Fixed Basket Methods for Compiling Consumer Price Indexes

Paul A. Armknecht
International Monetary Fund
Price Statistics Expert

Abstract

This article briefly presents the background and concerns that led to the development of two technical volumes related to the production of the CPI and the most common fixed basket approaches statistical offices use to compile the CPI. The article demonstrates the application formulas of both long-term and short-term price changes for fixed basket indexes. It also corrects the calculations and conclusions of a previous article by Msokwa. The article concludes that both the long-term and short-term (modified) Laspeyres price index formulas provide the exact same index values when properly calculated. The more serious issue with fixed basket approaches occurs when the elementary (first level) price indexes are calculated using unweighted averages. In such cases, geometric averages should be employed rather than arithmetic averages.

Key words: Consumer Price Index, long-term price index, short-term price index, modified Laspeyres, elementary price index.

1. Introduction to CPI compilation methods

The International Monetary Fund (IMF), the International Labour Organization (ILO), and the UN Economic Commission for Europe (UNECE) produced two consumer price index publications—*Consumer Price Index Manual: Theory and Practice.*2004 (*CPI Manual*)and *Practical Guide to Producing Consumer Price Indices,* 2009 (*Guide*)—that serve as the technical reference documents for countries to use in compiling the CPI. This article briefly presents the background and concerns that led to the development of these complementary volumes and the most common fixed basket approaches statistical offices use to compile the CPI. The article demonstrates the application formulas of both long-term and short-term price changes for fixed basket indexes. It also corrects the calculations and conclusions of a previous article by Msokwa.

The standard fixed basket price index methods used in most countries today date back 90 years to those proposed by W.C. Mitchell (1927) and G.H. Knibbs (1924). Index number theory has advanced substantially, particularly in the past 30 years, to provide us with better information on what our target index number formula should be. In particular, various approaches have been used to evaluate index number formula and derive those best suited for inflation measures. The research presented in the *CPI Manual* has resulted in improvements for fixed basket formulas and identified target indexes that are symmetric averages of standard formulas. The latter target indexes are the Fisher, Törnqvist, and Walsh price indexes, but these can only be produced in final form with a lag because they require both current and past weight information. Thus, this article primarily considers fixed basket indexes where the weight data are derived from some past period.

In addition, different formulas are used at different stages of aggregation. At the elementary or first stage where prices are first combined to form an index many countries will not use weights. At the second and higher levels, weights are applied, but these weights generally relate to some period in the past that becomes less representative with the passage of time. When compared to the target indices(Fisher, Walsh, or Törnqvist), it becomes apparent that the indices produced in practice are of substantially lower quality than the target indices. The new *Manual* and *Guide* discuss these issues thoroughly and provide approaches that countries can implement over time to move closer to the target measures.

2. Index number theory and practice differ

In consumer utility theory, consumers will maximize the satisfaction they receive from the purchase of goods and services given the constraints of their household budgets. Consumers make choices that can be measured by expenditure surveys when consumer markets are in equilibrium. These surveys reflect the levels of utility consumers have revealed to be their preferences.

The *CPI Manual* shows that the Laspeyres price index serves has a potential upward bias when compared to each of the target indexes and provides an upper bound in the measurement of consumer inflation. This occurs because, in part, the Laspeyres index assumes purchases are made in fixed quantities based on the optimal decisions from some previous period's experience. The standard Laspeyres price index formula is:

$$P_{Las}^{t} = \frac{\sum Q_i^0 P_i^t}{\sum Q_i^0 P_i^0} \qquad (1)$$

The quantities remain the same as in the base period, *0*, and no attempt is made to allow substitution of products or services in response to more current economic conditions. Also, items that have relatively larger (smaller) price increases have greater (lesser) implicit importance in the index calculation. The upper level substitution bias can be mitigated by frequent weight updates such as annually or biannually.

The *CPI Manual* also demonstrates that the Paasche index, which uses current period weights, has a potential downward bias compared to the target indexes. The formula for the Paasche index is:

$$P_{Pa}^{t} = \frac{\sum Q_i^t P_i^t}{\sum Q_i^t P_i^0} \qquad (2)$$

The downward bias can occur because the fixed weights in the current period, *t*, reflect current purchasing patterns after substitution and give more importance to those items that have experienced relatively smaller price changes and are purchased in larger quantities than in the base period.

Diewert (1976, 1983) has shown that the true cost of living index, which is a targeted measure of inflation according to the Boskin Commission and the *CPI Manual*, lies somewhere between the Laspeyres and Paasche indexes. He suggests that the Fisher ideal price index is a strong candidate for the best approximation of the cost of living index. It is the geometric mean of the Laspeyres and Paasche indices:

$$P_F^t = (P_{Las}^t \times P_{Pa}^t)^{1/2}$$

This choice can be justified from several perspectives: (1) the basket for the Fisher index represents the average over the period (both the base and the reference periods); (2) this index has more desirable statistical properties than either the Laspeyres or the Paasche indexes; and (3) it coincides with the dictates of economic theory.

