Selection of CPI formula in three steps

- How to aggregate elementary indices into higher-level price indices

Carsten Boldsen, UNECE

Three steps to go through

1. Consider the purpose(s) of the CPI

2. Select a target price index

3. Select formula for calculation of the regular CPI

1. The purpose(s) of the CPI

Theory

Measurement of inflation – pure price index, fixed basket, cost of goods index (COGI)

$$I_{0:t}^{Lowe} = \frac{\sum p_t^i q_b^i}{\sum p_0^i q_b^i}$$

Measurement of cost of living - cost of living index (COLI)

$$I_{0:t}^{COLI} = \frac{C\left(U, p_t^i\right)}{C\left(U, p_0^i\right)}$$

- There are many basket indices and many COLIs,
- Some basket indices are also good estimates of COLIs

Practise

- We don’t have information on all prices and quantities
- CPI calculated in two steps → need for expenditure shares for elementary aggregates to calculate higher-level indices
- In practice CPI is often used for different purposes
- Agreement on the main purpose(s) is useful for deciding on coverage and calculation methods

2. Select target price index for the CPI

What is a target price index?

Price index based on observable information about prices and quantities/expenditures and which can be calculated in practice

Advantages of having a target index:

- Provides reference & guidance for compilation of the regular CPI
- Necessary to quantify the size of any bias, the difference between what is ***actually measured*** and what ***should be measured***
- Can be used for improvements of the regular CPI
- Useful for documentation and explanation to users

Some fixed basked target indices (different versions of the Lowe index)

Walsh

$$P_{0:t}^{W} = \frac{\sum p_t^i \sqrt{q_0^i \cdot q_t^i}}{\sum p_0^i \sqrt{q_0^i \cdot q_t^i}} = \sum w_W^i \cdot \left(\frac{p_t^i}{p_0^i} \right)$$

$$w_W^i = \frac{\sqrt{\left(w_0^i \cdot w_t^i\right) / \left(p_t^i / p_0^i\right)}}{\sum \sqrt{\left(w_0^i \cdot w_t^i\right) / \left(p_t^i / p_0^i\right)}}$$

Marshall-Edgeworth

$$P_{0:t}^{ME} = \frac{\sum p_t^i \left(q_0^i + q_t^i\right) / 2}{\sum p_0^i \left(q_0^i + q_t^i\right) / 2} = \sum w_{ME}^i \left(\frac{p_t^i}{p_0^i} \right)$$

$$w_{ME}^i = \frac{v_0^i + \left(v_t^i / \left(p_t^i / p_0^i\right)\right)}{\sum \left(v_0^i + \left(v_t^i / \left(p_t^i / p_0^i\right)\right)\right)}, v_t^i = \frac{p_t^i q_t^i}{\sum p_t^i q_t^i}$$

Laspeyres

$$P_{0:t}^{La} = \frac{\sum p_t^i q_0^i}{\sum p_0^i q_0^i} = \sum w_0^i \cdot \left(p_t^i / p_0^i\right), w_0^i = \frac{p_0^i q_0^i}{\sum p_0^i q_0^i}$$

Some Cost of living target indices

Fisher price index

$$P_{0:t}^{F} = \left(P_{0:t}^{La} \cdot P_{0:t}^{Pa}\right)^{1/2}$$

Törnqvist price index

$$P_{0:t}^{T} = \prod \left(\frac{p_t^i}{p_0^i} \right)^{\left(w_0^i + w_t^i\right)/2}$$

Lloyd-Moulton

$$P_{0:t}^{LM} = \left[\sum w_0^i \left(\frac{p_t^i}{p_0^i} \right)^{1-\sigma} \right]^{1/(1-\sigma)}$$

- The Walsh price index can also be derived as a COLI (*CPI Manual*, chp 17)
- Any index could be a COLI, depending on the preferences

- All the target indices can be calculated by use of observable expenditure shares and prices

- Walsh, Fisher and Törnqvist price indices are ***superlative indices***:

  "Fisher, Walsh and Törnqvist price indices approximate each other very closely using "normal" time series data. This is a very convenient result since these three index number formulae repeatedly show up as being "best" in all the approaches to index number theory. Hence, this approximation result implies that it normally will not matter which of these indices is chosen as the preferred target index for a consumer price index." (*CPI Manual*, p 313)

