Bayesian Multiregional Population Forecasting: England

Arkadiusz Wiśniowski* and James Raymer**

Presentation prepared for the
Conference of European Statisticians
Joint Eurostat/UNECE Work Session on Demographic Projections
Geneva, 18-20 April 2016

*University of Manchester
**Australian National University

Contents

- Aims
- Background
- Modelling framework
- Illustration: population forecasts for England
- Results
- Summary and future work

Aims

- Develop a methodology for forecasting population using multiregional forecasting framework (Rogers 1975)
- Fully probabilistic forecasts (measures of uncertainty)

Background: Multistate (multiregional) population models

Background: Multistate (multiregional) population models

- Rogers (1975)
- Flexible platform for modelling and analysing subnational population change
- Combining all four population components by age, sex and region and between states (origin and destination)

Background: Multistate (multiregional) population models

- Rogers (1975)
- Flexible platform for modelling and analysing subnational population change
- Combining all four population components by age, sex and region and between states (origin and destination)
- Various approaches for single countries (see Rees & Turton 1998 and Gullickson 2001 for probabilistic models; e.g. Rogers & Willekens 1986; Wilson & Rees 2005 and Wilson & Bell 2007 for reviews)
- ... or groups of countries/regions (e.g. Kupiszewski & Kupiszewska 1998; Bijak et al. 2007)

Background: Modelling and forecasting age schedules

- Imposition of empirical tables of fixed rates
- Parametric approaches, e.g. Gompertz-Makeham, Heligman-Pollard, Coale-McNeil, Rogers-Castro
- Functional models: Hyndman et al.
- Relational approaches, e.g. Brass, Lee-Carter, Girosi-King, Haberman and Renshaw, Czado et al.
- Lee-Carter (1992) and extensions are widely used for forecasting mortality (Booth)

Modelling framework

Modelling framework

- Counts of demographic events Y_{rast} by region (r), age (a), sex (s) and time (t) follow a Poisson distribution

$$Y_{rast} \sim \text{Poisson}\left(\mu_{rast} E_{rast}\right)$$

  where E denotes exposure (population at risk)

- Logarithm of the rate μ_{rast} follows normal distribution

$$\log \mu_{rast} \sim \text{Normal}\left(M, \tau\right)$$

  where mean M is a model for the contingency table by region, age, sex, and time.

- Model M contains time effect κ – time series model used to forecast

$$\kappa_t \sim \text{Normal}\left(\phi_0 + \phi_1 \kappa_{t-1}, \tau_\kappa\right)$$

Age, sex and regional schedules over time

- Multiplicative components (log-linear) model
  - Modelling of contingency tables, e.g. migration by origin and destination and other characteristics (Smith et al. 2010, Raymer et al. 2011)
  - Decomposing a multidimensional array (ODAST) into lower dimensional arrays

Age, sex and regional schedules over time

- Multiplicative components (log-linear) model
  - Modelling of contingency tables, e.g. migration by origin and destination and other characteristics (Smith et al. 2010, Raymer et al. 2011)
  - Decomposing a multidimensional array (ODAST) into lower dimensional arrays
- Bilinear model (forecasting mortality: Lee & Carter 1992)
  - Has been adopted to fit various settings, including fertility and migration
  - Decomposes age-time schedule into average age schedule over time, age-specific changes in schedule in response to changes in time effect (subject to contraints on the model parameters)
  - Time-series models are used to model and forecast time effect

Illustration: England

Illustration: England

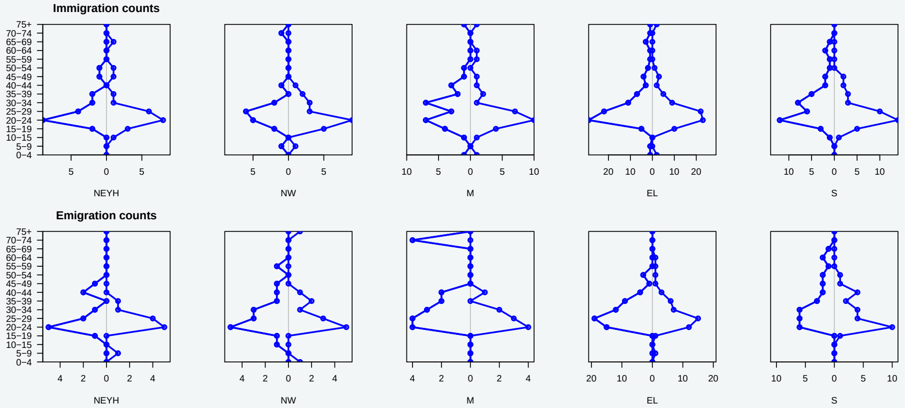
- Five origin and five destination regions: (1) North East with Yorkshire and Humber, (2) North West, (3) Midlands (West Midlands and East Midlands), (4) East and London, and (5) South (Southwest and Southeast)
- Baseline population: 2007, forecasting horizon: 2037
- 16 five-year age groups $(0-4,\ldots,75+)$