Similarly, the *CPI Manual* notes that the Törnqvist and Walsh price indexes are also appropriate targets for inflation measures using cost-of-living proxy measures. The Törnqvist price index is a weighted geometric average of price relatives where the weights are the average expenditure shares in the base and current periods:

$$P_T^t = \left(\frac{p_i^t}{p_i^0}\right)^{(s_i^0 + s_i^t)/2}$$

The Walsh price index also uses information from both the base and current periods, but the weights are the geometric average of the quantities in the two periods:

$$P_W^t = \frac{\sum \sqrt{q_i^0 q_i^t}\left(\frac{p_i^t}{p_i^0}\right)}{\sum \sqrt{q_i^0 q_i^t}}$$

Johnson, Reed, and Stewart discuss how the U.S. Bureau of Labor Statistics compiles a research price index (Chained Consumer Price Index for all urban consumers, CCPI-U) using the Törnqvist approach. This was suggested by Armknecht in 1996. However, the CCPI-U is revised each year for the two prior years as new weights become available from the consumer expenditure survey. Statistics Sweden produces their CPI using a Walsh index (Ribe, 2004), but it is subject to annual revisions also.

For most statistical agencies that produce the CPI, it is impractical to produce a Fisher (or other target indexes such as the Törnqvist or Walsh) price index because of limitations in getting current expenditure data. In addition, many countries have policies not to revise the CPI once published. Nonetheless, Armknecht and Silver (2014) have demonstrated that it may be possible to closely approximate the Fisher and Törnqvist price indexes by averaging an upward biased fixed weight index that uses arithmetic averages of price relatives with a fixed weight index that uses geometric averages of price relatives. Such indexes can be produced in real time using available data. For most practical purposes, however, statistical agencies continue to use Laspeyres-type price indexes when compiling their CPI.

3. The Laspeyres Index in Practice

3.1 Long-Term Laspeyres Price Index

The concerns with current index methods arise from the fact that, in practice, the index numbers in use often do not correspond to those espoused in theory or those in countries' published methodology documents. For example, many countries say they use a standard Laspeyres index as shown in equation (1), but the actual formula used is different. For a Laspeyres index, the price reference (base) period must be the same as the weight reference period.

The value in the denominator ($\sum Q_i^0 P_i^0$) is the expenditures on consumer purchases in the reference period *0*, when the price index has a value of 100. The numerator ($\sum Q_i^0 P_i^t$) represents the estimated value of purchasing the same basket of items in the current time period, *t*.

Equation (1) uses quantities as weights, but the data compiled from the household budget surveys (HBS) is usually the value of expenditure and the weights that are used are these expenditure weights (*w*) or their shares (*s*). From equation (1), we derive expenditure weight formulas as the following:

$$P_{Las}^t = \frac{\sum w_i^0 \frac{p_i^t}{p_i^0}}{\sum w_i^0}, \text{where } w_i^0 = q_i^0 \times p_i^0 \qquad (3)$$

Equation (3) can be interpreted two ways. First, it is a weighted average of the long-term price relatives ($\frac{p_i^t}{p_i^0}$)using the HBS expenditures (w_i^0)as the weights. Alternatively, the numerator is the value of the updated expenditure from the base period to the current period called the current cost weight, i.e., what it costs in period *t* to purchase the same item in the base period *0*. The denominator is the cost weight in the base period. The price index is the ratio of the current cost weight to the base period cost weight.

The Laspeyres formula can also be expressed in terms of expenditure shares:

$$P_{Las}^t = \sum s_i^0 \frac{p_i^t}{p_i^0} \qquad (4)$$

$$\text{where } s_i^0 = \frac{q_i^0 \times p_i^0}{\sum q_i^0 \times p_i^0} \text{and} \sum s_i^0 = 1$$

Equation (4) shows that the Laspeyres price index can be expressed as a share weighted average of the long-term (L-T) price changes of the items in the CPI basket. Equations (3) and (4) are the versions of the Laspeyres index used in CPI compilation.

Often, the weight reference period is, in fact, earlier than the price reference period. Consider a weight reference period of *b*, where *b* precedes period *0*. Practically all countries' CPI use an HBS that was conducted in the past to derive the CPI weights. This occurs because the HBS reference period usually covers an annual period and it takes time to process, edit, and compile the HBS data. The fixed base index with past period weights can be expressed as follows:

$$P_Y^t = \frac{\sum w_i^b \frac{p_i^t}{p_i^0}}{\sum w_i^b}, \text{where } w_i^b = q_i^b \times p_i^b \qquad (5)$$

$$= \sum s_i^b \frac{p_i^t}{p_i^0} \text{ and } \sum s_i^b = 1 \qquad (6)$$

This formulation of the fixed base index is a Young index. It is not a Laspeyres index because the weight reference period, *b*, and the price reference period, *0*, are different. If the weights from the HBS are updated for price change from period *b* to the price reference period *0*, the formula in use is a Lowe index. The weights in the Lowe index are derived as follows:

$$w_i^{b \to 0} = w_i^b \times \frac{p_i^0}{p_i^b}$$

These weights are used in the L-T formula to produce the Lowe price index.

$$P_{Lo}^t = \frac{\sum w_i^{b\to 0} \frac{p_i^t}{p_i^0}}{\sum w_i^{b\to 0}} = \sum s_i^{b\to 0} \frac{p_i^t}{p_i^0} \qquad (7)$$

Given these differences in approaches for introducing new weights, countries may often refer to their CPI as being a "Laspeyres-type index" because they are using a fixed basket.

