STATISTICAL COMMISSION and
ECONOMIC COMMISSION FOR EUROPE

CONFERENCE OF EUROPEAN STATISTICIANS

Joint Consultation on the European Comparison Programme
(Geneva, 31 March-2 April 2003)

WORKING PAPER No 8

ENGLISH ONLY

RECENT METHODOLOGICAL ISSUES

EQUI-REPRESENTATIVITY AND SOME MODIFICATIONS OF THE EKS METHOD AT THE BASIC HEADING LEVEL*

Submitted by Statistics Austria

* Prepared by S. Sergeev, Statistics Austria.

Content

Introduction

The calculation of PPPs at the basic heading level is closely linked to the approaches using to collect the basic price data. The ECP reform and the participation of the CC in the Eurostat comparison led to substantial changes, which had an impact on the structure and the content of input data. Obviously, the set of 31 countries is more heterogeneous than the former set of 18 EU / EFTA countries. Respectively, the numerical procedures applied should be adjusted („fine tuning“) to the new situation. Additionally a critical review of the methods is necessary taking into account the forthcoming ICP 2004 round. Many regions did not participate in the ICP a long time and they need the methodological and technical support (transfer of knowledge) from the regions like the ECP where international comparisons are carried out regularly. The ICP regions have some degree of freedom to choice the most appropriate methods. In this sense, all methods applying presently within the ECP should be cleaned from weaknesses to be recommended (and to be competetive with other methods like the CPD method for the point in question) for other ICP regions.

The EKS method using presently within the ECP for the calculation of the BH-PPPs was one of the topics of the discussions during the recent World Bank ICP Conference (Washington, March 2002). It was mentioned that this approach can produce biased results by some circumstances. These arguments are reasonable and some modifications are desirable. The author of this paper did the same conclusion during the preparation of the software for the ECP'96/II and a short respective notice was written at the beginning of 1997. It was planed to discuss this topic during the Eurostat Meeting of the Working Group on „GDP Volume Comparison“ (18th-19th November 1998). However this was postponed due to more urgent problems of the reformed Eurostat comparison. Nevertheless it was agreed after the Washington ICP meeting to consider this point and, in effect, the presentation of the modified method (mainly from a theoretical point of view) was done during the Eurostat Working Party on Purchasing Power Parities (LUX, 12 - 13 June 2002). The participants appreciated generally the efforts but it was agreed that a broad set of numerical experiments on the basis of actual data from Surveys should be done to check the possible impact of the proposed modifications on the ECP results. The requested experiments were carried for all six consumer Eurostat Surveys (E99-1, E99-2, E00-1, E00-2, E01-1, E01-2). The results of the experiments were discussed during the Eurostat PPP Working Party PPP (LUX, 18 November 2002). It was agreed that the modified method should be included in the PPP regulation as an alternative method. The present paper contains the analysis of the calculation of bilateral PPPs at the BH level as input data for the EKS method (traditional EKS as well as modified, so-called, EKS-S) and an analysis of the numerical experiments done on the basis of the Surveys from the reformed ECP.

What means „equi-representativity“in reality?

The ECP countries collect prices from a common multilateral basket and PPPs at the basic heading level are obtained on a multilateral basis using the EKS-method . Information on representativity of priced products is also taken into account in the calculation of basic heading parities and the term “equi-representativity” is used very broadly. The exact meaning of the term „equi-representativity“ is the crucial point.

Sometimes the equi-representativity of a multilateral basket is presented as the request that all participating countries should have an equal or similar No. of items representative for their markets in the common basket. Such intention is understandable but this is an incorrect principle because „rich“ countries have much broader set of products than „poorer“ countries and a multilateral basket should not be oriented on the “poorest” country. In this sense it is impossible to establish an equi-representative multilateral basket for a heterogeneous set of the countries.

The same principle refers to the multilateral set of collected prices. The common item list is an offer for the price collection. It is not necessary and not possible that all countries collect prices for all items from item list. The common set of prices does not necessarily have to be equi-representative for each country in the sense of an equal number of priced or representative items. Of course, a multilateral item list should contain representative products for each country and multilateral set of collected prices should contain prices for representative products for each countries but the term “equi-representativity” should refer to the method of the PPP calculation but not to product basket. The approach applied for the PPP calculation eliminates the impact of a possible non-equi-representativity of a common basket and a set of collected prices.

