in period t as $S(t) \equiv \{n: p_{tn} > 0\}$ for t = 1,2,...,T. It is assumed that these sets are not empty; i.e., at least one product is purchased in each period. For each product n, define the set of periods t where product n is present as $S^*(n) \equiv \{t: p_{tn} > 0\}$. Again, assume that these sets are not empty; i.e., each product is sold in at least one time period. Define the integers N(t) and T(n) as follows:

$$(45)\quad N(t) \equiv \Sigma_{n \in S(t)}\, 1; \qquad t = 1,...,T;$$

$$(46)\quad T(n) \equiv \Sigma_{t \in S^*(n)}\, 1; \qquad n = 1,...,N.$$

[38] The method was originally devised for making price comparisons across countries and is known as the Country Product Dummy multilateral method. It is due to Summers (1973). A weighted version of this model (with missing observations) was first applied in the time series context by Aizcorbe, Corrado and Doms (2000). The unweighted time series version is due to Court (1939) as we shall see.

If all N products are present in period t, then N(t) = N; if product n is present in all T periods, then T(n) = T.

The economic model that is consistent with the Time Dummy Product multilateral method is the following one:

(47) $p_{tn} = \pi_t \alpha_n$; $t = 1,\ldots,T;\ n \in S(t)$

where π_t is the period t price level and α_n is a quality adjustment parameter for product n. If all products were available in all periods, equations (47) tells us that prices for the group of products in scope are moving in a proportional manner. This is consistent with purchasers of the N products having the linear utility function, $f(q) = \alpha \cdot q \equiv \Sigma_{n=1}^{N} \alpha_n q_n$ where $\alpha \equiv [\alpha_1,\ldots,\alpha_N]$ and $q \equiv [q_1,\ldots,q_N]$.[39] Now take logarithms of both sides of equations (47), add error terms e_{tn} to the resulting equations and we obtain the following system of estimating equations:

(48) $\ln p_{tn} = \rho_t + \beta_n + e_{tn}$; $t = 1,\ldots,T;\ n \in S(t)$

where $\rho_t \equiv \ln \pi_t$ for $t = 1,\ldots,T$ and $\beta_n \equiv \ln \alpha_n$ for $n = 1,\ldots,N$. Note that equations (48) form the basis for the *time dummy hedonic regression model*, which is due to Court (1939).[40]

Estimates for the unknown parameters ρ_t and β_n that appear in equations (48) can be found by solving the following least squares minimization problem:

(49) $\min_{\rho,\beta} \{\Sigma_{t=1}^{T} \Sigma_{n \in S(t)} [\ln p_{tn} - \rho_t - \beta_n]^2\}$.

[39] See Diewert (2020d) for further explanation of the underlying economic model. It can be seen that this approach will only be adequate if the products are very close substitutes since a linear utility function implies that the products are perfect substitutes.

[40] This was Court's (1939; 109-111) hedonic suggestion number two. He chose to transform equations (47) by the log transformation because the resulting regression model fit his data on automobiles better. Diewert (2003) also recommended the log transformation on the grounds that multiplicative errors were more plausible than additive errors.

In order to obtain a unique solution to (49), we need to impose a normalization on the parameters.[41] Choose the normalization $\rho_1 = 0$ (which corresponds to $\pi_1 = 1$). Denote the resulting solution by $\rho^* \equiv [1,\rho_2^*,...,\rho_T^*]$ and $\beta^* \equiv [\beta_1^*,...,\beta_N^*]$. Use these estimates to form estimates for $\pi_t^* \equiv \exp[\rho_t^*]$ for t = 1,...,T and $\alpha_n^* \equiv \exp[\beta_n^*]$ for n = 1,...,N. It turns out that these estimates satisfy the following equations:

(50) $\pi_t^* = \Pi_{n\in S(t)} [p_{tn}/\alpha_n^*]^{1/N(t)}$; t = 1,...,T;

(51) $\alpha_n^* = \Pi_{t\in S^*(n)} [p_{tn}/\pi_t^*]^{1/T(n)}$; n = 1,...,N.

Note that p_{tn}/α_n^* is a *quality adjusted price* for product n in period t and p_{tn}/π_t^* is the corresponding *inflation adjusted price* for product n in period t. Thus the period t estimated price level, π_t^*, is the geometric mean of the quality adjusted prices for products that are available in period t and the estimated quality adjustment factor for product n, α_n^*, is the geometric mean of all of the inflation adjusted prices for product n over all periods. Note that if the set of available products in periods r and t is the same, then $\pi_t^*/\pi_r^* = [\Pi_{n\in S(t)}(p_{tn}/p_{rn})]^{1/N(t)}$ which is the Jevons index defined over the products that are present in both periods. These price levels generated by this method have satisfactory axiomatic properties.[42] There are some additional choices that the statistical agency will have to make if it uses this method; i.e., it is necessary to decide on the length of the window of observations T and it is necessary to decide on how to link the results of the latest window of estimates with the previous window of estimates for the price levels. The agency should be able to resolve these issues by experimenting with the different choices for the window length and for linking the price level estimates for a new window to the estimates of the previous window.

[41] We also need to impose a full rank condition on the X matrix generated by the linear regression model defined by equations (48) and $\rho_1 = 0$; see Diewert (2020c).

[42] See Diewert (2020c). It turns out that the price levels satisfy an *identity test* so if prices are equal in periods r and t, then $\pi_r^* = \pi_t^*$.

Case 2: Price and Quantity Data are Available

Method 4: Apply the Section 4 Carry Forward Methodology

Little additional explanation is required here; just apply the methodology explained in section 4 to the elementary index context. Diewert, Fox and Schreyer (2017) have more details on how to apply the carry forward methodology for Paasche, Laspeyres, Fisher and Törnqvist indexes in the case of two observations.

Method 5: Apply the Weighted Time Product Dummy Multilateral Method

The basic economic model is still the price proportionality model defined by equations (47) above but now we assume that we have expenditure or quantity information on household purchases in addition to price information. With this extra information, it is preferable to take the economic importance of each commodity into account and replace the least squares minimization problem defined by (49) with the following weighted least squares minimization problem:[43]

$$(52)\ \min_{\rho,\beta} \{\Sigma_{t=1}^{T} \Sigma_{n\in S(t)} s_{tn}[\ln p_{tn} - \rho_t - \beta_n]^2\}$$

where the period t expenditure share on commodity n is $s_{tn} \equiv p_{tn}q_{tn}/p^t\cdot q^t$ for t = 1,...,T and $n\in S(t)$.[44] Again, we need to make the normalization $\rho_1 = 0$ to obtain a unique solution ρ^* and β^* to (52). It turns out the solution will satisfy the following equations, which are the weighted counterparts to equations (50) and (51):[45]

$$(53)\ \pi_t^* = \exp[\Sigma_{n\in S(t)} s_{tn}\ln(p_{tn}/\alpha_n^*)]\ ; \qquad t = 1,...,T;$$

$$(54)\ \alpha_n^* = \exp[\Sigma_{t\in S^*(n)} s_{tn}\ln(p_{tn}/\pi_t^*)/\Sigma_{t\in S^*(n)} s_{tn}]\ ; \qquad n = 1,...,N.$$

[43] Rao (1995) (2004) (2005; 574) was the first to consider this model using expenditure share weights. However, Balk (1980; 70) suggested this class of models much earlier using different weights.
[44] See Diewert (2020c) for a discussion on the merits of different choices for the weights.
[45] See Diewert (2020c).

From (53) and (54), it can be seen that the period t estimated price level, π_t^*, is now a weighted geometric mean of the quality adjusted prices for products that are available in period t and the estimated quality adjustment factor for product n, α_n^*, is now a weighed geometric mean of all of the inflation adjusted prices for product n over all periods. Note that if the set of available products in periods r and t is the same, π_t^*/π_r^* will not collapse to a weighted Jevons index unless the expenditure shares in the two periods under consideration are equal.

Once the estimates for the π_t^* and α_n^* have been computed, we have the usual two methods for constructing period by period aggregate price and quantity levels, P^t and Q^t for t = 1,...,T. The way to see this is to consider the underlying equations (47) which were the equations $p_{tn} = \pi_t\alpha_n$ for t = 1,...,T and $n\in S(t)$. Take this equation for some n and t and multiply both sides of it by the observed quantity, q_{tn}, and sum the resulting equations. We obtain the following equations using the fact that $q_{tn} = p_{tn} \equiv 0$ for $n\notin S(t)$:

$$
\begin{aligned}
(55)\quad p^t\cdot q^t &= \Sigma_{n\in S(t)}\, p_{tn}q_{tn} && t = 1,\dots,T \\
&= \pi_t\Sigma_{n\in S(t)}\, \alpha_n q_{tn} \\
&= \pi_t\Sigma_{n=1}^{N}\, \alpha_n q_{tn} && \text{since } q_{tn} = 0 \text{ if n does not belong to } S(t) \\
&= \pi_t\alpha\cdot q^t.
\end{aligned}
$$

Because equations (47) will not hold exactly, with nonzero errors e_{tn}, equations (55) will only hold approximately. However, the approximate versions of equations (55) allow us to form period t price and quantity aggregate levels, say P^t and Q^t, in two separate ways: the π_t^* estimates that are part of the solution to (52) can be used to form P^{t*} and Q^{t*} via equations (56) and the α_n^* estimates that are part of the solution to (52) can be used to form the aggregates P^{t**} and Q^{t**} via equations (57):[46]

$$(56)\quad P^{t*} \equiv \pi_t^*\,;\quad Q^{t*} \equiv p^t\cdot q^t/\pi_t^*\,; \qquad t = 1,\dots,T;$$

[46] Note that the price level P^{t**} defined in (57) is a quality adjusted unit value index of the type studied by de Haan (2004).

(57) $Q^{t**} \equiv \alpha^*\cdot q^t$; $P^{t**} \equiv p^t\cdot q^t/\alpha^*\cdot q^t$; $\quad$ t =1,...,T.

Define the error terms $e_{tn} \equiv \ln p_{tn} - \ln\pi_t^* - \ln\alpha_n^*$ for t = 1,...,T and n = 1,...,N. If all $e_{tn} = 0$, then P^{t*} will equal P^{t**} and Q^{t*} will equal Q^{t**} for t = 1,...,T. However, if the error terms are not all equal to zero, then the statistical agency will have to decide on pragmatic grounds on which option to choose to form the aggregate price and quantity levels.[47]

It should be noted that $P^{t**} \equiv p^t\cdot q^t/\alpha^*\cdot q^t$ is a *quality adjusted unit value price level*.[48] There is also an inequality between P^{t*} and P^{t**} that is due to de Haan and Krsinich (2018; 763). From (53) and (56), we have $P^{t*} = \exp[\Sigma_{n\in S(t)}\, s_{tn}\ln(p_{tn}/\alpha_n^*)]$, which is a share weighted geometric mean of the period t quality adjusted prices, p_{tn}/α_n^*, for products that are actually present in period t. From (57), we have P^{t**} equal to the following expression:

$$
\begin{aligned}
(58)\ P^{t**} &\equiv p^t\cdot q^t/\alpha^*\cdot q^t \qquad\qquad t = 1,...,T\\
&= \Sigma_{n\in S(t)}\, p_{tn}q_{tn}/\Sigma_{n\in S(t)}\, \alpha_n^* q_{tn}\\
&= \Sigma_{n\in S(t)}\, p_{tn}q_{tn}/\Sigma_{n\in S(t)}\, \alpha_n^* (p_{tn})^{-1}p_{tn}q_{tn}\\
&= [\Sigma_{n\in S(t)}\, s_{tn}(p_{tn}/\alpha_n^*)^{-1}]^{-1}\\
&\leq P^{t*}
\end{aligned}
$$

since a share weighted harmonic mean of the quality adjusted prices present in period t is always equal to or less than the corresponding share weighted geometric mean using Schlömilch's inequality (see Hardy, Littlewood and Polyá (1934; 26)). Note that $P^{t**} \leq P^{t*}$ implies that $Q^{t**} \geq Q^{t*}$ for t = 1,...,T.

The axiomatic properties of the price levels π_t^* are studied in Diewert (2020c). They are reasonably good.