- Walsh, Marshall-Edgeworth, Laspeyres = expenditure weighted arithmetic average of elementary indices – fixed basket indices

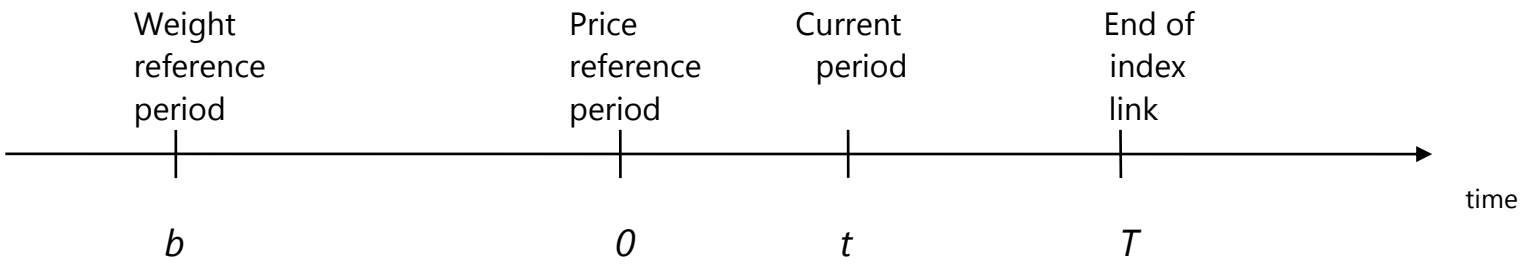
- LM can be calculated in real time when W_0 is available, but substitution elasticity difficult to estimate

3. Select formula for the regular CPI

Principle: choose calculation formula which provides best estimate of the target index

Constraints have to be taken into account

The typical situation



- Weights for the current period will usually be available only after the period
- Weights for the price reference month will usually not be available in real time
- Weights often refer to a period of a year or more, while prices are recorded monthly

➡ **The target indices cannot be calculated in real time**

➡ **An estimate has to be used for calculating the regular CPI**

What are the options?

Calculate the CPI as the expenditure weighted average of the elementary price indices

Expenditure weighted arithmetic average of the elementary price indices:

$$P_{0:t} = \sum w_b^j P_{0:t}^j$$

Important decision: Whether to price-update the weights from b to 0?

Price-updating:
- Lowe price index (not a Laspeyres index),
- Underlying quantities are kept constant from b to 0, maintain fixed basket
- If relative quantities remain constant (elasticity of substitution = 0) Lowe will be a god estimate of the target index

Not price-updating:
- Young price index
- Expenditure shares are kept constant from b to 0
- If expenditure shares remain constant (elasticity of substitution = 1) Young will be a god estimate of the target index

Expenditure weighted geometric average of elementary price indices (Geometric Young):

$$P_{0:t}^{GY} = \prod w_b^j P_{0:t}^j$$

Lloyd-Moulton constant elasticity of substitution cost of living index:

$$P_{0:t}^{LM} = \left[\sum w_0^i \left(P_{0:t}^j \right)^{1-\sigma} \right]^{1/(1-\sigma)}$$

- **Difficult to estimate elasticity of substitution σ**

Compare the CPI with the target index

- Calculate the target index for periods where prices and weights are available
- Compare the CPI with the target index