Equi-representativity means that the applied computational procedures principally enable the calculation of PPPs which are representative (non-biased) for each participating country on the basis of common set of items.

Two features are used for this aim:

- attribution of the sign of representativity (asterisk=*)[1] in a given country for representative items priced. The correct attribution of asterisks is very important in the context of equi-representativity[2].

- principle of graduality, this means that it is not necessary to compare bilaterally each country with each country[3]. The countries with (very) different consumption patterns (without common priced items) can be compared indirectly, via third and more similar countries.

The creation of a general item list for a survey within the Eurostat comparison is based on the proposals from all countries and the countries are obliged to propose at least one representative product per BH. It means that the Item List for a Survey should include at least one representative item for each country. Obviously, to be useable for the PPP calculation, this product should be priced, at least, in one other country. The number of asterisked products are usually very different and depends on the development of national markets. To achieve the equi-representativity of the

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[1] The representative products are marked by asterisk (‘*’). Therefore this approach was named as a method of „asterisks“.

[2] It is necessary to underline that the asterisks indicate the representativity of items within a concrete BH but not within an aggregate (eg. „Consumption of HH“, etc. as a whole). The representativity of BH within the aggregates as a whole is taken into account during the aggregation procedure by the use of BH expenditure data.

[3] It seems that the Eurostat comparison 1975 was an unique one where was a special attempt to establish the bilateral sub-sets of products (multinational teams of experts visited the countries for this purposes) and carry out direct bilateral BH comparisons for each pair of countries. However it is necessary to keep in mind that the EU consisted to that time point of nine relatively homogeneous countries.

basic heading results (PPPs), a two-stage procedure was implemented by Eurostat. For each pair of countries, two parities are calculated:

The first, Laspeyres-type, parity is obtained as the geometric mean of the price ratios for the products characteristic of the base country, regardless of whether the products are representative or not of the partner country.

The second, Paasche-type, parity is calculated as the geometric mean of the price ratios for the products characteristic of the partner country, no matter if they are characteristic of the base country.

The geometric average of these two parities (Fisher-type) is taken as a bilateral BH-PPP for a given pair of countries. This Fisher's PPP is usually regarded as an equi-representative bilateral PPP because even if one country selects more products representative of its consumption than another, the two-stage procedure used for calculation helps to reduce the influence of this difference on the final result.

It is declared sometimes that this procedure is successful if the item list includes at least one representative item for each country and this product is priced, at least, in one other country. It is a simplification. First of all, it is not desirable that a result depends on a singular item (the probability of different errors is very high in this case). Secondly, this is not fully correctly from pure mathematical point of view. One example with 3 products and 3 countries given below illustrates this thesis:

	Country A	**Country B**	**Country C**
Product 1	Price A1 *	Price B1	----
Product 2	Price A2	----	Price C2 *
Product 3	----	Price B3 *	Price C3

This example satisfies all requests describe above: all countries have priced items with asterisks and these asterisked items were priced at least in one other country. However, as we can see, bilateral F-PPP can't be calculated for any pair of the countries due to missing counterparts (either Laspeyres or Paasche type indices). The given situation is, of course, an extreme case, which occurs not very oft in the practice but this example demonstrates that the reality is much more complicated than many our simplified assumptions and a very detailed analysis of many premises and traditional postulates is necessary. (It will be demonstrated in the section "Reservations and recommendations" that although input data from the example above do not satisfied the standard premises but nevertheless the "true" parities can be obtained in this specific case).

Analysis of the traditional EKS method

To obtain the transitive basic heading PPPs, the EKS method is applied to the matrix of bilateral Fisher-type PPP described above. The EKS PPP for any pair of countries is calculated as a geometric mean of direct Fisher parity and all indirect Fisher-PPPs. The computation of a matrix of binary PPPs on the basis of country prices taking into account their representativity (asterisks *) is the most important step. The present traditional Eurostat approach is analysed below and a modified method for calculation of binary basic heading PPPs is proposed.