[47] De Haan and Krsinich (2018) were the first to realize that the results of a hedonic regression would lead to two separate ways to define the resulting aggregate price and quantity levels. See also Diewert (2020c) (2020d). If the accurate measurement of price levels is the target, then it is probably best to use P^{t*}; if the target is to measure aggregate quantity levels (and hence welfare), then it is probably best to use P^{t**}.

[48] The term "quality adjusted unit value price index" was introduced by Dalén (2001). Its properties were further studied by de Haan (2004) (2010), Silver (2010) (2011), de Haan and Krsinich (2018), Von Auer (2014) and Diewert (2020c) (2020d).

The issues of choosing a window length T for this multilateral method remain unresolved; statistical agencies can experiment with different choices for T. There is also the issue of linking the present window with the previous window.

From the viewpoint of the economic approach to index number theory, the use of this method should be confined to situations where the products in scope are close substitutes since the underlying economic assumption is that the products are perfect substitutes, except for random errors. Quality adjusted unit value price levels are appropriate in this situation but if the products are not close substitutes, it would be preferable to use the inflation adjusted carry forward prices methodology suggested by Diewert, Fox and Schreyer (2017) if the target index is a superlative index. Finally, Method 5 should not be used at higher levels of aggregation where substitution between elementary index categories may be low. At the second stage of aggregation it would be preferable to use Fisher, Walsh or Törnqvist indexes if actual price and quantity data are available or use pseudo Fisher indexes if the quantity data can only be approximated.

Method 6: The Use of Quality Adjusted Unit Value Price Levels

From the discussion of Method 5 above, it is clear that quality adjusted unit values can be used as price levels, provided that the commodities in scope for the elementary aggregate are close substitutes. However, it is not necessary to use the Weighted Time Product Dummy multilateral index number method in order to obtain estimates for the quality adjustment parameters, the components of the vector α. If the group of products under consideration consists of highly substitutable products and all of the products were purchased in the pre-lockdown period 0, then simply set α equal to p^0, the (unit value) price vector for the products in the pre-lockdown period. If all of the products were purchased for a number of pre-lockdown periods, say periods 0, −1, −2 and −3, and the price vectors for these periods were p^0, p^{-1}, p^{-2} and p^{-3}, then define α as follows:

$$(59)\ \alpha \equiv (1/4)[(p_{01})^{-1}p^0 + (p_{-1,1})^{-1}p^{-1} + (p_{-2,1})^{-1}p^{-2} + (p_{-3,1})^{-1}p^{-3}].$$

Thus α is set equal to the average of past pre-lockdown price vectors for the commodities in the group of commodities under consideration but these vectors of past prices are deflated by the price of the first commodity in order to eliminate the effects of general inflation between past periods for the group of commodities. The first commodity should be chosen to be the commodity with the largest average expenditure share in the group of commodities. If there are missing prices in the pre-lockdown periods, then instead of using the α defined by (59), the α defined by the Time Product Dummy multilateral method (Method 3 above) could be used to estimate the quality adjustment parameters.

From the viewpoint of the economic approach to index number theory, the use of quality adjusted unit values as estimates for price levels should only be applied if the commodities in the elementary group of commodities are close substitutes.[49]

Method 7: Linking Based on Relative Price and Quantity Similarity

A desirable property of the Fisher price index between two periods is the fact that the Fisher index will equal unity if prices in the two periods are equal even if the quantities demanded in the two periods are not equal. Most multilateral methods do not satisfy this strong identity test; they tend to satisfy a weaker identity test that says that the relative aggregate price levels between any two periods in the window of observations will equal unity provided that both prices and quantities are identical in the two periods being compared.

There is a recently developed multilateral method that satisfies the above strong identity test and can deal with missing observations. The method is based on building a set of

[49] It is possible to cluster N highly substitutable commodities in scope into quality groups based on their price per unit of a dominant characteristic. Group the N products into low quality, medium quality and high quality products based on their relative prices in the pre-lockdown period. Then aggregate price levels for each of the three groups of products could be constructed by simply taking unit values (without quality adjustment) for each group of products. We would end up with three elementary indexes in place of the single elementary index. Then these three separate indexes could be aggregated up into a single index using a superlative index number formula. This is feasible because we are assuming the availability of price and quantity data for Method 6. The advantage of this method is that it avoids the need for imputation.

Fisher index bilateral comparisons where each comparison is based on linking the periods that have the most *similar relative price structures*.[50] Initially, periods 1 and 2 are linked by the usual bilateral Fisher price index. When the data of period 3 become available, the price and quantity data of period 3 are linked to the corresponding data of either period 1 or 2, depending on which of these two periods has the most similar structure of relative prices. The bilateral Fisher index is used to link period 3 with period 1 if the measure of relative price similarity between periods 1 and 3 is higher than the measure of relative price similarity between periods 1 and 2. If the measure of relative price similarity between periods 2 and 3 is higher than the corresponding measure for comparing periods 1 and 3, then the bilateral Fisher index is used to link period 3 with period 2. When the data of period 4 become available, the data for period 4 are linked to the data of periods 1,2 or 3, depending on which of these 3 prior periods gives rise to the highest measure of price similarity. And so on. In practice, measures of *relative price dissimilarity* are used to link the data of two periods, using the lowest measure of dissimilarity to do the linking. At the first stage of the network of comparisons, the two periods that have the most similar structure of relative prices is chosen. At the next stage of the comparison, look for a third period that had the most similar relative price structure to the first two periods and link in this third country to the comparisons of volume between the first two countries and so on.

A key aspect of this linking methodology is the choice of the measure of similarity (or dissimilarity) of the relative price structures of two countries. Various measures of the similarity or dissimilarity of relative price structures have been proposed by Allen and Diewert (1981), Kravis, Heston and Summers (1982; 104-106), Hill (1997) (2009), Aten and Heston (2009) and Diewert (2009). The dissimilarity measure recently proposed by Diewert (2020c; 72) seems to be the most promising but the method needs to be more thoroughly tested before it can be suggested to statistical agencies for general use. A major advantage of this new method of linking periods is that the strong identity test will always be satisfied; i.e., if prices in the current period are the same as the prices in a past

[50] Hill (2001) (2004) was an early pioneer in using this similarity of relative prices approach to multilateral index number theory in the time series context. The real time linking method described here is due to Diewert (2020c).

period, the estimated price levels pertaining to these two periods will always be identical even if quantities or expenditures are not identical.[51] There can never be a chain drift problem using this new multilateral method.

7. Other Measurement Problems

7.1. No Agency Employee Price Collection

Most statistical agencies have stopped sending employees to retail outlets to collect prices. Instead the agencies have switched to web scraping; i.e., they collect online prices over the internet. The collected prices will not be perfectly comparable with the previously collected in store prices. Cavallo (2017) did a large scale comparison of in store prices versus online prices (excluding transport costs) across 10 countries and found little difference between in store and online prices.[52] His results provide some justification for comparing a web scraped price for a specific product with a collected price for the same product in a prior period. Under lockdown conditions, home delivery of products purchased online will increase dramatically. On the other hand, household travel expenses will decrease due to fewer in store shopping trips. As these travel expenses are in scope for household expenditures, it may make sense to collect online prices that include delivery since the delivered price is the price that the consumer actually faces for the product. The higher price for the delivered product will be offset by lower household transportation costs. In general, we endorse the collection of web scraped data to replace previous data that were collected by agency employees. However, some care should be taken to not collect online prices for goods or services which were never actually consumed by any household. Examples of such services are be listed airline fares or listed prepaid holiday packages that are eventually cancelled.[53]

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[51] If the prices in the current period are proportional to the prices in a prior period, then the ratio of the current period price level to the prior period price level will be equal to the factor of proportionality. Another advantage of Diewert's method is that it is not necessary to choose a window length for his suggested multilateral method.

[52] From Cavallo (2017; 291), online prices over the comparable in-store prices were on average 4% lower. The average markup ranged from −13% for Japan to +5% for Australia. See also Cavallo (2013) and Cavallo and Rigobon (2016).

[53] How exactly should cancellation fees be treated in a CPI? An interesting question.

7.2 Lack of Information on Current Household Expenditure Weights

It will be very difficult for statistical agencies to find current period expenditure share or quantity weights for their elementary index categories. The problem is that the "representative" basket for each month is changing rapidly as the virus spreads and lockdown rules change to react to current conditions. Here are some possible ways for NSOs to obtain current information on household expenditures:

- Some countries (such as the US and the UK) have continuous household expenditure surveys. Usually, the sample size for such surveys is small so, for example, the US Bureau of Labor Statistics Consumer Expenditure Survey does not have a big enough sample size to allow monthly publication of the implied monthly weights. It publishes semi-annual estimates. The way forward here is to increase the sample size. For countries that currently do not have a continuous consumer expenditure survey, it is absolutely necessary that they start one.
- Some private companies collect consumer expenditure data (along with prices and quantities) on a continuous basis for a sample of households using scanner data. NSOs can purchase these data (at a fair price) or set up their own competing company if they are unable to establish a satisfactory consumer expenditure survey.
- National governments can appeal to their business communities to persuade large firms producing consumer products to donate their electronic data to the NSO.[54] Many large retailers around the world are already donating their data and it should be possible for more firms to be persuaded to do this. This information will help to produce a better CPI and it will also allow much better production accounts to be produced.
- Credit card companies collect information on household purchases of consumer goods and services. If the expenditure information could also be combined with product codes, this information would enable the construction of consumer price indexes by location and demographic group. For some countries, it may be

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[54] Given that national governments are going to be forced to give handouts to a large number of firms, getting them to cooperate with the government for a good purpose should be possible.

possible to access this information source. For other countries, it may not be possible for the statistical agency to access this information due to privacy concerns.[55]

7.3 Should the CPI be Revised?

From the materials presented in section 7.2 above, it can be seen that national statistical agencies will not be able to produce very accurate period t basket updates q^{bt} that approximate actual period t consumption q^t in a timely fashion (if they can produce them at all). However, in time, better estimates for actual consumption in past periods may become available.[56] The question then arises: should the CPI be revised in the light of improved information that becomes available after the release date? From a statistical point of view, the answer to this question is yes. However, for many countries, a monthly CPI must be provided to the public and no revisions are allowed.[57]

Scanner data along with the usual information on retail sales can be massaged to produce some rough and ready weights in real time. NSOs will simply have to announce that their new estimates for inflation and economic growth are only very approximate estimates. A country's national accounts are allowed to be revised and this revision process is generally accepted by the public, hence estimates of economic growth can be revised. This is not the case for the CPI. A country could produce at least two CPIs: one that is not revised and is based on available information at the month of production of the index and another that is allowed to be revised in the light of information that becomes available at a later date.

This strategy has been successfully used by the Bureau of Labor Statistics in the US where two indexes are released at the same time; the first one ("CPI-U") is not revisable

[55] See Carvalho et al. (2020) and Dunn, Hood, and Driessen (2020) for examples of how such information can be used to analyze changes in expenditure patterns.
[56] Smoothing a sample of collected monthly household expenditures per household (by taking a moving average for example) will probably lead to more accurate trend estimates for monthly household expenditures. But the trend can only be calculated after some months have passed.
[57] This no revision policy is probably due to the fact that the country's CPI is often used for indexation purposes in legally binding contracts.

and the second one ("C-CPI-U") is allowed to be revised (and approximates a superlative index after the last revision). The second CPI can be labeled as an analytic CPI and can be used by economic analysts who require more accurate historical information on inflation. The first type of traditional CPI produced under lockdown conditions will necessarily be much more inaccurate; it will be very difficult to obtain adequate approximations to actual consumption during the start of the lockdown period due to the absence of accurate survey information on consumer expenditures. Users need to be alerted to this problem.

In section 7.2 we attempted to anticipate the problems that many statistical agencies will face in trying to update their baskets to reflect the lockdown realities. We realize that new lockdown baskets will not be available to many, if not most, NSOs. Our conclusion boils down to this: if later information shows that the early lockdown indexes are very inaccurate, then set the current CPI price level to the best possible estimate possible even if it is necessary to use a different methodology than was used in the pre-lockdown periods. For the revisable CPI, new information should be used to revise previous indexes.