Illustration: England

- Five origin and five destination regions: (1) North East with Yorkshire and Humber, (2) North West, (3) Midlands (West Midlands and East Midlands), (4) East and London, and (5) South (Southwest and Southeast)
- Baseline population: 2007, forecasting horizon: 2037
- 16 five-year age groups $(0-4,\ldots,75+)$

- Interregional migration
  - Estimates from Raymer et al. (2011)
  - Based on Patient Register snapshots
  - Estimated using multiplicative component model (OA, DA, AS, OD)
  - Available for 1991-2007
- Births (ONS) 2000-2007
- Deaths (ONS) 2003-2007
- International emigration and immigration (ONS) 1991-2007
  - Low quality of detailed characteristics (region-age-sex)
  - Assume the average RAS-specific profile for all years

International migration data 2012



Modelling population components for England

Modelling fertility

- Births by region (r), age (a) and time (t) follow a Poisson distribution

$$Y_{rat} \sim \mathrm{Poisson}\left(\mu_{rat} E_{rat}\right)$$

- Logarithms of fertility rates μ_{rat} follow normal distribution

$$\log \mu_{rat} \sim \mathrm{Normal}\left(RA + A_1 + A_2 \kappa_t, \tau\right)$$

- A_1 – average age schedule, A_2 – changes of age schedule over time (Lee & Carter parameters)

Modelling fertility

- Births by region (r), age (a) and time (t) follow a Poisson distribution

$$Y_{rat} \sim \text{Poisson}\left(\mu_{rat} E_{rat}\right)$$

- Logarithms of fertility rates μ_{rat} follow normal distribution

$$\log \mu_{rat} \sim \text{Normal}\left(RA + A_1 + A_2 \kappa_t, \tau\right)$$

- A_1 – average age schedule, A_2 – changes of age schedule over time (Lee & Carter parameters)

Modelling fertility

- Births by region (r), age (a) and time (t) follow a Poisson distribution

$$Y_{rat} \sim \mathrm{Poisson}\left(\mu_{rat} E_{rat}\right)$$

- Logarithms of fertility rates μ_{rat} follow normal distribution

$$\log \mu_{rat} \sim \mathrm{Normal}\left(RA + A_1 + A_2 \kappa_t, \tau\right)$$

- A_1 – average age schedule, A_2 – changes of age schedule over time (Lee & Carter parameters)
- Time series model to forecast κ

$$\kappa_t \sim \mathrm{Normal}\left(\phi_0 + \phi_1 \kappa_{t-1}, \tau_\kappa\right)$$

Modelling mortality

- Deaths by region (r), age (a), sex (s) and time (t) follow a Poisson distribution

$$Y_{rast} \sim \text{Poisson}\left(\mu_{rast} E_{rast}\right)$$

- Logarithms of mortality rates μ_{rast} follow normal distribution

$$\log \mu_{rast} \sim \text{Normal}\left(RA + RS + AS_1 + AS_2 \kappa_{st}, \tau\right)$$

- AS_1, AS_2 – age profiles for males and females separately, with sex-specific κ
- Time series model to forecast κ

$$\kappa_{st} \sim \text{Normal}\left(\phi_0 + \phi_1 \kappa_{st-1}, \tau_\kappa\right)$$

Modelling mortality

- Deaths by region (r), age (a), sex (s) and time (t) follow a Poisson distribution

$$Y_{rast} \sim \text{Poisson}\left(\mu_{rast} E_{rast}\right)$$

- Logarithms of mortality rates μ_{rast} follow normal distribution

$$\log \mu_{rast} \sim \text{Normal}\left(RA + RS + AS_1 + AS_2 \kappa_{st}, \tau\right)$$