3.2 Short-Term Laspeyres Price Index

Many countries use a modified version of the Laspeyres index that compiles the index based on short-term (S-T) price changes rather than the long-term price changes presented in equations (3) and (4). This modified method involves a two-step estimation process that breaks down the price movements into short-term, period-to-period changes that are used to bring forward the index from the previous period. This approach makes it easier for statistical offices to introduce replacement items in the sample if the ones they have been tracking are no longer available. The S-T approach also enables the statistical offices to make quality adjustments as improvement (deterioration) is made to the sampled items. The statistical office only needs to collect the current and previous prices for the item in order to introduce it into the index. In using the L-T method, the base price will need to be adjusted for the changes in the quality of the items in the sample. Equation (3) can be modified as follows:

$$P_{Las}^t = \frac{\sum w_i^0 \frac{p_i^t}{p_i^0}}{\sum w_i^0} = \frac{\sum w_i^0 \left(\frac{p_i^1}{p_i^0} \times \frac{p_i^2}{p_i^1} \times \frac{p_i^3}{p_i^2} \times \cdots \times \frac{p_i^{t-1}}{p_i^{t-2}} \times \frac{p_i^t}{p_i^{t-1}}\right)}{\sum w_0} \qquad (8)$$

Noting that the recent cost of the item (its cost weight) for the previous period is:

$$w_i^{t-1} = w_i^0 \left(\frac{p_i^1}{p_i^0} \times \frac{p_i^2}{p_i^1} \times \frac{p_i^3}{p_i^2} \times \cdots \times \frac{p_i^{t-1}}{p_i^{t-2}}\right) \qquad (9)$$

Equation (4) can be expressed as follows using the previous cost weight and the current price relative (price change):

$$P_{Las}^t = \frac{\sum w_i^0 \left(\frac{p_i^1}{p_i^0} \times \frac{p_i^2}{p_i^1} \times \frac{p_i^3}{p_i^2} \times \cdots \times \frac{p_i^{t-1}}{p_i^{t-2}} \times \frac{p_i^t}{p_i^{t-1}}\right)}{\sum w_i^0} = \frac{\sum w_i^{t-1} \times \frac{p_i^t}{p_i^{t-1}}}{\sum w_i^0} \qquad (10)$$

This is the equation for one version of the modified Laspeyres index. The numerator is the updated cost weight from the base period to the current period. The denominator is the base period cost weight and the ratio of the two multiplied by 100 provides the estimate of the current month's Laspeyres price index.

An alternative version of the modified Laspeyres index is to calculate the current month's index using a weighted average of the current month's price relatives to bring forward the previous month's price index. The weights used in the calculations are the previous month's cost weights from equation (9). This version of the modified Laspeyres price index is expressed as follows:

$$P_{Las}^t = \frac{\sum w_i^{t-1} \left(\frac{p_i^t}{p_i^{t-1}}\right)}{\sum w_i^{t-1}} \times I_{t-1} \qquad (11)$$

Noting that:

$$I_{t-1} = \frac{\sum w_i^0 \left(\frac{p_i^1}{p_i^0} \times \frac{p_i^2}{p_i^1} \times \frac{p_i^3}{p_i^2} \times \cdots \times \frac{p_i^{t-1}}{p_i^{t-2}}\right)}{\sum w_i^0}$$

Equation (11) can be used to derive the following:

$$P_{Las}^t = \frac{\sum w_i^{t-1} \left(\frac{p_i^t}{p_i^{t-1}}\right)}{\sum w_i^{t-1}} \left(\frac{\sum w_i^0 \left(\frac{p_i^1}{p_i^0} \times \frac{p_i^2}{p_i^1} \times \frac{p_i^3}{p_i^2} \times \cdots \times \frac{p_i^{t-1}}{p_i^{t-2}}\right)}{\sum w_i^0}\right)$$

$$P_{Las}^t = \frac{\sum w_i^{t-1} \left(\frac{p_i^t}{p_i^{t-1}}\right)}{\sum w_i^{t-1}} \left(\frac{\sum w_i^{t-1}}{\sum w_i^0}\right) \qquad (12)$$

Equation (12) shows that the Laspeyres index can be modified to calculate the price index in two steps. The first step is the calculation of the short-term relative that is then used to bring forward the previous period index which can be expressed as the ratio of the previous period aggregate cost weight ($\sum w_i^{t-1}$) to the base period aggregate cost weight ($\sum w_i^0$).