7.4 The Stockpiling Problem

Lockdowns have led governments to limit trips to retail outlets for purchases of food and other essential goods such as pharmaceutical products. These regulations plus the reactions of households to cut down on their shopping trips to limit the risk of infection have led households to accumulate large *stockpiles* of essential storable goods. Thus at the initial stages of a lockdown, there will be a large increase in purchases of storable goods but actual consumption of these goods will be far less. In other words, it becomes necessary to distinguish actual household *consumption* of storable goods from the *acquisition* of the goods. In principle, the national statistical agency will have to decide between these two approaches to the production of a CPI.

From a welfare point of view, it is monthly consumption of goods and services which is most relevant but it will generally be more convenient to stick to an acquisitions approach to the measurement of consumption. If the actual consumption approach to the scope of the CPI is chosen, then in principle, the stocks of storable items need to be measured at the beginning and end of each period.[58] If the acquisitions approach to storable goods is taken, then household purchases of essential storable goods at the beginning of the lockdown period will be very much larger than pre-lockdown purchases of the same goods. Once the lockdown has been in place for a month or two, then purchases of storables should fall back to pre-lockdown levels. But the problem here is that the assumption of a constant basket equal to a pre-lockdown basket for all post lockdown periods is going to be a rather poor assumption.

7.5 The Problem of Free Dwelling Rent

As the duration of the lockdown continues, an increasing number of tenants will not be able to pay their rents. However, due to government regulations or due to the forbearance of landlords, they will not be evicted. If there is a policy of rent forgiveness, then these nonpaying tenants are getting free rent. The question for the construction of a CPI is: what should be done under these circumstances?

Before we answer the above question, it will be useful to look at how rent indexes could be constructed for CPI purposes if conditions were "normal" with no lockdown. Consider the case where a rent subindex involves H rental properties; i.e., the target index is a rent index for H households who are living in rental housing. Typically, since the units of measurement for these units will be unique (since the location of each rental property is unique and location is an important rent determining characteristic). Thus suppose that the *observed rents* for the H properties in period t are $v_h^t > 0$ for $t = 0,1$ and $h = 1,...,H$.

[58] Real actual consumption of a storable good is equal to beginning of the period inventory stock plus new purchases of the good less end of period stock of the good. In principle, if actual consumption is the target concept, then household stocks of storables should be capitalized and added to household wealth. In normal times, the services provided by these storable stocks probably should not be added to the current flow of consumption unless one argues that these stocks are desirable in their own right as a form of insurance against future supply shocks. In times of a pandemic, such an argument seems reasonable. Note that not recognizing a flow of services from the storables stock is a different treatment from the treatment of the services that consumer durables provide over their useful lifetime. Stocks of consumer durables should also be capitalized and added to household wealth but the services that durables render during a month need to be recognized as part of actual consumption.

We need to decompose these values into price and quantity components. This can be done as follows: (i) define all period 0 prices for the rental properties equal to 1; (ii) set the corresponding period 0 quantities q_h^0 equal to the corresponding period 0 rental values v_h^0; (iii) set the corresponding period 1 quantities equal to $q_h^1 \equiv q_h^0(1-\delta_h)$ where $\delta_h \geq 0$ is the one period depreciation rate or quality adjustment parameter that adjusts the period 0 quantity q_h^0 for depreciation of the structure;[59] (iv) the period 1 constant quality price for the nth rental property p_h^1 is set equal to actual period 1 rent, v_h^1, divided by the corresponding period 1 quantity q_h^1. Thus we have the following definitions for h = 1,...,H:

(60) $p_h^0 \equiv 1$;

(61) $q_h^0 \equiv v_h^0$;

(62) $q_h^1 \equiv q_h^0(1-\delta_h) = v_h^0(1-\delta_h)$;

(63) $p_h^1 \equiv v_h^1/q_h^1 \quad = v_h^1/[v_h^0(1-\delta_h)]$.

Using the above assumptions (60)-(63), the Laspeyres and Paasche rent subindexes, P_L and P_P, can be written as follows:

$$(64)\ P_L \equiv \Sigma_{h=1}^H p_h^1 q_h^0/\Sigma_{h=1}^H p_h^0 q_h^0$$
$$= \Sigma_{h=1}^H v_h^1(1-\delta_h)^{-1}/\Sigma_{h=1}^H v_h^0$$
$$= \Sigma_{h=1}^H S_h^0[v_h^1(1-\delta_h)^{-1}/v_h^0]\ ;$$

$$(65)\ P_P \equiv \Sigma_{h=1}^H p_h^1 q_h^1/\Sigma_{h=1}^H p_h^0 q_h^1$$
$$= \Sigma_{h=1}^H v_h^1/\Sigma_{h=1}^H v_h^0(1-\delta_h)$$
$$= \{\Sigma_{h=1}^H S_h^1[v_h^1(1-\delta_h)^{-1}/v_h^0]^{-1}\}^{-1}$$

where the period t expenditure share of household h in total household expenditure on rental housing S_h^t are defined as follows:

[59] A more complete housing model would decompose the rental price of a dwelling unit into the sum of two parts: a structure component and a land component. Depreciation would apply only to the structure component.

(66) $S_h^t \equiv v_h^t/\Sigma_{i=1}^H v_i^t$; t = 0,1 and h = 1,...,H.

P_L is a base period share weighted *arithmetic average* of the depreciation adjusted rent ratios, $v_n^1(1-\delta_n)^{-1}/v_n^0$, and P_P is a current period share weighted *harmonic average* of the depreciation adjusted rent ratios. Usually, P_P will be less than P_L.[60] If the statistical agency is able to collect data on paid rents and can make estimates for monthly depreciation rates, then both P_L and P_P can be constructed and hence our preferred rent subindex, the Fisher index $P_F \equiv [P_L P_P]^{1/2}$, can also be constructed.

Suppose household h gets a rent holiday in period 1 so that $p_h^1 = v_h^1 = 0$ but q_h^1 still equals $q_h^0(1-\delta_h)$. We can calculate a reservation price p_h^{1*} which will be consistent with utility maximizing behavior on the part of household h during period 1; see Appendix C for the relevant methodology. In practice it will typically be difficult to estimate these reservation prices, but either carry forward prices or inflation adjusted carry forward prices can be taken as approximate reservation prices.[61]

In the case of rental housing, carry forward values from period 0 may provide an adequate approximation to the period 1 imputed values for rental properties that are available at no charge. If property h is rented at a zero price for period 1, define the prices and quantities for periods 0 and 1 using the assumption $v_h^1 = v_h^0$ as follows:

(67) $p_h^0 \equiv 1$; $q_h^0 \equiv v_h^0$; $q_h^1 \equiv q_h^0(1-\delta_h) = v_h^0(1-\delta_h)$; $p_h^1 \equiv v_h^0/q_h^1 = (1-\delta_h)^{-1}$.

[60] If rents are equal for the two periods so that $v_h^1 = v_h^0$ for h = 1,...,H, then the expenditure shares will also satisfy $S_h^1 = S_h^0$ for h = 1,...,H and by Schlömilch's inequality, $P_P \le P_L$; see Hardy, Littlewood and Polya (1934; 26).

[61] Appendix C also addresses the problems that arise if a partial rent payment is made under the assumption that the remaining rent owed is forgiven. If instead, the unpaid portion of rent is merely deferred, then the analysis of this situation is much more complex: should the deferred rent be added to the regular rent payment in a future period when the deferred rent is paid back to the landlord? Or will it be necessary to create a new category in the System of National Accounts that keeps track of these deferred rent liabilities?

If all H properties had rent forgiveness in period 1, the Laspeyres and Paasche price indexes would take the following form using the carry forward values for the imputed period 1 values of rental services:[62]

(68) $P_L \equiv \Sigma_{h=1}^{H} p_h^1 q_h^0 / \Sigma_{h=1}^{H} p_h^0 q_h^0$

$= \Sigma_{h=1}^{H} S_h^0 (1-\delta_h)^{-1}$

> 1 ;

(69) $P_P \equiv \Sigma_{h=1}^{H} p_h^1 q_h^1 / \Sigma_{h=1}^{H} p_h^0 q_h^1$

$= \{\Sigma_{h=1}^{H} S_h^1 [v_h^1 (1-\delta_h)^{-1} / v_h^0]^{-1}\}^{-1}$

$= \{\Sigma_{h=1}^{H} S_h^1 (1-\delta_h)\}^{-1}$ using $v_h^1 = v_h^0$ for h = 1,...,H

> 1.

Thus using carry forward values as imputed values for free rental housing in period 1 leads to Laspeyres and Paasche indexes which are greater than 1. It is the depreciation of the structure portion of rents that leads to this result.[63]

7.6 How Should Scanner Data be Combined with Web Scraped Data?

Many statistical agencies now have access to scanner data from some retailers. How exactly should the indexes which are generated by the use of these data be combined with traditional price data collected by statistical agency employees or by the use of web scraped data?

In general, it is preferable if the contribution of these two sources of price data be combined in an index which weights the prices according to their economic importance;

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[62] It is also the case that under the assumption that $v_h^1 = v_h^0$ for h = 1,...,H, we have $P_L > P_P > 1$. Thus the more appropriate Fisher index will lie between P_L and P_P.

[63] It will not be easy to determine appropriate depreciation rates because the ratio of land to structure will differ across various types of rental property. Hedonic regression techniques can be used to determine appropriate depreciation rates; see Diewert, de Haan and Hendricks (2015), Diewert and Shimizu (2015) (2017) and Diewert, Nishimura, Shimizu and Watanabe (2020). The hedonic regression techniques used in these papers can be used to provide decompositions of rent into land and structure components and they also provide estimates for structure depreciation rates.

i.e., to their shares of expenditure in the elementary category under consideration. It is not a problem to calculate expenditure shares for the scanner data but the web scraped data will not come with the associated expenditure data and so weighting the two sources of data by their relative quantities or expenditure shares will not be possible. In the end, some rough explicit or implicit estimate of the relative economic importance of the two sources of data will have to be made. Area specialists in NSOs will have to provide approximate weights for each elementary category that uses the two sources of information.

8. Conclusion

The current pandemic conditions have never been experienced since the establishment of modern economic statistics, hence responses to ensure the continued production of high quality economic statistics on key variables in these conditions have not previously been rigorously laid out.

The current recommended approaches for dealing with products disappearing due to lockdowns or consumers stockpiling draw on the standard response to disappearing products in any period. That is, the advice is essentially to continue with current practice as if nothing has happened, using imputed prices for missing goods and expenditure weights from a pre-lockdown period. For example, Eurostat advice for construction of the Harmonized Index of Consumer Prices (HICP) includes the following:[64]

1. "The HICP weights are updated at the beginning of each year and are kept constant throughout the year. Thus, the weights will not change this year as a result of the impact of the COVID-19 on expenditures."
2. All sub-indices for the product classifications "will be compiled even when for some categories no products are available on the market."

[64] The HICP is the European Union's standardised measure of consumer price inflation that each member country must construct. See Appendix C for an extended quote of the advice, along with information on the recommendations from the IMF, UNECE and U.S. Bureau of Labor Statistics.

We have demonstrated that following the advice on price imputations will lead to overstated estimates of changes in real consumption and understated estimates of changes in the cost of living. We have pointed out the problems of using expenditure weights which are irrelevant for the periods under consideration. In addition, we have considered a range of other difficult measurement problems that arise, such as how to deal with “rent holidays”.

Three steps that NSOs can take to provide as much information as possible on price indexes during a time of lockdown are:

1. Collect whatever prices are available, including from non-traditional sources. For missing prices, use inflation adjusted carry forward prices.[65]
2. Urgently start a program to obtain current expenditure weights for the consumption basket.
3. Produce a revisable CPI as an analytical series that can be updated as new methodology is developed and new data sources are exploited.