- AS_1, AS_2 – age profiles for males and females separately, with sex-specific κ
- Time series model to forecast κ

$$\kappa_{st} \sim \text{Normal}\left(\phi_0 + \phi_1 \kappa_{st-1}, \tau_\kappa\right)$$

Modelling interregional migration

- Migration events by origin (o), destination (d), age (a), sex (s) and time (t) follow a Poisson distribution

$$Y_{odast} \sim \text{Poisson}\left(\mu_{odast} E_{oast}\right)$$

- Logarithms of out-migration rates μ_{odast} follow normal distribution

$$\log \mu_{odast} \sim \text{Normal}\left(OA + DA + OS + DS + OD_1 + OD_2 \kappa_t, \tau\right)$$

- OD_1 – average OD 'schedule', OD_2 – changes of OD schedule over time
- Time series model to forecast κ

$$\kappa_t \sim \text{Normal}\left(\phi_0 + \phi_1 \kappa_{t-1}, \tau_\kappa\right)$$

Modelling interregional migration

- Migration events by origin (o), destination (d), age (a), sex (s) and time (t) follow a Poisson distribution

$$Y_{odast} \sim \text{Poisson}\left(\mu_{odast} E_{oast}\right)$$

- Logarithms of out-migration rates μ_{odast} follow normal distribution

$$\log \mu_{odast} \sim \text{Normal}\left(OA + DA + OS + DS + OD_1 + OD_2 \kappa_t, \tau\right)$$

- OD_1 – average OD 'schedule', OD_2 – changes of OD schedule over time
- Time series model to forecast κ

$$\kappa_t \sim \text{Normal}\left(\phi_0 + \phi_1 \kappa_{t-1}, \tau_\kappa\right)$$

Modelling international emigration

- Logarithms of emigration rates Y_{rast}/E_{rast} follow normal distribution

$$\log \frac{Y_{rast}}{E_{rast}} \sim \text{Normal}\left(RSA + \kappa_t, \tau\right)$$

- RSA – reflects proportioning of the data
- Time series model to forecast κ

$$\kappa_t \sim \text{Normal}\left(\phi_0 + \phi_1 \kappa_{t-1}, \tau_\kappa\right)$$

Modelling international immigration

- Logarithms of immigration counts Y_{rast} follow normal distribution

$$\log Y_{rast} \sim \text{Normal}\left(RSA + \kappa_t, \tau\right)$$

- Time series model to forecast κ

$$\kappa_t \sim \text{Normal}\left(\phi_0 + \phi_1 \kappa_{t-1}, \tau_\kappa\right)$$

Prior distributions

Prior distributions

- Hierarchical specification permits borrowing of strength

$$OA \sim \text{Normal}\left(A, \tau_1\right), \qquad DA \sim \text{Normal}\left(A, \tau_2\right)$$
$$OS \sim \text{Normal}\left(S, \tau_3\right), \qquad DS \sim \text{Normal}\left(S, \tau_4\right)$$
$$A \sim \text{Normal}\left(0, \tau_A\right)$$
$$S \sim \text{Normal}\left(0, \tau_S\right)$$

- Weakly informative priors for variance

$$\tau \sim \text{Normal}\left(0, 10^4\right) \mathbb{1}(\tau > 0)$$

- Priors for changing profiles (OD_1, OD_2, A_1, A_2)
  $\Rightarrow$ as in Wiśniowski et al. (2015)
- Weakly informative priors for autoregressive model

$$\phi_0 \sim \text{Normal}\left(0, 10^4\right), \qquad \phi_1 \sim \text{Normal}\left(0, 10^4\right) \mathbb{1}(0 \leq \phi_1 \leq 1)$$

Prior distributions

- Hierarchical specification permits borrowing of strength

$$OA \sim \text{Normal}(A, \tau_1), \qquad DA \sim \text{Normal}(A, \tau_2)$$
$$OS \sim \text{Normal}(S, \tau_3), \qquad DS \sim \text{Normal}(S, \tau_4)$$
$$A \sim \text{Normal}(0, \tau_A)$$
$$S \sim \text{Normal}(0, \tau_S)$$