Example – The Danish CPI 1996-2006

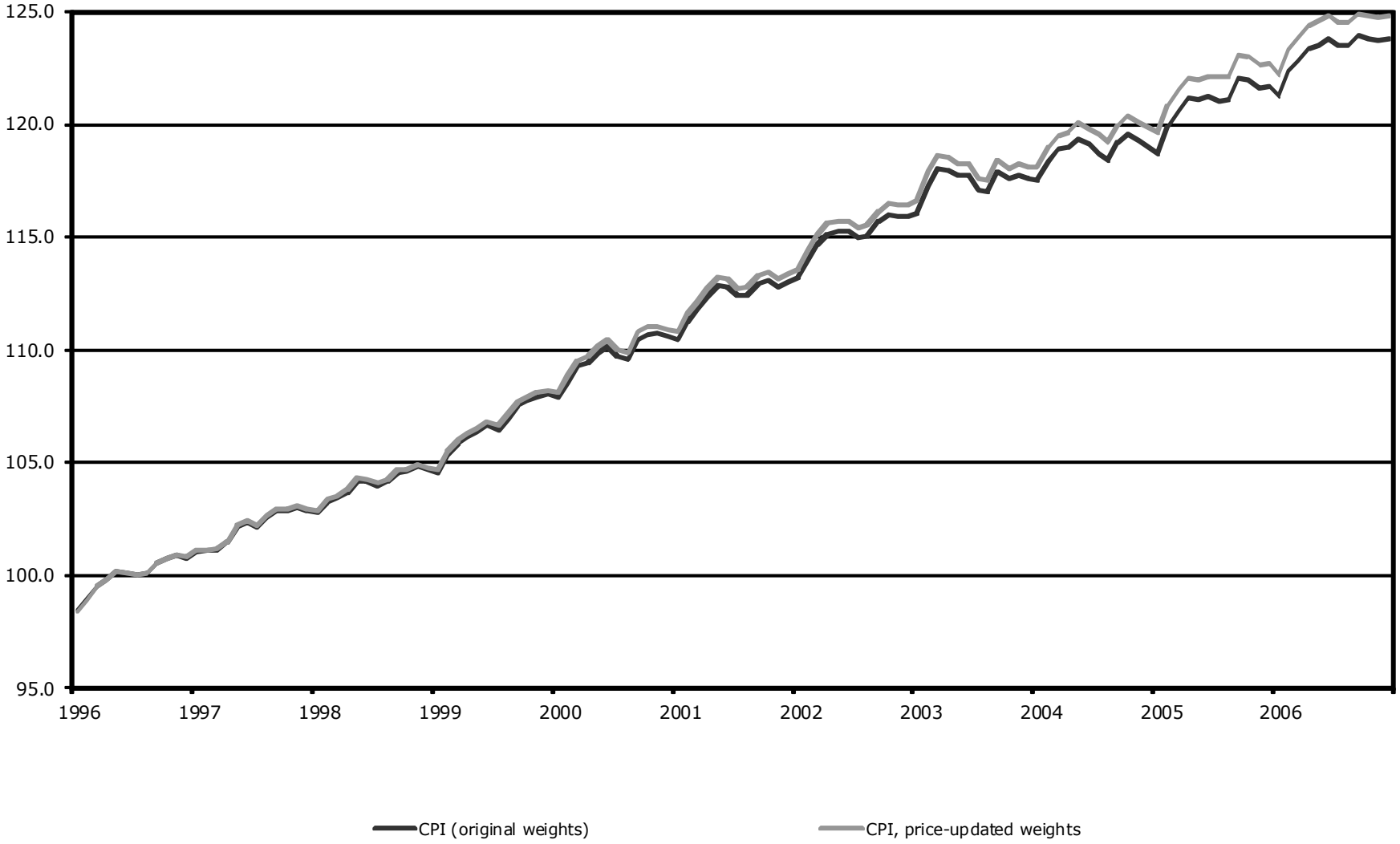
Different target indices 1996 – 2003

	1996	1999	2003	1996-1999	1999-2003	1996-2003
	Index, 1996=100			*Average annual rate of change in %*		
Walsh	100,00	106,39	117,07	2,09	2,42	2,28
M-E	100,00	106,35	117,10	2,07	2,43	2,28
Fisher	100,00	106,34	117,05	2,07	2,43	2,27
Törnqvist	100,00	106,38	117,08	2,08	2,42	2,28

The CPI and the Walsh index, 1996 – 2003

	1996	1999	2003	1996-1999	1999-2003	1996-2003
	Index, 1996=100			*Average annual rate of change in %*		
CPI	100,00	106,64	117,49	2,17	2,45	2,33
Walsh	100,00	106,39	117,07	2,09	2,42	2,28
Difference				0,08	0,03	0,05

The CPI with original and price-updated weights (1996 = 100)



CPI, Walsh and Geometric Walsh (1996 = 100)