The U.S. Bureau of Labor Statistics (BLS) approach to dealing with the coronavirus pandemic is very much in line with the approach advocated in this paper;[66] i.e., statistical agencies need to move away from a fixed basket Lowe index and attempt to produce approximations to Laspeyres and Paasche indexes (and hence Fisher indexes)[67] so that consumer price inflation is measured by an index that is relevant to current consumer expenditure patterns. The example set by the BLS shows that it is not impossible to produce household expenditure information in real time. Given that lockdown conditions may apply in varying degrees for up to 18 months, it is important to have information on current period household expenditure patterns so that meaningful estimates of consumer price inflation can be produced during the lockdown periods.

[65] While we favour using reservation prices, we acknowledge that currently it is unlikely that NSOs will be able to estimate these in a timely fashion.
[66] See Appendix C for more on their recommendations.
[67] The production of approximate Walsh or Törnqvist indexes as alternatives to the Fisher index is also possible. Note that the BLS produces an approximation to the Törnqvist index in real time.

It is unlikely that expenditure patterns will revert to the pattern that prevailed in periods just before the first lockdown period. Hence the argument that inflation adjusted carry forward pricing will produce an accurate index when the lockdown period ends is dubious. But what is clear is that the use of adjusted carry forward prices will not produce an accurate CPI within the lockdown period. New basket information is required in order to produce a meaningful CPI within the lockdown period. Thus establishing a continuous consumer expenditure survey is key to producing a meaningful CPI during these turbulent times.

Appendix A: Eurostat, IMF, UNECE and U.S. Bureau of Labor Statistics Advice

National and international statistical agencies are attempting to provide advice to national price statisticians on how to construct a CPI under lockdown conditions. Below, we give a sampling of this advice.

Here is the advice from Eurostat to European Union countries on how to calculate the EU's Harmonized Index of Consumer Prices (HICP):

"The compilation of the HICP in the context of the COVID-19 crisis is guided by the following three principles:

- Stability of the HICP weights,
- Compilation of indices covering the full structure of the European version of the Classification of Individual Consumption According to Purpose (ECOICOP),
- Minimizing the number of imputed prices and sub-indices.

The first principle ensures that there will be no change in the sub-index weights used in the compilation of the HICP during the year, which is the standard practice. The HICP sub-indices are aggregated using weights reflecting the household consumption expenditure patterns of the previous year. The HICP weights are updated at the beginning of each year and are kept constant throughout the year. Thus, the weights will not change this year as a result of the impact of the COVID-19 on expenditures.

The second principle means that all sub-indices for the full ECOICOP structure will be compiled even when for some categories no products are available on the market. In such cases, prices do not exist and they should be replaced with imputed prices. Sub-indices consisting of both imputations and observed prices should be compiled and aggregated using the standard HICP compilation procedures.

Finally, the third principle underlines the idea that, whenever possible, missing price observations should be replaced by price quotes obtained from other sources. Price collection can fail because of restrictions that do not allow price collectors to visit sampled outlets, because outlets have been closed down or because it is impossible to offer certain services (e.g. flights). Possible sources to replace the missing prices in case manual price collection activities are restricted are the following:

- Outlets' websites,
- Telephone and email enquiries.

Some NSIs may also have access to scanner data that, although not yet integrated into the HICP production system, could be used for the replacement of the missing prices. However, the replacements using scanner data should be done with care as this could entail a change of outlet to a different category or market segment." Eurostat (2020).

Thus the Eurostat advice boils down to carry on as if nothing has happened; i.e., keep the existing fixed basket methodology, use web scraped or telephone interview price data to replace previously personally collected data and if a consumption category disappears due to shut down restrictions, use carry forward imputed prices. This goes against the conclusions of this paper, but the Eurostat advice is understandable. In the short run, it is impossible for Eurostat to change their methodology to a variable basket approach. Thus the hope is probably that the lockdown period will be brief and when economies return to "normal", the fixed basket approach will again be satisfactory and the use of the old fixed basket and imputation methods will enable price comparisons with the pre-lockdown period to be reasonably accurate.

Here is the advice from the UNECE:

"Price collection may be restricted due to closed outlets or price collectors may not be allowed to work or enter outlets. It may also be that outlets do not provide the usual set of prices through other channels (e.g. on paper or via e-mail) and/or there may be shortage of staff in the main office to receive and process the prices that are received. Alternative modes of price collection include telephone, e-mail, online prices and scanner data. However, it may be difficult to ensure a minimum coverage of all products (goods and services). In particular, this may be the case for products for which price collectors usually collect prices. This could, for example, be the case for clothing and fresh food in many countries. In such cases the statistical office may have to rely on collecting a minimum of prices for the most important or the most representative products.

Compilation

For imputation of observations the general recommendation is to follow a bottom-up approach. This means that the first choice is to impute missing prices with observed price developments of similar products or products that are expected to have similar price developments. If such product prices are not available, the next choice will be to impute the missing prices with the average price development of the product group or the elementary aggregate to which the product belong. If these are not available, the closest available higher-level price index should be used for the imputation.

In some instances it may not be possible to collect prices for specific product groups or elementary aggregates or even indices above the elementary aggregate level. In such cases the price development of the product group or the elementary aggregate may be imputed by the price development of similar product groups or elementary aggregates. If this is not possible, the price development may be imputed by the higher-level index in which the product group or the elementary aggregate enters. However, imputation of a missing elementary aggregate by the overall CPI may also be justified. This corresponds to leaving the elementary aggregate out of the calculation of the CPI. This may be the preferred option if households' expenditures on an elementary aggregate is assessed to be zero or close to zero. In some countries this may be the case for e.g. international travels, domestic airline travels, child care and sports and cultural events.

These are general recommendations. National circumstances and knowledge of the developments for particular markets and products must be considered. In all cases, it is important to apply imputation methods that ensure the index reaches the correct level when again it becomes possible to collect prices and include them in the index." UNECE (2020)

The advice from the UNECE is similar to the advice from Eurostat but the last sentence in the above quotation provides an explicit explanation for the carry on as usual methodology; i.e., when things return to "normal", the post lockdown CPI indexes will be comparable to the pre-lockdown CPI index.

Here is the advice from the International Monetary Fund:

"When facing increased numbers of missing prices, it is important to remind that all temporarily missing prices should be imputed using one of the methods described in Consumer Price Index Manual: Concepts and Methods. As noted in the Manual, carrying forward, or repeating the last available price, should be avoided as it introduces a downward bias into the index. The imputation techniques described in the Manual do not introduce bias into the index. Imputations are self-correcting, which means that once a price can be collected, the index returns to the correct level. This is important so that the CPI continues to provide a reliable estimate of price change. The CPI is a critical input to economic policy making, particularly during periods of economic uncertainty.

If an entire index is missing, it is recommended to use the next level up in aggregation as the basis for making the imputation. For example, if all prices for oranges are missing, the index for citrus fruits can be used as the basis for making the imputation. If all citrus fruits are missing, the index for fruits is used as the basis for making the imputation. If all fruits are missing, the index for fruits and vegetables is used. If fruits and vegetables are missing, the index for food is used. If the index for food is missing, the index for food and non-alcoholic beverages is used. Finally, if all food and non-alcoholic beverages is missing, the All Items index is used as the basis to impute.

...

Users will continue to need data at the most detailed level. All indexes should continue to be published, even if they are imputed. As noted previously, all imputed indexes should be flagged and clearly noted for users. It is important for transparency that users are able to access the full set of data that are normally disseminated."

IMF (2020)

The above advice is in line with the guidance provided by Eurostat and the UNECE. It is more explicit in one respect in that it rules out simple carry forward pricing and endorses inflation adjusted carry forward prices. We also endorse inflation adjusted carry forward prices over the use of carry forward prices.

Finally, here is some information on how the US Bureau of Labor Statistics is planning to produce its CPI under current conditions:

"**How are prices collected for the CPI?** Price data used to calculate the CPI are primarily provided by two different surveys that are administered continuously each month:

- Commodities and Services Pricing Survey, an establishment survey of businesses selling goods and services to consumers, used to provide the price data for the CPI.
- Housing Survey, a survey of landlords and tenants used to provide rent data for CPI's shelter indexes.

Survey operations for CPI pricing surveys may be affected by limitations on data-collection staff, the availability of survey respondents, and the availability of items. Note that CPI data are collected throughout the entire month. Specifically, any given price in the CPI sample is collected in one of three defined pricing periods, corresponding roughly to the first 10, second 10, and final 10 days of the month. BLS uses several data-collection modes for CPI surveys that include telephone, internet, and automated electronic data capture. However, the majority of data are collected by personal visit. About 65 percent of CPI price data and 50 percent of CPI rent data are typically collected by personal visit. This type of collection has been suspended since March 16, 2020. (It was suspended on March 5th in the Seattle area.)

What happens if BLS cannot collect CPI data? The percentage of prices in the CPI sample that may be unavailable, either because the outlet is closed or the item is out of stock, is expected to increase. When BLS cannot obtain a price either because of data-collection limitations or the item being unavailable, it will generally be considered "temporarily unavailable." The CPI program has specific procedures for handling temporarily unavailable prices. Missing prices are generally imputed by the prices that are collected in the same or similar geographic area and item category. Essentially, the price movement of items that are not collected is estimated to be the same as those that are collected for a given item and geographic area.

Will data collection for CPI expenditure weights be affected? The Consumer Expenditure Survey (CE), a household survey capturing consumer spending data, is used to calculate relative importances (weights) of goods and services in the CPI market basket. CE in-person data collection ceased on March 19, 2020. CE data are collected by the U.S. Census Bureau through an agreement with BLS. The Census Bureau is transitioning to collecting these data through telephone. Changes to CE survey operations will not have an immediate impact on CPI data, but may have long-term impacts. These weights are used in the chained CPI index (C-CPI-U). The March 2020 weights will be incorporated in the final March 2020 chained CPI indexes, which are released in February 2021. BLS also incorporates the CE weights in an annual weight update to the CPI-U and CPI-W indexes. These weight updates will be effective with the January 2022 indexes, released in February 2022. BLS is working on mitigation strategies to reduce measurement error of CPI weights caused by a potential loss of CE survey data.

Under what circumstances would some data not be published? A CPI index is not published if it fails a data-quality standard known as an adequacy ratio. Specifically, if BLS fails to collect at least one price in a geographic area that account for more than half the geographic weight of the index, the index is not published. Even in months without disruptions, some minor indexes with small samples occasionally fail this standard and are not published. (One example is Repair of household items.) Data-collection disruptions would have to be extremely severe for major CPI indexes not to be published based on this standard. Data-collection disruptions may be more severe in some area than others, and it is possible that some data for metro areas may fail data quality-standards and not be published. BLS will continue to monitor data-collection disruptions." Bureau of Labor Statistics (2020)

The BLS approach to dealing with the COVID-19 pandemic is very much in line with the approach advocated in this paper.

Appendix B: Measuring Real Consumption when there are only Two Commodities

We show that the bias in a CPI that uses inflation adjusted carry forward prices will produce an inflation estimate for the first shut down period that is too low as compared to a cost of living index that uses reservation prices for the commodities that are not available. The companion estimate of real consumption (or welfare) that uses a Lowe index to deflate nominal household expenditures into an estimate for real consumption will be too high.

It may be useful for many readers to have a figure which can explain the underlying index number theory in a relatively simple way. In this Appendix we consider the case where M = 1 and N = 1; i.e., we have only one continuing commodity q and one unavailable commodity Q.

We assume that the household or group of households have preferences that can be represented by the *utility function*, f(q,Q), which is linearly homogeneous, increasing and concave in q,Q. The dual unit cost function for this utility function is c(p,P) where p and P are the positive prices for a unit of q and Q respectively.[68]

The *observed quantity data* for periods 0 and 1 are (q^0,Q^0) and (q^1,Q^1). The *observed price data* for period 0 are (p^0,P^0) but for period 1, only the price for a unit of the continuing commodity is observed, $p^1 > 0$; the price for the unavailable commodity is the *Hicksian reservation price* $P^{1*} > 0$. We assume that Q^1 is equal to 0:

(B1) $Q^1 = 0$.

The *period t utility level* attained by the household is denoted by u^t for t = 0,1. We have the following definitions:

(B2) $u^0 \equiv f(q^0,Q^0) > 0$; $u^1 \equiv f(q^1,0) > 0$.