- Weakly informative priors for variance

$$\tau \sim \text{Normal}\left(0, 10^4\right) \mathbb{1}(\tau > 0)$$

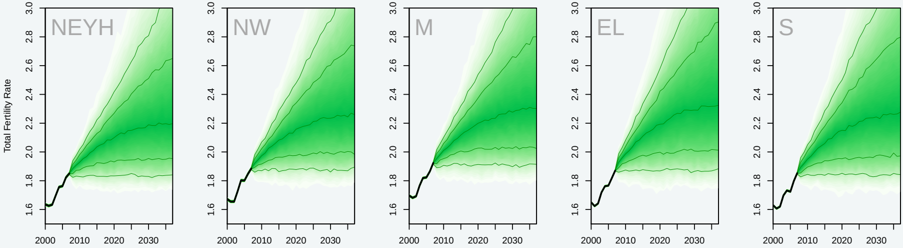
- Priors for changing profiles (OD_1, OD_2, A_1, A_2)
  $\Rightarrow$ as in Wiśniowski et al. (2015)
- Weakly informative priors for autoregressive model

$$\phi_0 \sim \text{Normal}\left(0, 10^4\right), \qquad \phi_1 \sim \text{Normal}\left(0, 10^4\right) \mathbb{1}(0 \leq \phi_1 \leq 1)$$

- ... except for fertility model

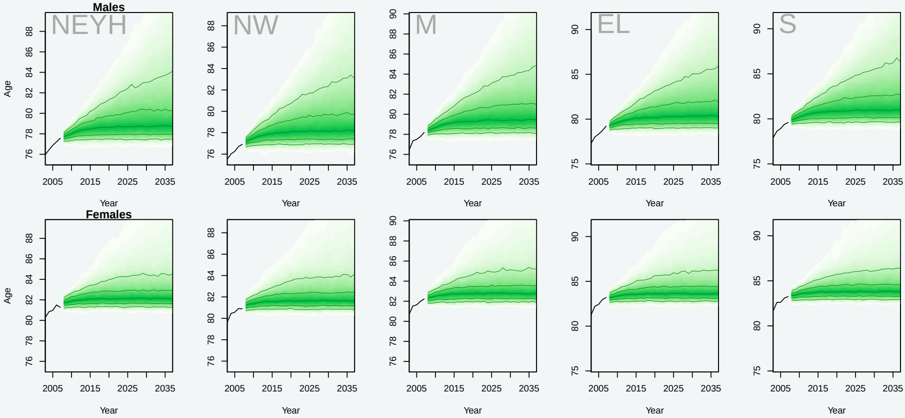
$$\phi_1 \sim \text{Normal}\left(0, \frac{1}{9}\right) \mathbb{1}(-1 \leq \phi_1 \leq 1)$$

Results

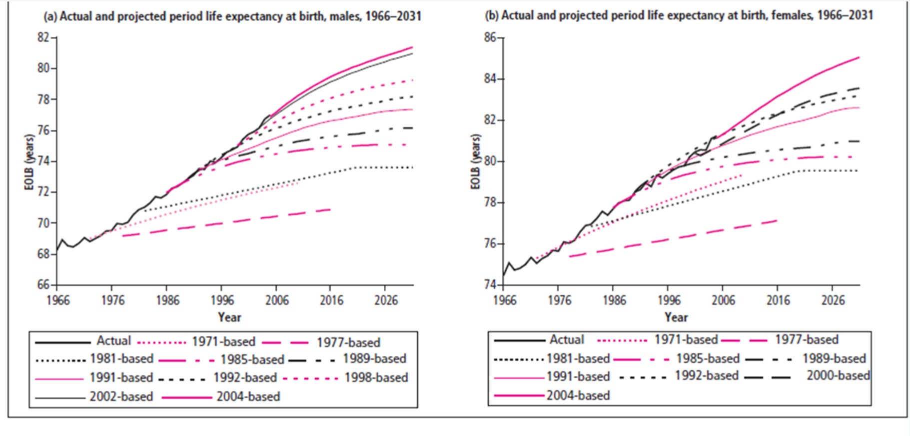


Note: bold black line – data,
green lines from bottom – 10^{th}, 25^{th}, 50^{th} (median), 75^{th} and 90^{th} percentile