Let us have for a selected pair of countries the following composition of price data for a given basic heading:

n11 - no. of priced items with „*“ in both countries (set **)

n10 - no. of priced items with „*“ in the 1st country and without „*“ in the 2nd country (set ***-)**

n01 - no. of priced items without „*“ in the 1st country and with „*“ in the 2nd country (set **-***)

According to the general agreement the items non-representative for both countries (without * in both countries) are not included in the calculation at all.

As it was indicated above, to obtain the equi-representativity of BH-PPPs, Eurostat uses a two-stage procedure. To understand better the main features of the approach, this procedure is described below in detail.

On the first stage for each pair of countries (named numerator country and denominator country) two parities are calculated:

a) A first parity is obtained as the geometric mean of the price ratios for the items representative of the denominator country (Laspeyres-type):

(1) $$L(j/h) = \left(\prod_{i=1}^{k} \frac{{}_{*h}P_j^i}{{}_{*h}P_h^i} \right)^{1/k} = \left(\prod_{i=1}^{(n11+n01)} \frac{{}_{*h}P_j^i}{{}_{*h}P_h^i} \right)^{1/(n11+n01)}$$

where

L(j / h) is a parity of Laspeyres-type between countries j and h;

${}_{*h}P_j^i$ and ${}_{*h}P_h^i$ - the prices of item i in countries j and h representative of the denominator country h,

k = (n11 + n01) is the number of items representative of the denominator country h (this set includes the items representative in both countries as well as the items representative in the denominator country only).

b) A second parity is obtained as the geometric mean of the price ratios for the items representative of the numerator country (Paasche-type):

(2) $$P(j/h) = \left(\prod_{l=1}^{m} \frac{{}_{*j}P_j^l}{{}_{*j}P_h^l} \right)^{1/m} = \left(\prod_{l=1}^{(n11+n10)} \frac{{}_{*j}P_j^l}{{}_{*j}P_h^l} \right)^{1/(n11+n10)}$$

where

P(j / h) is a parity of Paasche-type between countries j and h;

${}_{*j}P_j^l$ and ${}_{*j}P_h^l$ - the prices in countries j and h of item l representative of the numerator country j,

m = (n11 + n10) is the number of items representative of the numerator country j (this set includes the items representative in both countries as well as the items representative in the numerator country only).

The simple geometric mean (parity of Fisher-type) is calculated from the parities of Laspeyres- and Paasche – types during the second stage of calculation:

$$\text{F-PPP}(j / h) = [(L(j/h) * P(j/h)]^{1/2}$$

Other versions of BH-PPPs of Laspeyres, Paasche and Fisher types

The PPPs of Laspeyres and Paasche types presented above are based on geometric averages. Therefore speaking stricktly the combination of them is the PPP of the Tornqvist type which can be calculated directly instead of the two-stage calculation of the Fisher type-PPP:

$$T(j/h) = [\prod_{i=1}^{k+m} (\frac{P_{ij}}{P_{ih}})^{(w_{ij}+w_{ih})/2}]^{1/[\sum_{i=1}^{k+m}(w_{ij}+w_{ih})/2]} = [\prod_{i=1}^{k+m} (\frac{P_{ij}}{P_{ih}})^{(1/m+1/k)}]^{1/2}$$

This Tornqvist index is calculated on the basis of the same imaginary structures of countries as it is done by the traditional method: the weights of countries j and h (w_{ij}, w_{ih}) are equal to (=1/m) and (=1/k) respectively for representative items; the weights for non-representative items are equal to zero ($\Sigma w_{ij}=1$ and $\Sigma w_{ih}=1$).

It is possible also to use without loss of generality more traditional forms of the Laspeyres and Paasche indices (arithmetic and harmonic averages).