Denote the *observed expenditure on consumer goods and services* in period t by v^t. We have the following definitions:

(B3) $v^0 \equiv p^0q^0 + P^0Q^0$;

(B4) $v^1 \equiv p^1q^1 + P^{1*}Q^1$

$= p^1q^1$ using assumption (B1).

[68] The function c(p,P) is defined as $c(p,P) \equiv \min_{q,Q} \{pq + PQ : f(q,Q) \geq 1 ; q \geq 0, Q \geq 0\}$. The unit cost function is also increasing, linearly homogeneous and concave in p and P. We assume that it is also differentiable. For the early history of duality theory and its application to index number theory, see Diewert (1974) (1976) (2020a).

When the economic approach to index number theory is used, it is assumed that observed expenditures on consumer goods and services is equal to the minimum cost of achieving the utility level for the period under consideration. Using this approach we have the following equalities:

(B5) $v^0 = c(p^0,P^0)f(q^0,Q^0) = c(p^0,P^0)u^0$;
(B6) $v^1 = c(p^1,P^{1*})f(q^1,Q^1) = c(p^1,P^{1*})f(q^1,0) = c(p^1,P^{1*})u^1$.

Let $c_1(p,P)$ denote the partial derivative of $c(p,P)$ with respect to p; i.e., $c_1(p,P) \equiv \partial c(p,P)/\partial p$ and $c_2(p,P) \equiv \partial c(p,P)/\partial P$. The assumption of cost minimizing behavior on the part of households along with Shephard's Lemma implies that the following relationships will hold:[69]

(B7) $q^0 = c_1(p^0,P^0)u^0$;
(B8) $Q^0 = c_2(p^0,P^0)u^0$;
(B9) $q^1 = c_1(p^1,P^{1*})u^1$;
(B10) $Q^1 = c_2(p^1,P^{1*})u^1$
$= 0$ using assumption (B1).

Suppose the household spent *all* of its period t expenditure on consumer goods and services, v^t, on purchases of the continuing commodity q. Denote this period t hypothetical expenditure on purchases of q by q^{te}, the *budgetary equivalent expenditure* on the always available commodity. We have the following definitions:

(B11) $q^{0e} \equiv v^0/p^0$
$= [p^0q^0 + P^0Q^0]/p^0$ using (B3)

[69] See Shephard (1953; 11) or Diewert (2020a; 11). Shephard's Lemma implies that the consumer demand functions, q(u,p,P) and Q(u,p,P), regarded as functions of the consumer's utility level u and the prices, p and P that the consumer faces, are equal to $q(u,p,P) = c_1(p,P)u$ and $Q(u,p,P) = c_2(p,P)u$. Hicks (1946; 311-331) introduced this type of demand function into the economics literature and so these functions are known as *Hicksian demand functions*. They can be estimated using econometric techniques; see Diewert and Feenstra (2019).

$= q^0 + [P^0/p^0]Q^0.$

(B12) $q^{1e} \equiv v^1/p^1$

$= p^1q^1/p^1$ using (B4)

$= q^1.$

Since there are no purchases of the unavailable commodity in period 1, q^{1e} turns out to equal the observed consumption of the continuing commodity, which is q^1.

The minimum cost of achieving the utility level u^0 if the consumer faced the prices of period 1, p^1 and P^{1*}, is $c(p^1,P^{1*})u^0$. Since $f(q^0,Q^0)$ equals u^0, we see that (q^0,Q^0) is a feasible solution for this cost minimization problem and hence we have the following inequality:

(B13) $c(p^1,P^{1*})u^0 \leq p^1q^0 + P^{1*}Q^0.$

Similarly, the minimum cost of achieving the utility level u^1 if the consumer faced the prices of period 0, p^0 and P^0, is $c(p^0,P^0)u^1$. Since $f(q^1,Q^1) = f(q^1,0)$ equals u^1, we see that $(q^1,0)$ is a feasible solution for this cost minimization problem and hence we have the following inequality:

(B14) $c(p^0,P^0)u^1 \leq p^0q^1 + P^0Q^1 = p^0q^1.$

Now convert the hypothetical expenditures $c(p^1,P^{1*})u^0$ into purchases of q using the price of the continuing commodity for period 1, p^1, to obtain the hypothetical quantity q^{0*}:

(B15) $q^{0*} \equiv c(p^1,P^{1*})u^0/p^1$

$\leq [p^1q^0 + P^{1*}Q^0]/p^1$ using (B13)

$= q^0 + [P^{1*}/p^1]Q^0$

$\equiv q^{0e*}$

where q^{0e*} converts the period 0 purchases Q^0 of the disappearing commodity into equivalent amounts of the continuing commodity using the relative prices of period 1 and adds this amount to the period 0 actual purchases of the continuing commodity q^0.[70]

Convert the hypothetical expenditures $c(p^0,P^0)u^1$ into purchases of q using the price of the continuing commodity for period 0, p^0, to obtain the hypothetical quantity q^{1*}:

$$
\begin{aligned}
\text{(B16)}\quad q^{1*} &\equiv c(p^0,P^0)u^1/p^0 \\
&\leq p^0q^1/p^0 && \text{using (B14)} \\
&= q^1.
\end{aligned}
$$

Suppose that the consumption vectors, (q^0,Q^0) and (q^1,Q^1) are given for periods 0 and 1 along with a household utility function, $f(q,Q)$. Define the period 0 and 1 utility levels by $u^0 \equiv f(q^0,Q^0)$ and $u^1 \equiv f(q^1,Q^1)$. Finally, suppose that the reference prices $p > 0$ and $P > 0$ are given. The family of *Allen quantity indexes*,[71] $Q_A(p,P,u^0,u^1)$, is defined as follows:

$$\text{(B17)}\quad Q_A(p,P,u^0,u^1) \equiv C(u^1,p,P)/C(u^0,p,P)$$

where $C(u,p,P)$ is the household's minimum cost of achieving the utility level u if it faces the prices p,P. Since we have assumed that the utility function is linearly homogeneous,[72] the consumer's total cost or expenditure function, $C(u,p,P)$, factors into the product of the unit cost function, $c(p,P)$ times the utility level u; i.e., we have $C(u,p,P) = c(p,P)u$. If we substitute this factorization of the cost function into definition (B17), we find that the Allen quantity index collapses down to the utility ratio, u^1/u^0; i.e., we have the following equality for all choices of the reference prices, p and P:

$$\text{(B18)}\quad Q_A(p,P,u^0,u^1) \equiv C(u^1,p,P)/C(u^0,p,P)$$

---

[70] The hypothetical quantity q^{0e*} is useful when we define the Paasche quantity index; see Figure 1 below.

[71] Each choice of p and P generates a possibly different Allen quantity index that measures aggregate quantity change between periods 0 and 1. The definition of the Allen (1949) quantity index provides a useful way to cardinalize a measure of consumer utility.

[72] In the economics literature, the assumption of a linearly homogeneous utility function is sometimes called the assumption of homothetic preferences.

$= c(p,P)u^1/c(p,P)u^0$
$= u^1/u^0.$

Equations (B18) can be used to obtain alternative ways of estimating the Allen quantity index under our assumptions. These alternative expressions for u^1/u^0 can be illustrated by looking at Figure 1 below. Our first alternative way of expressing the utility ratio uses (B18) as follows:

(B19) $u^1/u^0 = c(p^0,P^0)u^1/c(p^0,P^0)u^0$
$= p^0q^{1*}/[p^0q^0 + P^0Q^0]$ using (B3), (B5) and (B16)
$= q^{1*}/[q^0 + (P^0/p^0)Q^0]$
$= q^{1*}/q^{0e}$ using (B11).

Our second alternative way of expressing the utility ratio uses (B18) as follows:

(B20) $u^1/u^0 = c(p^1,P^{1*})u^1/c(p^1,P^{1*})u^0$
$= p^1q^1/p^1q^{0*}$ using (B3), (B5) and (B15)
$= q^1/q^{0*}.$

The alternative quantities of the continuing commodity defined above, q^0, q^1, q^{0*}, q^{1*}, q^{0e}, q^{1e} and q^{0e*}, are all illustrated in Figure 1.

The ingredients that go into the construction of the Laspeyres and Paasche quantity indexes can also be illustrated in Figure 1. The *Laspeyres quantity index*, Q_L, is defined as follows:

(B21) $Q_L \equiv [p^0q^1 + P^0Q^1]/[p^0q^0 + P^0Q^0]$
$= p^0q^1/[p^0q^0 + P^0Q^0]$ using (B1), $Q^1 = 0$
$= s_q^0[q^1/q^0]$ where $s_q^0 \equiv p^0q^0/v^0$
$= q^1/[q^0 + (P^0/p^0)\,Q^0]$
$= q^1/q^{0e}$ using (B15)

$\geq q^{1*}/q^{0e}$ using (B16)

$= u^1/u^0$ using (B19).

Hence the Allen quantity index (equal to the utility ratio, u^1/u^0) is bounded from above by the ordinary Laspeyres quantity index Q_L. In Figure 1, Q_L is equal to q^1/q^{0e} and u^1/u^0 is equal to q^{1*}/q^{0e}. Q_L will typically have a large upward bias relative to the true Allen index under our assumptions.

The *Paasche quantity index*, Q_P^*, is defined as follows:[73]

(B22) $Q_P^* \equiv [p^1q^1 + P^{1*}Q^1]/[p^1q^0 + P^{1*}Q^0]$

$= p^1q^1/[p^1q^0 + P^{1*}Q^0]$ using (B1), $Q^1 = 0$

$= q^1/[q^0 + (P^{1*}/p^1)\, Q^0]$

$= q^1/q^{0e*}$ using (B15)

$\leq q^1/q^{0*}$ using (B15)

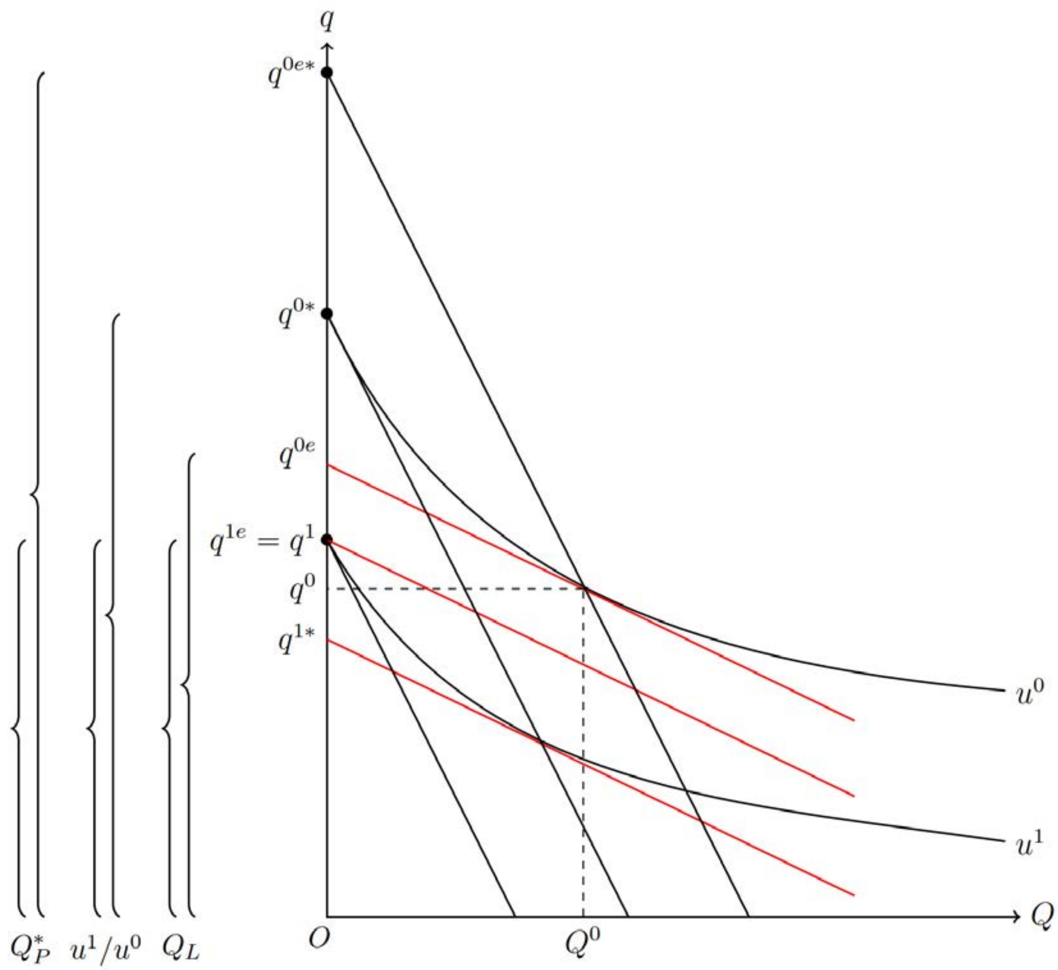
$= u^1/u^0$ using (B20).