4. Calculations using the L-T and S-T Laspeyres formulas

A previous article in this journal by Msokwa presented calculations of the L-T Laspeyres and the modified (S-T) Laspeyres index formulas. Table 1.1 in the Annex 1 presents the original data set used by Msokwa. Table 1.2 shows the results of calculating the index using equation (3) with long-term price relatives. The monthly data in the table are the ratios of the prices in the current month compared to those in the base period (Jan) expressed as indexes. In Table 1.3, the base period weights are updated using the long-term price relatives as shown in the numerator of equation (3), summed, and then divided by the total weight shown in the denominator of equation (3). Table 1.4 in Annex 1 presents the short-term price relatives calculated from Table 1.1. Table 1.5 shows the modified (two-step) Laspeyres index calculated using the updated cost weight method of equation (11). The weights are multiplied by the by the price relative for Feb. to derive the Feb. cost weight. Subsequently the Mar. price relative is used to multiply the Feb. cost weight and derive the Mar. cost weight. This chaining process is continued to derive the cost weight for each month through Dec.

Msokwa incorrectly calculated the modified (S-T) Laspeyres price index as only consisting of the short-term component of equations (11) and (12). The results that Msokwa presented in his calculations of the modified Laspeyres index appearing in annex 2 (Table A2.3) of his article represent the short-term price changes in each period. The correct Laspeyres index would be obtained by chaining the short-term relatives to obtain the long-term index. In Table 1.6 of Annex 1, the cost weights in Table 1.5 are normalized (sum to 1). These weights are next used in Table 1.7 to calculate the weighted average of the S-T price relatives from Table 1.4. When the aggregate S-T price relatives are chained together, they result in the calculation of the modified Laspeyres index. The aggregate S-T price relatives in the penultimate line of Table 1.7 correspond to the results presented by Msokwa in his Table A2.3. However, these are not the modified Laspeyres price indexes. The results presented in the final line of Table 1.7 in the Annex 1 are the correct modified Laspeyres price indexes. The aggregate Laspeyres price indexes derived in Tables 1.2, 1.3, 1.5, and 1.7 using equations (3), (11), and (12) show the same results. These equations and calculations demonstrate that both the long-term and modified Laspeyres price indexes yield the same price indexes.

5. Problems when using unweighted price indexes

The weights derived from the HBS as items in the CPI are typically for a commodity grouping such as cheese, butter, milk, etc. There is no identification of the specific brand or variety of the commodity and an associated weight. Statistical offices select a sample of individual transactions to represent each commodity, but there are typically no weights available at the transaction level. The statistical offices then use some method of averaging to produce an average price or an average price change to use in deriving the item or elementary index. (This level of computation is usually referred to as an elementary aggregate because it is the first level at which an index is compiled for aggregation to higher levels of the CPI.)

When weights are not available, the choice of the averaging method can be very important. The *CPI Manual*, chapter 20, shows that the larger the variation in the individual prices, the larger the difference among the standard averaging methods.

5.1 Arithmetic averages

The two methods used historically by statistical offices to calculate the elementary indexes are the ratio of average prices, known as the Dutot index, or the average of price relatives, known as the Carli index. The Dutot index uses the average prices of the sample of transactions in the current and base period to derive the current elementary index:

$$I_D^t = \frac{\sum p_i^t/n}{\sum p_i^0/n} = \frac{\sum p_i^t}{\sum p_i^0} \qquad (13)$$

The Dutot index has an implicit weighting associated with the base price levels (p_i^0). By multiplying equation (13) by (p_i^0/p_i^0) in both the numerator and denominator, the following equation can be derived:

$$I_D^t = \frac{\sum \left(\frac{p_i^t}{p_i^0}\right) p_i^0}{\sum p_i^0} \qquad (14)$$

The base prices serve as weights and the transactions with the largest base prices receive more importance than those with the smaller base prices in calculating price change over time. Normally items with the highest prices would have less weight. In order to avoid this potential bias in weighting, the sampled transactions should be homogeneous in terms of their base price levels or their long-term price changes.

The Dutot index can also be calculated using the short-term price relative method where the price changes from the monthly price relatives are chained from the base period to the current month. Equation (13) can be expressed as:

$$I_D^t = \frac{\sum p_i^t}{\sum p_i^{t-1}} \times \ I_D^{t-1} \qquad (15)$$

Annex 2 provides a table with an example of an item along with prices for a representative sample of seven transactions. The arithmetic average prices are derived along with the long-term (L-T) price relatives. The Dutot price index is calculated using the L-T price relatives. Next, the short-term price relatives are calculated from the monthly average prices. These are chained together and produce the same results for the Dutot price index. The results in Table 2.A show that, when the prices return to their original base period levels, the Dutot price index is 100. The second method traditionally used by statistical agencies has been the average of price relatives known as the Carli price index.

$$I_C^t = \frac{1}{n}\sum \frac{p_i^t}{p_i^o} \qquad (16)$$

The Carli index is similar to the Laspeyres index where each observation is equally weighted. As seen in Table 2.B, the Carli index using L-T price relatives produces slightly different results than the Dutot index. This is due to the fact that the Dutot index is implicitly weighted by the base period prices while the Carli index has equal weights. Both indexes return to 100 when the price levels return to their base price values.