A parity of Laspeyres-type can be obtained as the arithmetic mean of the price ratios for the items representative of the denominator country with equal weights (= 1/ k):

$$L(j/h) = \sum_{i=1}^{k} \frac{{}_{*h}P_j^i}{{}_{*h}P_h^i} / k = \sum_{i=1}^{k} \frac{{}_{*h}P_j^i}{{}_{*h}P_h^i} / (n11 + n01)$$

A parity of Paasche-type can be obtained as the harmonic mean of the price ratios for the items representative of the numerator country with equal weights (= 1/ m):

$$P(j/h) = m / \sum_{l=1}^{m} 1/(\frac{{}_{*j}P_j^l}{{}_{*j}P_h^l}) = (n11 + n10) / \sum_{l=1}^{m} 1/(\frac{{}_{*j}P_j^l}{{}_{*j}P_h^l})$$

The geometric averages (Laspeyres and Paasche type) are more transparent for the decomposition and the explanation of the compensatory effect. However the arithemetic and harmonic versions presented in this box refer more directly to the traditional forms of indices and they can be useful if a more complicated weighting system for items (like “very representative”, “representative”, “not very representative”, “non-representative”) should be used – see Annex 2.

The Fisher-type PPP is usually regarded as equi-representative bilateral PPP. At the first look, indeed if one country selects much more products representative of its consumption than another, then the separate calculation of Laspeyres and Paasche PPPs with the further unweighted averaging eliminate the influence of this difference. Nevertheless, this is an hypothesis only and in reality the two-stage procedure described above brings in some cases biased (not equi-representative) results. To illustrate this fact, the formulas (1) and (2) should be presented in a modified form.

A parity of Laspeyres-type can be rewritten as a weighted geometric average of two different sets of individual price ratios. Namely, the first set includes items representative of both countries h and j, the second set includes items representative of the country h but non-representative of country j:

(3) $$L(j/h) = \left[\left(\prod_{i1=1}^{n11} \frac{{}_{(*h*j)}P_j^{i1}}{{}_{(*h*j)}P_h^{i1}} \right) * \left(\prod_{i2=1}^{n01} \frac{{}_{(*h-j)}P_j^{i2}}{{}_{(*h-j)}P_h^{i2}} \right) \right]^{1/(n11+n01)} =$$

$$= [PPP_{j/h}(**)]^{n11/(n11+n01)} * [PPP_{j/h}(-*)]^{n01/(n11+n01)}$$

where

${}_{*h*j}P_j^{i1}$ and ${}_{*h*j}P_h^{i1}$ - the prices of item i1 in countries j and h representative of both countries **(**)**,

${}_{*h-j}P_j^{i2}$ and ${}_{*h-j}P_h^{i2}$ - the prices of item i2 in countries j and h representative of the country h but non-representative of country j **(-*).**

$PPP_{j/h}$**(**) –** average PPP between countries j and h for the set (**),

$PPP_{j/h}$**(-*) –** average PPP between countries j and h for the set (**-***).

A parity of Paasche-type can be rewritten in a similar form. In this case, the first set of items includes items representative of both countries (i.e. the same as in Laspeyres-parity but with an other weight) and the second set includes items representative of the country j but non-representative of country h:

(4) $$P(j/h) = \left[\left(\prod_{i1=1}^{n11} \frac{{}_{(*h*j)}P_j^{i1}}{{}_{(*h*j)}P_h^{i1}} \right) * \left(\prod_{i3=1}^{n10} \frac{{}_{(-h*j)}P_j^{i3}}{{}_{(-h*j)}P_h^{i3}} \right) \right]^{1/(n11+n10)} =$$

$$= [PPP_{j/h}(**)]^{n11/(n11+n10)} * [PPP_{j/h}(*-)]^{n10/(n11+n10)}$$

where

${}_{*h*j}P_j^{i1}$, ${}_{*h*j}P_h^{i1}$ - the prices of item i1 in countries j and h representative of both countries **(**)**,

${}_{-h*j}P_j^{i3}$, ${}_{-h*j}P_h^{i3}$ - the prices of item i3 in countries j and h representative of the country j but non-representative of country h **(*-)**.

$PPP_{j/h}$**(**) –** average PPP between countries j and h for the set (**),

$PPP_{j/h}$**(*-) –** average PPP between countries j and h for the set (***-**).