Hence the Allen quantity index is bounded from below by the ordinary Paasche quantity index Q_P^*. In Figure 1, Q_P^* is equal to q^1/q^{0e*} and u^1/u^0 is equal to q^1/q^{0*}. It can be seen that Q_P^* is well *below* the true utility ratio, $u^1/u^0 = q^1/q^{0*}$, since q^{0e*} is well above q^{0*}. It can also be seen that $Q_L = q^1/q^{0e}$ is well *above* $u^1/u^0 = q^1/q^{0*}$ since q^{0e} is well below q^{0*}.[74]

[73] We attach an asterisk to P_P because we require an estimate for the period 1 reservation price, P^{1*}, in order to evaluate the index using observable data.

[74] Figure 1 uses the preferences of period 0 to illustrate the decline in real consumption. If we used the preferences of the household for period 1, then the change in real consumption can be measured by the utility ratio $f(q^1,0)/f(q^0,0) = q^1/q^0$ using the linear homogeneity of $f(q,Q)$. For the example illustrated in Figure 1, we see that $q^1/q^0 > 1$. Thus using the post lockdown preferences, we have a utility increase whereas using the pre-lockdown preferences, we have $u^1/u^0 = f(q^1,0)/f(q^0,Q^0) < 1$, a utility decrease. This illustrates the need for more than one CPI and more than one estimate for real consumption growth as we transition from the pre-lockdown situation to the lockdown situation.

Figure 1: Welfare Measurement in the Two Good Case



There are two sets of three parallel lines in Figure 1. The slope of the red straight line that is tangent to the period 0 indifference curve at (q^0,Q^0) is equal to $-P^0/p^0$ while the slope of the black straight line that is tangent to the period 1 indifference curve at $(q^1,0)$ is equal to $-P^{1*}/p^1$. Note that $P^{1*}/p^1 > P^0/p^0$, which reflects that the (imputed) market clearing price for the unavailable commodity in period 1, P^{1*}, is much greater than the inflation adjusted carry forward price for the unavailable good, $(p^1/p^0)P^0$. However, if q and Q are *perfect substitutes*, then the period 0 indifference curve will coincide with the straight line (this line represents the period 0 budget constraint of the household) that is tangent to the period 0 indifference curve. In this case, it can be seen that q^{0e} will coincide with q^{0*} and

q^{0e*} while q^1 will coincide with q^{1*}. In this case, the Allen quantity index, u^1/u^0 will be equal to Q_L, Q_P^* and Q_F^*.

In the perfect substitutes case, the household utility function is equal to the linear function, $f(q,Q) \equiv \alpha_q q + \alpha_Q Q$ where α_q and α_Q are positive constants that reflect the relative utility to the household for the consumption of a unit of each commodity. The Allen quantity index, u^1/u^0, is then equal to $[\alpha_q q^1 + \alpha_Q Q^1]/[\alpha_q q^0 + \alpha_Q Q^0]$. The corresponding *consumer price index* is equal to $[v^1/v^0]/[u^1/u^0]$ which in turn is equal to $[v^1/v^0]/[(\alpha_q q^1 + \alpha_Q Q^1)/(\alpha_q q^0 + \alpha_Q Q^0)] = [v^1/(\alpha_q q^1 + \alpha_Q Q^1)]/[v^0/(\alpha_q q^0 + \alpha_Q Q^0)] = v_\alpha{}^1/v_\alpha{}^0$ where the period t *quality adjusted unit value* is defined as $v_\alpha{}^t \equiv v^t/(\alpha_q q^t + \alpha_Q Q^t)$ for t = 0,1. Thus in the perfect substitutes case, the Allen quantity index is equal to a quality adjusted unit value index, which can readily be calculated provided that estimates for α_q and α_Q are available. A simple choice for the α's is to set them equal to the corresponding base period prices; i.e., set $\alpha_q = p^0$ and $\alpha_Q = P^0$. In this case, the quality adjusted unit value index collapses down to the Paasche price P_P which will be defined below by (B36).

The point of this digression into quality adjusted unit value indexes is that this type of index can be readily implemented by statistical agencies without having to do any econometric estimation[75] and this type of index is perfectly acceptable even under lockdown conditions, provide that the group of products under consideration are close substitutes so that the household indifference surfaces for these products are close to being parallel planes. Thus the indifference curves in Figure 1 as drawn are applicable at *higher levels of aggregation* where the product groups under consideration are *not* close substitutes. In this case, it will be necessary to estimate reservation prices in order to

[75] Econometric estimation may lead to improved estimates for the α's. For example, suppose the statistical agency has information on the unit value prices for N highly substitutable commodities prevailing in a number of past periods. Denote the price vector for period t as $p^t \equiv [p_1{}^t,...,p_N{}^t]$ for t = 1,...,T. Then a reasonable estimator for the vector of quality adjustment factors is $\alpha \equiv (1/T)\,\Sigma_{t=1}{}^T\,(p_1{}^t)^{-1}p^t$; i.e., take the arithmetic average of past vectors of normalized product prices where the normalized vector of prices for period t is the period t vector of prices p^t divided by the price of the numeraire product 1. The numeraire product should be chosen to be the product with the largest average share of expenditures on the N products. As another example where econometrics can play an important role in estimating the α's, we note that most hedonic regression models can be interpreted as additive utility models; see Diewert (2020d).

approximate a cost of living index. If the products are close substitutes, then the indifference curves should be drawn as curves that are almost straight lines. In this case, quality adjusted unit values could be used instead of attempting to estimate preferences so that reservation prices can be calculated.

From (B21), it can be seen that Q_L is equal to the following observable expressions:

$$\text{(B23)}\ Q_L = q^1/[q^0 + (P^0/p^0)\,Q^0] = q^1/[q^0 + (P^0Q^0)/p^0] = q^1/[v^0/p^0] = [v^1/p^1]/[v^0/p^0] = [v^1/v^0]/[p^1/p^0].$$

Thus period 1 aggregate Laspeyres real consumption can be set equal to q^1 and period 0 aggregate Laspeyres real consumption can be set equal to $q^0 + (P^0/p^0)\,Q^0$ which is equal to $q^0 + v^0/p^0$ which in turn is equal to period 0 real consumption of continuing commodities, q^0, plus the value of period 0 consumption of disappearing commodities due to shutdowns, v^0, deflated by the period 0 price level for continuing commodities, p^0. The final equality in (B23) is also useful; it shows that Q_L is equal to the value ratio, v^1/v^0, divided by the consumer price index for the continuing commodity, p^1/p^0.

From (B22), it can be seen that Q_P^* is equal to the following expression (which cannot be evaluated unless an estimate for the reservation price P^{1*} is available):

$$\text{(B24)}\ Q_P^* = q^1/[q^0 + (P^{1*}/p^1)\,Q^0].$$

Note the similarity of this expression for Q_P^* to the decomposition of Q_L given by the first equality in (B23): the price ratio for soon to be unavailable products to the price of continuing products in period 0, P^0/p^0, that is used in (B22) is replaced by the reservation price for unavailable products to the price of continuing products in period 1, P^{1*}/p^1.

The *inflation adjusted carry forward price for unavailable products* in period 1, P^{1A}, is defined as follows:

(B25) $P^{1A} \equiv (p^1/p^0)P^0$.

The approximate Paasche quantity index that results if we use P^{1A} in place of the true reservation price P^{1*} is $Q_P{}^A$ defined as follows:

$$
\begin{aligned}
\text{(B26) } Q_P{}^A &\equiv q^1/[q^0 + (P^{1A}/p^1)\, Q^0] && \\
&= q^1/[q^0 + (P^0/p^0)\, Q^0] && \text{using (B25)} \\
&= Q_L && \text{using (B23)} \\
&\geq u^1/u^0 && \text{using (B21).}
\end{aligned}
$$

Thus using inflation adjusted prices in our highly simplified model leads to an approximation to the Paasche quantity index that is exactly equal to the observable Laspeyres quantity index defined by (B21). Typically, the strict inequality will hold in (B26) so both Q_L and $Q_P{}^A$ will have upward biases as compared to the economic Allen index, u^1/u^0.

Using definition (B21), we have the following alternative way of expressing Q_L:

$$
\begin{aligned}
\text{(B27) } Q_L &\equiv [p^0q^1 + P^0Q^1]/[p^0q^0 + P^0Q^0] && \\
&= p^0q^1/[p^0q^0 + P^0Q^0] && \text{using (B1), } Q^1 = 0 \\
&= p^0q^0[q^1/q^0]/v^0 && \text{using (B3)} \\
&= s_q{}^0[q^1/q^0]
\end{aligned}
$$

where $s_q{}^0 \equiv p^0q^0/v^0$ is the expenditure share of the continuing commodity in period 0.

Using definition (B22), we have the following alternative way of expressing $Q_P{}^*$:

$$\text{(B28) } Q_P{}^* \equiv [p^1q^1 + P^1Q^1]/[p^1q^0 + P^{1*}Q^0]$$

$= p^1q^1/[p^1q^0 + P^{1*}Q^0]$ using $Q^1 = 0$

$= p^1q^1/[(p^1/p^0)p^0q^0 + (P^{1*}/P^0)P^0Q^0]$

$= [p^1q^1/v^0]/[(p^1/p^0)(p^0q^0/v^0) + (P^{1*}/P^0)(P^0Q^0/v^0)]$

$= [p^1q^1/p^0q^0][p^0q^0/v^0]/[(p^1/p^0)s_q^0 + (P^{1*}/P^0)s_Q^0]$

$= [p^1/p^0][q^1/q^0]s_q^0/[(p^1/p^0)s_q^0 + (P^{1*}/P^0)s_Q^0]$

$= s_q^0[q^1/q^0]/\{s_q^0 + s_Q^0[(P^{1*}/P^0)/(p^1/p^0)]\}$

where $s_Q^0 \equiv P^0Q^0/v^0$ is the period 0 expenditure share of the commodity which will disappear in period 1. If $P^{1*}/P^0 > p^1/p^0$, then $Q_P^* < s_q^0[q^1/q^0] = Q_L$.[76]

From (B21) and (B22), we see that relative to the true index, u^1/u^0, Q_L has an upward bias and Q_P^* has a downward bias. This suggests that taking an average of Q_L and Q_P^* would lead to an index which could provide a closer approximation to the true index. Define the Fisher (1922) quantity index, Q_F^*, as the geometric mean of Q_L and Q_P^*:

(B29) $Q_F^* \equiv [Q_LQ_P^*]^{1/2}$

$= \{[[s_q^0(q^1/q^0)]^2/[s_q^0 + s_Q^0[(P^{1*}/P^0)/(p^1/p^0)]]\}^{1/2}$ using (B27) and (B28)

$= s_q^0(q^1/q^0)/\{s_q^0 + s_Q^0[(P^{1*}/P^0)/(p^1/p^0)]\}^{1/2}$

$= Q_L/\{s_q^0 + s_Q^0[(P^{1*}/P^0)/(p^1/p^0)]\}^{1/2}$ using (B27)

$= Q_L/m(\rho)$

where

(B30) $\rho \equiv (P^{1*}/P^0)/(p^1/p^0)$ and

(B31) $m(\rho) \equiv [s_q^0 + s_Q^0\rho]^{1/2}$.