A Carli price index can also be calculated using the short-term relative method. The chained Carli is calculated using the following formula:

$$I_{Cc}^t = \frac{1}{n}\sum \frac{p_i^t}{p_i^{t-1}} \times \ I_{Cc}^{t-1} \qquad (17)$$

Table 2.C in Annex 2 presents the S-T price relatives for the seven sampled varieties and the average price relatives for each month. When these price relatives are chained, they produce different results than those for the fixed base Carli using the average of L-T price relatives. The chained Carli price index has a definite upward bias. When the variety prices return to the base price levels, the index in the example has increased to 111.9 when we expect it to be 100. This chained version of the Carli index should not be used by statistical agencies for calculating elementary level indexes in the CPI.

5.2 Geometric averages

With the introduction of the *CPI Manual* in 2004, a major emphasis was placed on using geometric averaging when weights are not available for the individual transactions in the CPI elementary indexes. The geometric price index is known as the Jevons price index and is calculated either as the ratio of the geometric average prices or as the product of the price relatives with each weighted exponentially:

$$I_J^t = \frac{\prod(p_i^t)^{\frac{1}{n}}}{\prod(p_i^o)^{\frac{1}{n}}} = \prod\left(\frac{p_i^t}{p_i^o}\right)^{\frac{1}{n}} \qquad (18)$$

In Table 2.A of Annex 2 the geometric mean is calculated for each month and used to compile the elementary Jevons index by the ratio of average prices. In Table 2.B, the geometric mean of the L-T price relatives is used to calculate the Jevons index. The Jevons indexes are the same using both the ratio of averages and average of L-T relatives. This is much different than the results using the arithmetic means of average prices (Dutot index) or the average of L-T price relatives (Carli index) which differ consistently in the examples in Tables 2.A and 2.B with a maximum difference of almost 3 percent in October.

The Jevons index can also be calculated using the chained S-T price relative method:

$$I_J^t = \prod\left(\frac{p_i^t}{p_i^o}\right)^{\frac{1}{n}} = \prod\left[\left(\frac{p_i^t}{p_i^{t-1}}\right)^{\frac{1}{n}} \times \left(\frac{p_i^{t-1}}{p_i^o}\right)^{\frac{1}{n}}\right] = \prod\left[\left(\frac{p_i^t}{p_i^{t-1}}\right)^{\frac{1}{n}}\right] \times I_J^{t-1} \quad (18')$$

The Jevons index provides different estimates than either the Dutot or the Carli. Like the Dutot index. The Jevons yields the same index numbers whether using the L-T price relative method or the S-T relative method as shown in Tables 2.A.The Jevons index also provides the same index numbers despite the method used as is seen in Tables 2.A, 2.B, and 2.C. This property does not hold true for the Carli index. The chained S-T Carli index in our sample data is always equal to or greater than the L-T Carli index.

The Jevons index level will always be equal to or less than the Dutot index because a geometric average is always equal to or less than an arithmetic average. However, this does not hold for the price changes. For example, in the months of July through October and again in December the S-T price relatives for the Jevons index are larger than those for the Dutot index.

The *CPI Manual* (chapter 20) strongly encourages the use of the Jevons price index for calculating elementary indexes where weights are unavailable. It notes that the Dutot price index should only be used in cases where the sample of transactions is homogeneous with respect to base price levels or price trends. The *Manual* strongly discourages the use of the S-T Carli price index because of its known upward bias. The S-T method for the Jevons index will easily accommodate replacement transactions or adjustments for quality changes. As mentioned earlier, the statistical office will only need to collect prices for the current and previous periods to enter in the system. If the L-T method is used, quality adjustments will involve changing the base price of the transaction for the value of the quality change.

6. Conclusions

Comparing the results of the aggregate indexes presented in Tables 1.2, 1.3, 1.5, and 1.7, one must conclude that both the long-term method and modified method for compiling the Laspeyres price index yield the same results. This differs significantly from Msokwa's conclusions. In Section 7 of his article Msokwa concludes:
"… The results were different when using the modified method on the same figures; the following were the result; for the month of April the index was 80.28 less than 100 and for the month of August the index was 104.91 more than 100, though the prices for all items were the same as January in both months. When it happened that the prices for September and October were same for all items, the indices, however were different with 129.78 and 100 respectively. (Table A2.3 in Annex 2)" (p. 73).