As it can be seen from the formulas (3) and (4), a parity of Fisher-type is calculated, in general, on the basis of three different sets of items:

$$\text{F-PPP}(j/h) = [\mathbf{L}(j/h) * \mathbf{P}(j/h)]^{1/2} = \{[PPP_{j/h}(**)]^{n11/(n11+n01)} * [PPP_{j/h}(-*)]^{n01/(n11+n01)} *$$
$$* [PPP_{j/h}(**)]^{n11/(n11+n10)} * [PPP_{j/h}(*-)]^{n10/(n11+n10)}\}^{1/2}$$

The influence of each set (weights) on the final result (Fisher-PPP) is the following:

a) Weight for the set of items with „**“:

(5) $$DS(**) = 0.5 * n11 * \left(\frac{1}{n11 + n10} + \frac{1}{n11 + n01}\right) = 0.5 * n11 * \frac{(2 * n11 + n10 + n01)}{(n11 + n10) * (n11 + n01)}.$$

Items representative in both countries (**) are included in the calculation twice and they bring double contribution and they yield therefore the most reliable price ratios in accordance with the concept of representativity.

b) Weight for the set of items with „*-“:

(6) $$DS(*-) = 0.5 * \left(\frac{n10}{n11 + n10}\right)$$

c) Weight for the set of items with „-*“:

(7) $$DS(-*) = 0.5 * \left(\frac{n01}{n11 + n01}\right)$$

It is obviously that equi-representativity can be obtained only if the contributions (weights) of each of the two sets of items exceptionally representative for <u>one</u> country are equal. In this case non-representativity of each set is compensated by opposite influence of another set. The formulae (6) and (7) show that the weights for two sets of unilaterally representative items are equal in two cases:

a) if **<u>n10 = n01</u>** or b) if **<u>n11 = 0</u>**.

The equi-representativity is distorted in all other cases. Therefore an old theoretical Eurostat recommendation - to exclude direct Fisher-PPPs that are liable to lead to results of lower quality (those are with high Laspeyres / Paasche ratio - see, Eurostat 1989 and Eurostat 1996) – was correct. However, this theoretical principle was not used during the recent Eurostat comparisons due to practical considerations. There are several reasons for the obtaining of high Laspeyres / Paasche ratios. It seems, not all of them should lead to the ignorance of direct PPPs. If there is relatively considerable number of items representative in both countries or the sets of unilaterally non-representative items compensate each other then one can believe that direct comparison even on the basis of limited data brings a more true PPP (even with high L/P ratio) than the indirect PPP obtained via third countries.

The present paper describes a modification of the standard EKS method which can be labeled as „Method of reciprocal compensation of non-representativity“ (the recent Eurostat PPP WP / November ‘02 named this method as the EKS-S method).

Method of reciprocal compensation of non-representativity

To achieve equi-representative bilateral results in a general case (no. of unilateral representative products are different, etc.) it is proposed to calculate at the first stage three separate PPPs in explicit form instead of two indices of Laspeyres and Paasche-type (in principle, as it was demonstrated above, these three PPPs are present in Laspeyres and Paasche Indices in an implicit form):

(8)
$$PPP^{**}(j/h) = \left(\prod_{i1=1}^{n11} \frac{{}_{(*h*j)}P_j^{i1}}{{}_{(*h*j)}P_h^{i1}} \right)^{1/n11}$$

where

PPP^{**} (j/h) - a parity calculated on the basis of prices ${}_{*h*j}P_j^{i1}$ and ${}_{*h*j}P_h^{i1}$ only, i.e the prices in countries j and h of items representative of both countries **(**)**,

(9)
$$PPP^{-*}(j/h) = \left(\prod_{i2=1}^{n01} \frac{{}_{(*h-j)}P_j^{i2}}{{}_{(*h-j)}P_h^{i2}} \right)^{1/n01}$$

where

PPP^{-*} (j/h) - a parity calculated on the basis of prices ${}_{*h-j}P_j^{i2}$ and ${}_{*h-j}P_h^{i2}$, i.e. the prices in countries j and h of items representative of country h only **(-*).**

(10)
$$PPP^{*-}(j/h) = \left(\prod_{i3=1}^{n10} \frac{{}_{(-h*j)}P_j^{i3}}{{}_{(-h*j)}P_h^{i3}} \right)^{1/n10}$$

where

PPP^{*-} (j/h) - a parity calculated on the basis of prices ${}_{-h*j}P_j^{i3}$ and ${}_{-h*j}P_h^{i3}$, i.e. the prices in countries j and h of items representative of country j only **(*-).**