[76] The economic approach to index number theory will always imply that $P^{1*}/P^0 \geq p^1/p^0$ or $P^{1*} \geq (p^1/p^0)P^0$ which is the inflation adjusted carry forward price for the disappearing product. The economic approach is consistent with $P^{1*} = (p^1/p^0)P^0$ but this equality will hold only if the consumer's indifference curves are parallel straight lines so that q and Q are perfect substitutes after quality adjustment. The perfect substitutes condition is unlikely to hold under a widespread lockdown.

Thus ρ is a *relative inflation rate*; it is the price index for locked out commodities, P^{1*}/P^0, divided by the price index for always available commodities, p^1/p^0, and m(ρ) is the square root of the weighted mean of 1 and ρ, where the weights are the base period expenditure shares, s_q^0 for 1 and s_Q^0 for ρ. Thus if ρ > 1, then the weighted mean of 1 and ρ will be greater than 1 and thus the square root of this weighted mean will also be greater than 1 and thus Q_F^* will be less than the Laspeyres quantity index, Q_L. If ρ = 1, then m(ρ) = 1 and $Q_F^* = Q_L$.

It is useful to provide an additive approximation to m(ρ). It can be seen that m(ρ) can be interpreted as the geometric mean of 1 and the function $s_q^0 + s_Q^0\rho$. An approximation to this geometric mean is the corresponding arithmetic mean. Thus we have the following approximation to m(ρ):[77]

$$\text{(B32)}\quad m(\rho) \approx (½)(1) + (½)(s_q^0 + s_Q^0\rho) = (½)[1 + s_q^0] + (½)s_Q^0\rho.$$

Using (B29) and (B32), we have the following approximation to the Fisher quantity index:

$$\text{(B33)}\quad Q_F^* \approx s_q^0(q^1/q^0)/(½)\{[1 + s_q^0] + [s_Q^0(P^{1*}/P^0)/(p^1/p^0)]\}.$$

Up to this point, we have assumed that the household's utility function, f(q,Q) was linearly homogeneous, increasing and concave. If we are willing to assume that f(q,Q) has a certain functional form, then it can be shown that the Fisher quantity index Q_F^* defined by (B29) is exactly equal to the true Allen quantity index, u^1/u^0, under the assumption that the household utility function has this certain functional form[78] and the

[77] The first order Taylor series approximation to m(ρ) around the point ρ = 1 is also equal to the right hand side of (A32). If ρ = 1, then the right hand side of (A32) is also equal to 1; if ρ > 1, then the right hand side of (A32) is also greater than 1.

[78] This functional form is $f(q,Q) \equiv [a_{11}q^2 + 2a_{12}qQ + a_{22}Q^2]^{1/2}$. The parameters for this function are the a_{ij}. These parameters must satisfy some restrictions in order to satisfy the concavity and monotonicity conditions for f(q,Q). These conditions are described in Diewert (1976), Diewert and Hill (2010) and Diewert and Feenstra (2019). This functional form is *flexible*; i.e., it can approximate an arbitrary linearly

household is minimizing the cost of achieving the utility level $u^1 \equiv f(q^1,0)$ in period 1 and the utility level $u^0 \equiv f(q^0,Q^0)$ in period 0. Thus under the assumption that f(q,Q) has the required functional form, we have the following exact equality:

(B34) $u^1/u^0 = Q_F^* = [Q_L Q_P^*]^{1/2}$.

It is useful to develop formulae for the Laspeyres and Paasche price indexes that are counterparts to the formulae for the Laspeyres and Paasche quantity indexes given by (B27) and (B28).

Define the Laspeyres price index P_L^* as follows:

(B35) $P_L^* \equiv [p^1q^0 + P^{1*}Q^0]/[p^0q^0 + P^0Q^0]$
$= [(p^1/p^0)p^0q^0 + (P^{1*}/P^0)p^0Q^0]/v^0$ using (B3)
$= (p^1/p^0)s_q^0 + (P^{1*}/P^0)s_Q^0$
$= (p^1/p^0)\{s_q^0 + [(P^{1*}/P^0)/(p^1/p^0)]s_Q^0\}$.

Define the Paasche price index P_P as follows:

(B36) $P_P \equiv [p^1q^1 + P^{1*}Q^1]/[p^0q^1 + P^0Q^1]$
$= p^1q^1/p^0q^1$ using (B1), $Q^1 = 0$
$= p^1/p^0$.

The Fisher (1922) price index P_F^* is defined as the geometric mean of P_L^* and P_P:

(B37) $P_F^* \equiv [P_L^*P_P]^{1/2}$
$= (p^1/p^0)\{s_q^0 + [(P^{1*}/P^0)/(p^1/p^0)]s_Q^0\}^{1/2}$ using (B35) and (B36)
$= (p^1/p^0)m(\rho)$ using (B30) and (B31)
$\approx (p^1/p^0)\{(½)[1 + s_q^0] + (½)s_Q^0[(P^{1*}/P^0)/(p^1/p^0)]\}$ using (B32).

homogeneous, twice continuously differentiable function to the second order around any point (q,Q). This functional form has a linear utility function as a special case; see Diewert (2020a; 15).

where $s_Q^0 \equiv P^0Q^0/v^0$ is the period 0 expenditure share of the commodity which will disappear in period 1. If $P^{1*}/P^0 > p^1/p^0$, then $P_L^* > P_F^* > P_P$.

Using the above definitions, it is straightforward to show that the following well known equalities hold:

(B38) $v^1/v^0 = P_L^*Q_P^* = P_PQ_L = P_F^*Q_F^*$.

In order to calculate Q_F^* defined by (B29) or P_F^* defined by (B37), we need an estimate for the reservation price P^{1*}. It is possible to generate an estimate for P^{1*} if an estimate for the elasticity of demand for the products is known or could be estimated using econometric techniques. In order to accomplish this task, we first require some preliminary material.

Recall definitions (B7)-(B10) which defined the Hicksian demand functions[79] for the two goods evaluated at the period 0 and 1 data for the household. For general positive prices, p and P, and positive utility level u, the Hicksian demand functions, q(u,p,P) and Q(u,p,P), can be defined in terms of the first order partial derivatives of the consumer's unit cost function, c(p,P) as follows:

(B39) $q(u,p,P) \equiv c_1(p,P)u$;

(B40) $Q(u,p,P) \equiv c_2(p,P)u$

where $c_2(p,P) \equiv \partial c(p,P)/\partial P$. The *Hicksian elasticity of demand* for Q as a function of P evaluated at u^0,p^0,P^0, η^0, is defined as follows:

(B41) $\eta^0 \equiv [P^0/Q^0]\partial Q(u^0,p^0,P^0)/\partial P$

$= P^0c_{22}(p^0,P^0)u^0/Q^0$ differentiating both sides of (B40) with respect to P and evaluating the derivatives at (u^0,p^0,P^0)

[79] See Hicks (1946; 311-331).

$= P^0c_{22}(p^0,P^0)u^0/c_2(p^0,P^0)u^0$ using (B8)

$= P^0c_{22}(p^0,P^0)/c_2(p^0,P^0)$

≤ 0

where the inequality follows from the concavity of the unit cost function in p and P. We will assume that the inequality in (B41) is strict so that $\eta^0 < 0$; i.e., we assume that as the price of Q goes up, consumers buy less of it and more of q in order to keep their utility or welfare level constant. Below, we will find it convenient to work with the negative of η^0, which we define as η^{0*}. Thus we have the following inequality:

(B42) $\eta^{0*} \equiv - P^0c_{22}(p^0,P^0)u^0/Q^0 > 0.$

It turns out that it will be useful to find the period 0 reservation price for the disappearing commodity which we label as P^{0*}. Thus P^{0*} is the price which will make the demand for q equal to 0 in period 0; i.e., P^{0*} is the solution to the following equation:

(B43) $0 = Q(u^0,p^0,P^{0*})$

$= c_2(p^0,P^{0*})u^0$ using (B40).

The reason why it is useful to find the period 0 reservation price P^{0*} is due to the fact that if we can determine P^{0*}, then we can determine the period 1 reservation price P^{1*}; in fact, the following equation holds:

(B44) $P^{1*}/p^1 = P^{0*}/p^0.$

Using (B43), we see that the following equation holds:[80]

(B45) $0 = c_2(p^0,P^{0*})$

[80] Since c(p,P) is linearly homogeneous in (p,P), Euler's Theorem on homogeneous functions implies that the first order derivatives of this function are homogeneous of degree 0 in (p,P). Thus we have $c_2(\lambda p,\lambda P) = c_2(p,P)$ for all $\lambda > 0$. Choose $\lambda = 1/p$ and we obtain the equation $c_2(p,P) = c_2(1,P/p)$.

$= c_2(1,P^{0*}/p^0)$ using the linear homogeneity of c(p,P)

$= c_2(1,P^{1*}/p^1)$ if (B44) holds

$= c_2(p^1,P^{1*})$ using the linear homogeneity of c(p,P)

$= c_2(p^1,P^{1*})u^1$

$= Q(u^1,p^1,P^{1*})$

$\equiv Q^1.$

Thus if we set P^{1*} equal to the inflation adjusted carry forward reservation price $(p^1/p^0)P^{0*}$, this will be the correct reservation price for period 1.

We can find an approximation to P^{0*} by equating the first order Taylor series approximation to $Q(u^0,p^0,P)$ around the point P^0 to zero. Call this approximation to P^{0*}, P^{0**}. Thus we need to solve the following equation for P^{0**}:[81]

(B46) $0 = Q(u^0,p^0,P^0) + [\partial Q(u^0,p^0,P^0)/\partial P][P^{0**} - P^0]$

$= Q^0 + c_{22}(p^0,P^0)u^0[P^{0**} - P^0]$

where the second equality follows by differentiating $Q(u^0,p^0,P^0)$ defined by (B8). Solving the equation (B46) for P^{0**} leads to the following estimate for P^{0*}:

(B47) $P^{0**} - P^0 \equiv - Q^0/c_{22}(p^0,P^0)u^0$

$= P^0/\eta^{0*}$ using definition (B42).

Divide both sides of (B47) through by P^0 and we obtain the following expression for P^{0**}/P^0:

(B48) $P^{0**}/P^0 = 1 + 1/\eta^{0*} > 1.$

[81] The approximation methodology used here is similar to the methodology used by Hausman (1981) (2003), Diewert (1998), Diewert and Feenstra (2019), Diewert, Fox and Schreyer (2019) and Brynjolfsson, Collis, Diewert, Eggers and Fox (2020).

The inequality in (B48) follows because we assumed $\eta^{0*} > 0$. From (B44), we see that $P^{1*} = (p^1/p^0)P^{0*}$. Approximate P^{0*} by P^{0**} and define the approximate reservation price for period 1 as:

$$\text{(B49)}\quad P^{1**} \equiv (p^1/p^0)P^{0**} = (p^1/p^0)P^0(1 + 1/\eta^{0*}) \qquad \text{using (B48).}$$

The last equation in (B49) can be rearranged to give us the following approximation to the term $(P^{1*}/P^0)/(p^1/p^0)$ which appears in the formulae for Q_P^*, Q_F^*, P_L^* and P_F^*:[82]

$$\text{(B50)}\quad \rho^* \equiv (P^{1**}/P^0)/(p^1/p^0) = 1 + 1/\eta^{0*}.$$

The above analysis shows that if an estimate for the elasticity of demand η^{0*} can be obtained, then estimates for the bias in the Laspeyres quantity index Q_L defined by (B27) relative to the Paasche and Fisher quantity indexes defined by (B28) and (B29) can be calculated. Similarly, if an estimate for the elasticity of demand η^{0*} exists, then estimates for the bias in the Paasche price index P_P defined by (B36) relative to the Laspeyres and Fisher quantity indexes defined by (B35) and (B37) can be calculated.