These cited results represent the weighted monthly price relatives, not the aggregate price index from the modified (two-step) formula. These monthly relatives, when chained together, actually provide the correct price index as shown in Table 1.7 of Annex 1 to this article. This error in calculation of the modified Laspeyres also affects the other major conclusion by Msokwa:

“These results indicates(sic) that fixed basket weight Laspeyres’ method yields results that are consistent with the economic and index number theories while modified Laspeyres’ method does not. The most striking part is that when prices of the current period happen to be the same as the base period prices the index number computed by the modified Laspeyres’ formula does not yield to 100 (the base period price index).” (p. 73). As the correct calculations in Table 1.7 demonstrate, the modified Laspeyres formulas do, in fact, return the price index to 100 when the prices return to their base period levels over time. The recommendations in Section 8 of the Msokwa article raising concerns about the modified Laspeyres methods are also called into question because they are based on formulas and calculations that are not correct. In fact, the use of the modified (two step) formula should be encouraged because it enables statistical offices to make replacements for missing items more easily and to update the sample for new items that have gained significantly in importance to the consumer market.

The more serious issue with the modified (two-step) index formula occurs when weights are not available. The arithmetic average of S-T price relatives (Carli index) has an upward bias and should not be used for compiling elementary level indexes in the CPI. The arithmetic average of prices (Dutot index) also has an issue related to the homogeneity of the sample transactions. The formula implicitly uses the base prices of the sample transactions as weights. It should only be used in cases where the base prices are homogeneous in terms of their levels.

The best approach for calculating unweighted elementary indexes in the CPI is to use the Jevons price index that is geometric average of the transaction price levels or geometric average of transactions’ price relatives. They are mathematically equivalent and so they yield the same index results. The short-term version of the Jevons index is usually recommended because it facilitates the replacement of transactions, the introduction of new products, and the adjustments needed to make quality changes.

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Annex 1: Corrected Examples for CPI Methods (presented in Msokwa article)

Table 1.1: Prices and weights

Item	Wgt (Wo)	Jan (Po)	Feb	March	April	May	June	July	August	Sept	Oct.	Nov	Dec
A	25	120	125	140	146	120	150	110	120	165	165	167	170
B	40	300	310	342	348	300	310	290	300	351	351	354	360
C	25	405	450	455	464	405	475	400	405	486	486	490	500
D	55	90	96	115	123	90	96	85	90	126	126	130	135
Total	145												

Table 1.2: Long-term Laspeyres price index using price relatives

Item	Wgt (Wo)	Jan (Po)	Feb	March	April	May	June	July	August	Sept	Oct.	Nov	Dec
A	25		104.167	116.667	121.667	100.000	125.000	91.667	100.000	137.500	137.500	139.167	141.667
B	40		103.333	114.000	116.000	100.000	103.333	96.667	100.000	117.000	117.000	118.000	120.000
C	25		111.111	112.346	114.568	100.000	117.284	98.765	100.000	120.000	120.000	120.988	123.457
D	55		106.667	127.778	136.667	100.000	106.667	94.444	100.000	140.000	140.000	144.444	150.000
Total	145												
Agg Index		100	106.082	119.401	124.569	100.000	110.739	95.324	100.000	129.776	129.776	132.195	135.711

Table 1.3: Updated cost weight method for L-T Laspeyres price index

Item	Wgt (Wo)	Jan (Po)	Feb	March	April	May	June	July	August	Sept	Oct.	Nov	Dec
A	25		26.042	29.167	30.417	25.000	31.250	22.917	25.000	34.375	34.375	34.792	35.417
B	40		41.333	45.600	46.400	40.000	41.333	38.667	40.000	46.800	46.800	47.200	48.000
C	25		27.778	28.086	28.642	25.000	29.321	24.691	25.000	30.000	30.000	30.247	30.864
D	55		58.667	70.278	75.167	55.000	58.667	51.944	55.000	77.000	77.000	79.444	82.500
Total	145		153.819	173.131	180.625	145.000	160.571	138.219	145.000	188.175	188.175	191.683	196.781
Agg Index			106.082	119.401	124.569	100.000	110.739	95.324	100.000	129.776	129.776	132.195	135.711

Table 1.4: Month-to-month price changes in terms of price relatives

Item	Wgt (Wo)	Jan (Po)	Feb	March	April	May	June	July	August	Sept	Oct.	Nov	Dec
A			1.0417	1.1200	1.0429	0.8219	1.2500	0.7333	1.0909	1.3750	1.0000	1.0121	1.0180
B			1.0333	1.1032	1.0175	0.8621	1.0333	0.9355	1.0345	1.1700	1.0000	1.0085	1.0169
C			1.1111	1.0111	1.0198	0.8728	1.1728	0.8421	1.0125	1.2000	1.0000	1.0082	1.0204
D			1.0667	1.1979	1.0696	0.7317	1.0667	0.8854	1.0588	1.4000	1.0000	1.0317	1.0385

Table 1.5: Modified (two-step) Laspeyres index using S-T price relatives to update cost weights