Life Expectancy

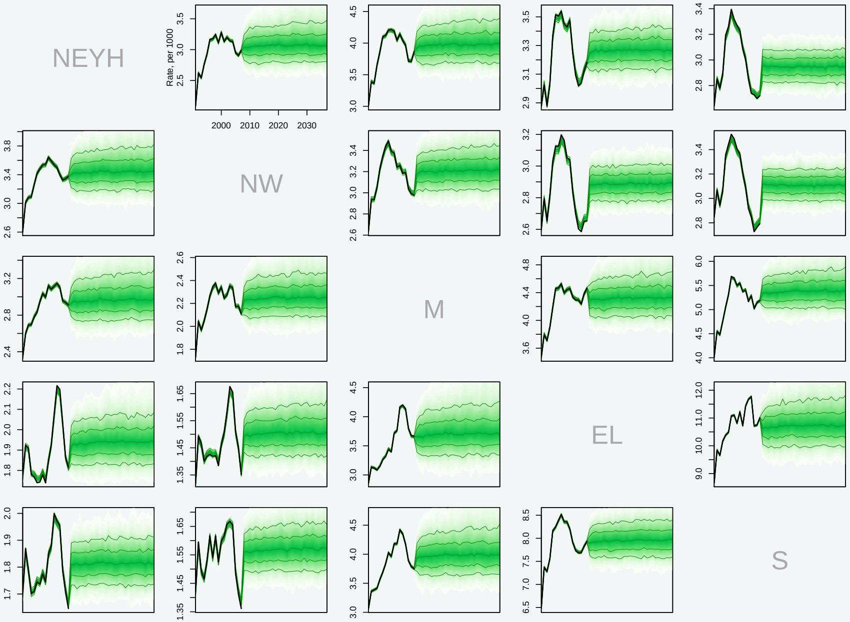


Life Expectancy

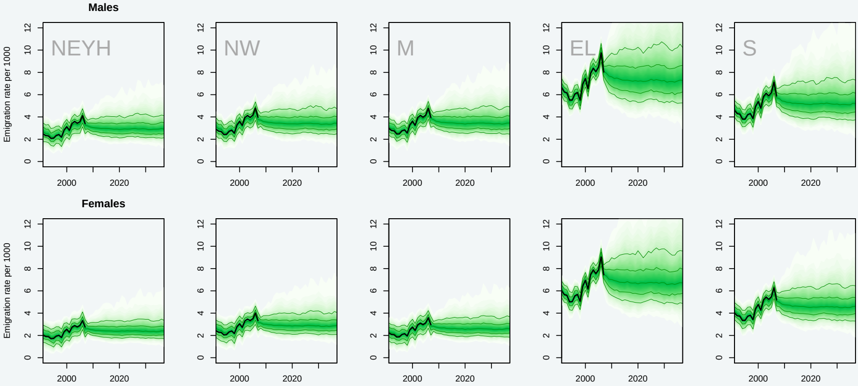


Source: Shaw (2007) 50 years of UK National population projections: how accurate have they been? Population Trends. 128: p8-23.

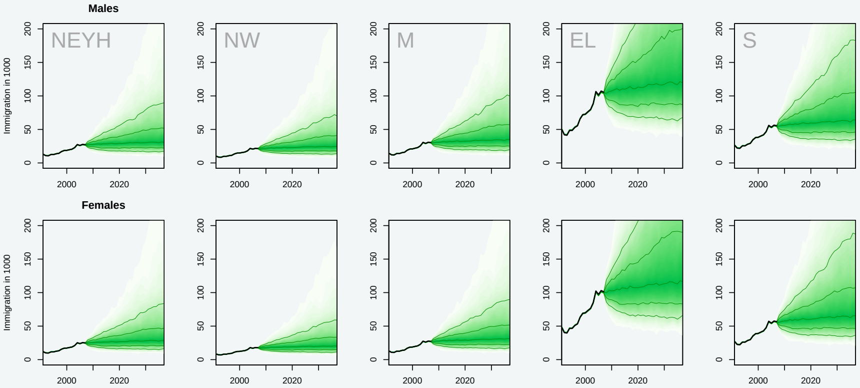
Internal migration (out-migration rates per 1000)



International emigration (rates per 1000)



International immigration (counts in 1000)