To illustrate the possible magnitudes of the differences between the Laspeyres quantity index for consumption defined by (B27) relative to the Fisher quantity index defined by (B37), let $s_q^0 = ½$, $s_Q^0 = ½$ (so that ½ of all consumption producing industries are shut down), $p^1/p^0 = 1$ (so that there is no inflation in the prices of continuing commodities) and $\eta^{0*} = ½$. In this case, $\rho^* = (P^{1**}/P^0)/(p^1/p^0) = 1 + 1/\eta^{0*} = 1 + 2 = 3$ and $m(\rho^*) = [s_q^0 + s_Q^0\rho^*]^{1/2} = [½ + (½)3]^{1/2} = 2^{1/2} = 1.414$.[83] Then $Q_L/Q_F^* = m(\rho^*) \approx 1.414$. Hence the *Laspeyres quantity index will overstate real consumption* (from a welfare perspective) by about 40%.

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[82] See (A28), (A29), (A35) and (A37).

[83] If we use the approximation to $m(\rho)$ defined by the right hand side of (A32), we find that this approximate estimate is equal to 1.5 as compared to the true estimate equal to $2^{1/2} \approx 1.414$.

The bias in the Paasche price index (which is equal to the Laspeyres and Paasche price indexes for just the continuing commodities in our simple model) relative to the Fisher index will go in the other direction and understate cost of living inflation. Thus $P_P/P_F^* = 1/m(\rho^*) = 1/1.414 = 0.707$. Thus the Paasche price index (which is also the carry forward price index in our simple model) will understate cost of living inflation by about 30%. Carry forward basket type indexes which have a basket which is approximately proportional to the period 0 quantity vector will have biases (relative to a welfare or cost of living perspective) similar to the biases in the Laspeyres quantity index and in the Paasche price index. Our simple model analysis indicates that the bias in fixed basket type price indexes will be very large if the pre-shutdown share of expenditure for commodities that are ultimately unavailable during lockdown periods is large.

Appendix C: Defining Reservation Prices Under Lockdown Conditions

We use the same notation as was used in section 2 of the main text with one exception: we now assume that some components of the Q^1 vector could be positive. We also assume that the analysis here is applied to a single household. An example where the quantity consumed of a lockdown affected commodity is positive in a lockdown period is rental housing where the tenant is given a rent holiday. Thus in this case, if the first commodity in Group 2 is rental housing, then $Q_1^1 > 0$ but the observed price is $P_1^1 = 0$. However, the utility value of its free rent to the household is not 0; it is a positive reservation price, $P_1^{1*} > 0$. Our problem here is to find a way to estimate this reservation price. Once we have found this price, it is appropriate to add $P_1^{1*}Q_1^1$ to the observed expenditures, $p^1 \cdot q^1$, in order to better approximate the value of actual consumer expenditure in period 1. Hence this Appendix has some relevance to the construction of the national accounts during lockdown periods as well as being relevant for the construction of a cost of living index.

Restating the notation used in section 2, denote the period t price and quantity vectors for always available products by $p^t \equiv [p_1^t,...,p_M^t] >> 0_M$ and $q^t \equiv [q_1^t,...,q_M^t] >> 0_M$ for $t = 0,1$. The lockdown affected price and quantity vectors for period 0 are $P^0 \equiv [P_1^0,...,P_N^0] >> 0_N$

and $Q^0 \equiv [Q_1^0,...,Q_N^0] >> 0_N$. The observed price for the locked down commodities in period 1 is $P^1 = 0_N$, and the corresponding observed quantity vector is $Q^1 \equiv [Q_1^1,...,Q_N^1] > 0_N$. Most of the components of Q^1 will be zero but some components could be positive; i.e., the government may be providing some goods and services free of charge or it may legislate rent holidays for tenants which will lead to 0 prices. We provide a methodology for finding estimates for the vector of reservation prices P^{1*}.

We first consider the consumer's period 0 cost minimization problem. Suppose the household utility function is $u = f(q,Q)$ where $f(q,Q)$ is nonnegative, increasing and concave in the components of the vectors q and Q. Define the period 0 utility level as $u^0 \equiv f(q^0,Q^0) > 0$. We assume that (q^0,Q^0) solves the following period 0 cost minimization problem:

$$\text{(C1)}\ C(u^0,p^0,P^0) \equiv \min_{q,Q} \{p^0\cdot q + P^0\cdot Q : f(q,Q) \geq u^0;\ q \geq 0_M;\ Q \geq 0_N\}$$
$$= p^0\cdot q^0 + P^0\cdot Q^0$$

where C(u,p,P) is the consumer's cost function.[84]

Define the *consumer's conditional cost function*, $C^*(u,p,Q)$, for $u \geq 0$, $p >> 0_M$ and $Q \geq 0_N$ as follows:

$$\text{(C2)}\ C^*(u,p,Q) \equiv \min_q \{p\cdot q : f(q,Q) \geq u;\ q \geq 0_M\}.$$

When solving the cost minimization problem defined by (C2), the consumer minimizes the cost associated with the purchase of always available goods and services, p·q, subject to achieving at least the utility level u, conditional on having on hand, the vector of sometimes available goods and services Q.[85] We note that if the consumer's utility

[84] The cost function is used to define true cost of living indexes such as $C(u^0,p^1,P^{1*})/C(u^0,p^0,P^0)$ and $C(u^1,p^1,P^{1*})/C(u^1,p^0,P^0)$.

[85] Under our regularity conditions on f(q,Q), it can be shown that $C^*(u,p,Q)$ has the following properties: it is increasing in u for fixed p and Q, it is nondecreasing, concave and linearly homogeneous in p for fixed u

function f(q,Q) is estimated using available econometric techniques for data in the pre-lockdown period, then an empirical estimate for the conditional cost function $C^*(u,p,Q)$ defined by (C2) can be obtained.

We assume that the observed period 1 consumption vector q^1 solves the conditional cost minimization problem (C2) when (u,p,Q) equals (u^1,p^1,Q^1). Using this assumption, we have the following equality:

$$\text{(C3)}\quad p^1 \cdot q^1 = C^*(u^1,p^1,Q^1).$$

Assuming that $C^*(u^1,p^1,Q)$ is differentiable with respect to the components of Q at the point $Q = Q^1$, define the vector of *period 1 reservation prices for the goods and services* subject to lockdown restrictions, P^{1*}, as the negative of the vector of first order partial derivatives of $C^*(u^1,p^1,Q^1)$ with respect to the components of Q; i.e., define P^{1*} as follows:

$$\text{(C4)}\quad P^{1*} \equiv -\nabla_Q C^*(u^1,p^1,Q^1) \geq 0_N$$

where the inequality in (C4) follows from the fact that $C^*(u,p,Q)$ is nonincreasing in the components of Q.

We use the reservation prices defined by (C4) as the prices for the lockdown affected goods and services for period 1. Consider the following period 1 (regular) cost minimization problem where $u^1 \equiv f(q^1,Q^1)$:

$$\begin{aligned}\text{(C5)}\quad C(u^1,p^1,P^{1*}) &\equiv \min_{q,Q} \{p^1 \cdot q + P^{1*} \cdot Q : f(q,Q) \geq u^1;\ q \geq 0_M;\ Q \geq 0_N\} \\ &= \min_Q \{P^{1*} \cdot Q + \min_q \{p^1 \cdot q : f(q,Q) \geq u^1;\ q \geq 0_M\} : Q \geq 0_N\} \\ &= \min_Q \{P^{1*} \cdot Q + C^*(u^1,p^1,Q) : Q \geq 0_N\} \qquad \text{using definition (C2).}\end{aligned}$$

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and Q, it is convex in (u,Q) for fixed p and it is nonincreasing in Q for fixed p and u. If in addition f(q,Q) is linearly homogeneous in q,Q, then $C^*(u,p,Q)$ is linearly homogeneous in (u, Q).

The first order necessary conditions for Q^1 to solve the final cost minimization problem[86] in (C5) are the following conditions:

$$\text{(C6)}\quad P^{1*} + \nabla_Q C^*(u^1,p^1,Q^1) = 0_N.$$

But conditions (C6) are equivalent to equations (C4), which were used to define the reservation prices P^{1*}. Hence we have the following:

$$\begin{aligned}\text{(C7)}\quad C(u^1,p^1,P^{1*}) &= \min_Q \{P^{1*}\cdot Q + C^*(u^1,p^1,Q) : Q \geq 0_N\} && \text{using (C5)}\\ &= P^{1*}\cdot Q^1 + C^*(u^1,p^1,Q) && \text{using (C6)}\\ &= P^{1*}\cdot Q^1 + p^1\cdot q^1 && \text{using (C3).}\end{aligned}$$

The above algebra shows that if the consumer had the “income” $p^1\cdot q^1 + P^{1*}\cdot Q^1$ to spend on the commodities in scope for the lockdown period 1 and faced the prices p^1 and P^{1*}, then the consumer would minimize the cost of achieving the actual utility level $u^1 \equiv f(q^1,Q^1)$ by choosing the observed consumption vector, (q^1,Q^1). Thus the observed lockdown restricted consumption vector would be freely chosen by the consumer facing the price vectors (p^1,P^{1*}) in order to minimize the cost of achieving the actual period 1 utility level, u^1.

Our conclusion at this point is that if the target CPI is a cost of living index, then the reservation price vector P^{1*} should be used to value period 1 locked down products in place of the observed vector of zero prices, $P^1 = 0_N$. Laspeyres, Paasche and Fisher indexes can be formed using the reservation prices P^{1*}.

If we make the further assumption that the utility function $f(q,Q)$ is the square root of a quadratic form in the components of q and Q, then since this functional form is exact for

[86] These conditions are also sufficient for Q^1 to solve the last minimization problem since $C^*(u,p,Q)$ is a convex function of Q under our regularity conditions.

the Fisher index using the reservation prices, the utility ratio, u^1/u^0, would be equal to this Fisher index.

There is only one aspect of the reservation price methodology presented here which is not standard: the methodology above shows how theoretical reservation prices can be obtained for products that are supplied at zero prices during the lockdown period but the corresponding quantities held by consumers may be positive instead of being set equal to zero. If there are such products so that $Q^1 > 0_N$, then $P^{1*}{\cdot}Q^1 > 0$ and this imputed expenditure on lockdown affected products should be added to the expenditures on always available goods and services during period 1, $p^1{\cdot}q^1$, to form overall consumer expenditures or "income".

This has implications for the System of National Accounts: actual consumer expenditures on goods and services during lockdown periods, which are equal to $p^1{\cdot}q^1$, need to be augmented by imputed expenditures on lockdown affected goods and services that are provided at zero prices but have marginal utilities that are above zero, $P^{1*}{\cdot}Q^1$. For lockdown period products n that are simply not available, the corresponding quantity Q_n^1 will equal 0 so $P_n^{1*}Q_n^1$ will also equal 0 and there is no need to add $P_n^{1*}Q_n^1$ to actual expenditures. But for some government services provided at zero cost and for rental housing that is temporarily supplied at a zero price, we need to add these imputed expenditures to actual expenditures to get an accurate picture of actual real consumption and welfare.

There is an example in Diewert and Feenstra (2019) that shows how the Fisher functional form can be estimated econometrically and it also calculates the resulting reservation prices for missing products in a grocery store example. So it is possible to estimate reservation prices, but the estimation of reservation prices *at scale* is not possible at the present time. The actual reservation prices will have to be approximated by inflation adjusted carry forward prices or by simple unadjusted carry forward prices. The analysis in Appendix A above shows that inflation adjusted carry forward prices can have a substantial downward bias relative to their true reservation prices.

Finally, the above methodology can be modified to deal with a related problem associated with the measurement of housing rents. Many urban areas have some form of rent control imposed on price increases for rental housing. If these rent controls have been in place for long periods of time, the actual price paid for a rented dwelling unit in period t, say P_n^t, could be well below a current market price P_n^{t*}, which would likely approximate the appropriate period t reservation price. Thus in order to calculate a cost of living index, the actual price paid P_n^t should be replaced by its higher opportunity cost price P_n^{t*} and the resulting Laspeyres, Paasche and Fisher indexes should be calculated using these higher prices.[87] In the national accounts, the actual rental income and expenditure, $P_n^tQ_n^t$, will be recorded in both the production and household accounts but the imputed housing expenditures *above* the recorded expenditures, $P_n^{t*}Q_n^t - P_n^tQ_n^t > 0$, needs to be added to household expenditures on consumer goods and services.

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[87] This replacement should only be done if there are local rent controls that have led to artificially low rental prices.

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