Item	Wgt (Wo)	Jan (Po)	Feb	March	April	May	June	July	August	Sept	Oct.	Nov	Dec
A	25		26.042	29.167	30.417	25.000	31.250	22.917	25.000	34.375	34.375	34.792	35.417
B	40		41.333	45.600	46.400	40.000	41.333	38.667	40.000	46.800	46.800	47.200	48.000
C	25		27.778	28.086	28.642	25.000	29.321	24.691	25.000	30.000	30.000	30.247	30.864
D	55		58.667	70.278	75.167	55.000	58.667	51.944	55.000	77.000	77.000	79.444	82.500
Total	145		153.819	173.131	180.625	145.000	160.571	138.219	145.000	188.175	188.175	191.683	196.781
Agg Index	145	100	106.082	119.401	124.569	100.000	110.739	95.324	100.000	129.776	129.776	132.195	135.711

Table 1.6: Normalized monthly updated weights from updated cost weights

Item	Wgt (Wo)	Jan (Po)	Feb	March	April	May	June	July	August	Sept	Oct.	Nov	Dec
A	0.1724		0.1693	0.1685	0.1684	0.1724	0.1946	0.1658	0.1724	0.1827	0.1827	0.1815	0.1800
B	0.2759		0.2687	0.2634	0.2569	0.2759	0.2574	0.2797	0.2759	0.2487	0.2487	0.2462	0.2439
C	0.1724		0.1806	0.1622	0.1586	0.1724	0.1826	0.1786	0.1724	0.1594	0.1594	0.1578	0.1568
D	0.3793		0.3814	0.4059	0.4161	0.3793	0.3654	0.3758	0.3793	0.4092	0.4092	0.4145	0.4192
Total	1.0000		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 1.7: Weighted price relatives using normalized weights to produce a chain Laspeyres price index

Item	**Wgt (Wo)**	**Jan (Po)**	**Feb**	**March**	**April**	**May**	**June**	**July**	**August**	**Sept**	**Oct.**	**Nov**	**Dec**
A	0.1724		0.1796	0.1896	0.1757	0.1384	0.2155	0.1427	0.1809	0.2371	0.1827	0.1849	0.1848
B	0.2759		0.2851	0.2965	0.2680	0.2215	0.2851	0.2408	0.2894	0.3228	0.2487	0.2508	0.2504
C	0.1724		0.1916	0.1826	0.1654	0.1384	0.2022	0.1538	0.1809	0.2069	0.1594	0.1607	0.1610
D	0.3793		0.4046	0.4569	0.4342	0.3045	0.4046	0.3235	0.3979	0.5310	0.4092	0.4222	0.4304
Agg price chg		1.00000	1.0608	1.1255	1.0433	0.8028	1.1074	0.8608	1.0491	1.2978	1.0000	1.0186	1.0266
Agg chain index		100.000	106.082	119.401	124.569	100.000	110.739	95.324	100.000	129.776	129.776	132.195	135.711

Annex 2: Examples of Alternate Unweighted Price Index Measures

Table 2.A: Dutot and Jevons Price Indexes Using Averages of Prices

	Base	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.	Jan.
Item A							**Prices**							
Variety 1	2.36	2.09	1.93	2.28	2.05	2.09	2.18	2.75	2.70	2.67	2.60	2.73	2.21	2.36
Variety 2	5.02	5.38	5.12	4.45	4.08	4.03	7.12	9.48	6.28	5.57	4.80	4.75	4.48	5.02
Variety 3	5.34	5.07	5.09	5.52	6.29	5.02	5.36	7.95	6.17	5.93	5.40	6.55	6.79	5.34
Variety 4	6.00	5.73	4.27	4.92	4.75	5.15	6.06	8.17	7.44	6.42	4.77	5.49	5.32	6.00
Variety 5	6.12	6.39	5.50	5.46	5.86	6.08	6.31	7.10	6.40	6.97	6.12	5.70	5.08	6.12
Variety 6	2.80	2.72	2.82	2.96	2.85	2.78	3.33	4.36	3.14	3.24	3.14	3.11	2.61	2.80
Variety 7	6.21	5.45	6.95	6.88	5.27	5.29	9.91	9.23	5.08	5.85	5.29	6.67	5.14	6.21
Arithmetic average price	**4.84**	**4.69**	**4.52**	**4.64**	**4.45**	**4.35**	**5.75**	**7.01**	**5.32**	**5.23**	**4.59**	**5.00**	**4.52**	**4.84**
L-T Price Relative	*1.000*	*0.970*	*0.935*	*0.959*	*0.920*	*0.899*	*1.189*	*1.448*	*1.099*	*1.082*	*0.949*	*1.033*	*0.934*	*1.000*
Dutot Index (L-T Ratio of Average Prices)	**100.0**	**97.0**	**93.5**	**95.9**	**92.0**	**89.9**	**118.9**	**144.8**	**109.9**	**108.2**	**94.9**	**103.3**	**93.4**	**100.0**
S-T Price Relative		*0.970*	*0.964*	*1.025*	*0.959*	*0.977*	*1.323*	*1.218*	*0.759*	*0.985*	*0.877*	*1.089*	*0.904*	*1.071*
Dutot Index (Chained S-T Ratio of Average Prices)	**100.0**	**97.0**	**93.5**	**95.9**	**92.0**	**89.9**	**118.9**	**144.8**	**109.9**	**108.2**	**94.9**	**103.3**	**93.4**	**100.0**
Geometirc average price	***4.55***	***4.38***	***4.20***	***4.38***	***4.17***	***4.10***	***5.22***	***6.49***	***5.01***	***4.97***	***4.42***	***4.76***	***4.23***	***4.55***
Jevons Index (L-T Ratio of Geometric Average Prices)	**100.0**	**96.3**	**92.3**	**96.1**	**91.7**	**90.1**	**114.6**	**142.5**	**110.2**	**109.2**	**97.0**	**104.5**	**93.0**	**100.0**