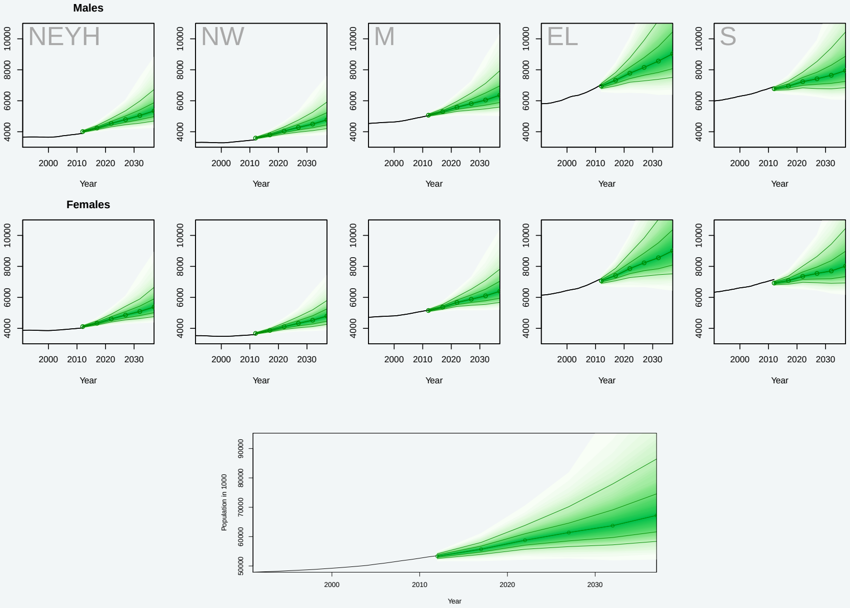
Multiregional population forecasts

- Cohort-component model based on classical approach by Rogers (1975), Rogers & Ledent (1976) and Ledent (1978)
- Open population: emigration rates and immigration counts (Rees 1986; Wiśniowski et al. 2015)
- Limitations

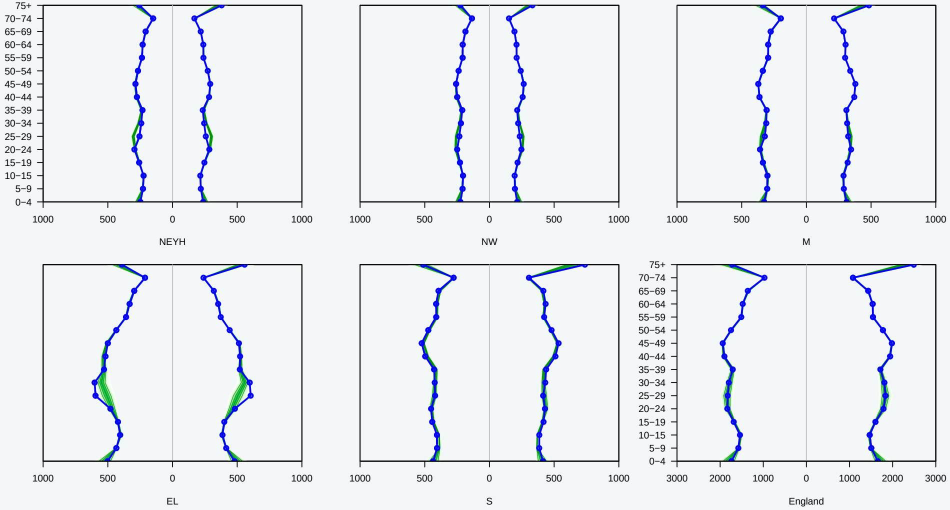
Multiregional population forecasts

- Cohort-component model based on classical approach by Rogers (1975), Rogers & Ledent (1976) and Ledent (1978)
- Open population: emigration rates and immigration counts (Rees 1986; Wiśniowski et al. 2015)
- Limitations
  - Simple approximations for proportions of person-years lived in a period
  - Rates are specific to the region where events occur
  - Oldest age group is 75+
  - Flows amongst constituency countries neglected

Population totals



Population pyramids 2012–2037



Population data (blue) - 2012

Summary

- Combination of multiplicative component and bilinear models offers a flexible approach to forecasting population components
- Flexible framework allowing incorporation of expert opinion
- Bayesian framework allows coherent integration of uncertainty
- Limitations: large number of components, large arrays $\rightarrow$ computationally intensive

Next steps

- Test various models for population components (DIC as model selection criterion)
- Incororate correlations between components (immigration and emigration) and sexes (mortality) (Li & Lee 2005; Wiśniowski et al. 2015)
- Produce forecasts of mortality of older age groups (e.g. Bijak et al. 2015)
- Improve model for international migration (Wiśniowski et al. 2016, IMEM)
- Add longer time series
- Increase number of regions

Acknowledgements

Authors would like to thank members of the Modelling Team at the ESRC Centre for Population Change, University of Southampton: Jakub Bijak, Jon J. Forster and Peter W. F. Smith, for their contribution to the early developments of the modelling framework.