Table 2.B: Carli and Jevons Price Indexes Using Averages of L-T Price Relatives

Item A							**L-T Price Relatives**							
Variety 1	1.000	0.888	0.816	0.968	0.869	0.888	0.927	1.166	1.147	1.134	1.101	1.160	0.937	1.000
Variety 2	1.000	1.072	1.019	0.886	0.813	0.803	1.417	1.888	1.250	1.109	0.956	0.945	0.893	1.000
Variety 3	1.000	0.949	0.953	1.033	1.178	0.939	1.002	1.487	1.154	1.109	1.011	1.225	1.271	1.000
Variety 4	1.000	0.955	0.712	0.820	0.792	0.857	1.009	1.361	1.240	1.069	0.794	0.915	0.886	1.000
Variety 5	1.000	1.044	0.898	0.892	0.957	0.992	1.031	1.160	1.046	1.138	1.000	0.931	0.829	1.000
Variety 6	1.000	0.974	1.008	1.058	1.018	0.995	1.191	1.557	1.122	1.158	1.124	1.111	0.932	1.000
Variety 7	1.000	0.877	1.118	1.108	0.848	0.852	1.595	1.485	0.817	0.941	0.852	1.073	0.827	1.000
Arithmetic average of L−T price relatives	*1.000*	*0.966*	*0.932*	*0.966*	*0.925*	*0.904*	*1.167*	*1.444*	*1.111*	*1.094*	*0.977*	*1.051*	*0.939*	*1.000*
Carli Index (L-T Arithmetic Changes)	**100.0**	**96.6**	**93.2**	**96.6**	**92.5**	**90.4**	**116.7**	**144.4**	**111.1**	**109.4**	**97.7**	**105.1**	**93.9**	**100.0**
Geometric average of price relatives	*1.000*	*0.963*	*0.923*	*0.961*	*0.917*	*0.901*	*1.146*	*1.425*	*1.102*	*1.092*	*0.970*	*1.045*	*0.930*	*1.000*
Jevons index (L-T Geometric Changes)	**100.0**	**96.3**	**92.3**	**96.1**	**91.7**	**90.1**	**114.6**	**142.5**	**110.2**	**109.2**	**97.0**	**104.5**	**93.0**	**100.0**

Table 2.C: Carli and Jevons Price Indexes Using Chained S-T Price Relatives

Item A							**S−T Price Relatives**							
Variety 1	1.000	0.888	0.920	1.185	0.898	1.022	1.043	1.259	0.983	0.989	0.971	1.054	0.808	1.067
Variety 2	1.000	1.072	0.950	0.870	0.917	0.988	1.765	1.332	0.662	0.887	0.862	0.989	0.945	1.120
Variety 3	1.000	0.949	1.004	1.084	1.140	0.797	1.067	1.484	0.776	0.961	0.912	1.212	1.037	0.787
Variety 4	1.000	0.955	0.745	1.152	0.966	1.082	1.177	1.349	0.912	0.862	0.743	1.152	0.969	1.128
Variety 5	1.000	1.044	0.860	0.993	1.074	1.037	1.039	1.126	0.901	1.089	0.879	0.931	0.891	1.206
Variety 6	1.000	0.974	1.035	1.050	0.962	0.978	1.197	1.308	0.721	1.032	0.970	0.989	0.839	1.073
Variety 7	1.000	0.877	1.275	0.991	0.765	1.005	1.872	0.931	0.550	1.151	0.906	1.259	0.771	1.209
Arithmetic average of S−T price relatives	*1.000*	*0.966*	*0.970*	*1.046*	*0.960*	*0.987*	*1.309*	*1.255*	*0.786*	*0.996*	*0.892*	*1.084*	*0.894*	*1.084*
Carli Index (Chained S-T Arithmetic Changes)	**100.0**	**96.6**	**93.7**	**98.0**	**94.1**	**92.9**	**121.5**	**152.6**	**120.0**	**119.5**	**106.5**	**115.4**	**103.2**	**111.9**
Geometric average of S-T price relatives	*1.000*	*0.963*	*0.958*	*1.042*	*0.954*	*0.983*	*1.272*	*1.244*	*0.773*	*0.991*	*0.889*	*1.077*	*0.890*	*1.075*
Jevons index (Chained S-T Geometric Changes)	**100.0**	**96.3**	**92.3**	**96.1**	**91.7**	**90.1**	**114.6**	**142.5**	**110.2**	**109.2**	**97.0**	**104.5**	**93.0**	**100.0**