The general concept underlining the two-stage approach approach is that PPP^{**} is unbiased figure, PPP^{-*} is an overestimated value (relatively „true" PPP - biased upwards), PPP^{*-} is an underestimated value (biased downward). Therefore it is logically to believe that, to obtain a non-biased binary PPP during the averaging, these biases (up and down) should be equal, i.e. they should compensate each other. It is possible if the equal weights are assigned to the PPP^{-*} and the PPP^{*-}. So, three PPPs indicated above can be averaged with the following weights[4]:

a) Weight for the set of items with „**":

(11)
$$DM(**) = \frac{2*n11}{2*n11+n10+n01}$$

b) Weights for the set of items with „***-**" and for the set of items with „**-***":

(12)
$$DM(*-) = DM(-*) = 0.5*\frac{n10+n01}{2*n11+n10+n01}$$

It is clear from the comparison of the formulae (11) – (12) with the formulae (5) - (7) that the new weights are based partly on the old idea: Items (**) received double weight relatively Items (***-**) and (**-***) – so, an imaginary sum of total representativity can be calculated as (2*n11+n10+n01). However the modified method assigns the

[4] The abbreviation **DM** means **D**eal (amount) **M**odified; an opposite term is **DS** - **D**eal (amount) **S**tandard.

equal weights for the PPP of the set of items with „***-**“ and for the PPP of the set of items with „**-***“ and, in effect, the necessary compensate effect is obtained. Additionally, the modified method presents the weights in more explicit form.

Other possible weighting systems

If the qualitative indicators (like “representative” / “non-representative”) are used then each system of quantative weights attributed to them will be inevitable partly arbitrary. Obviously, this concerns also the systew of weights proposed above. However the present system of asterisks is arbitrary itself it is not an unique. For example, the ESCAP 1985 used 3 types of items: very representative (weight of 3), moderate representative (weight of 2) and non-representative (weight of 1; but these items were used in a few cases only).

The weights **DM** are calculated on the basis of no. of prices for all three sets. There are some proposals to increase the weight of parity for the set of items representative in both countries (**) as the most reliable part. So, S.Varjonen (OECD) proposed the following system of weights (he named this modification as the EKS-S* method but it can be named as the EKS-S-V modification):

$$w_{11} = \frac{n_{11}}{n_{11} + MIN(n_{10}, n_{01})}; \quad w_{10} = w_{01} = 0.5 \times \frac{MIN(n_{10}, n_{01})}{n_{11} + MIN(n_{10}, n_{01})}.$$

Of course, this system is possible (and the EKS-S method uses also this approach when either n10 = 0 or n01 = 0 – only the PPP for the set (**) is used in these extreme cases). There are other possibilities to increase the weight for PPP(**). For example, it is possible to give the weight = 3 (but not 2 as presently) to the representative items and the weight = 1 to non-representative items. However, it seems, that the weighting system itself is not the most important factor taking into account the general premises of the applied approach:

- basic headings comprise the sets of more or less homogeneous products, therefore variation of partial PPPs within the BHs should be not very high
- the bias (underestimation / overestimation relatively “true” PPP) of partial PPPs is more or less equal for all unilaterally representative items (*-) / (-*); it means practically that all asterisked products are more or less equally representative in different countries and, respectively, non-asterisk products are equally unrepresentative.

Of course, these premises are very strong for the actual situations but all traditional considerations about the equi-representativity (a possibilty to have for a country only one representative item within BH), etc. are correct only by the premises mentioned above. If they are not valid then the whole approach shoud be revised (see, for example, Annex 2).

Therefore if we want to work within the present concept then it is important that the parities for the sets **(*-)** and **(-*)** should have equal weights (to have a compensatory effect) but an obvious priority for a higher weight for the set (**) is not especially necessary. If no. of items representative in both countries is high but the variation of partial PPPs within the set (**) is also high or these PPP are overlapped with the partial PPPs for the sets (-*) / (*-) then it is clear that some of our premises are not in accordance with the reality. Some adjustments with the weights calculated on the basis on no. of prices can’t help in this case. The analysis of the variation of partial PPPs with an estimation of their reliability (see Annex 1) and possible use of some criteria (similar with L/P ratio - see page 13) between the PPPs for the sets PPP(*-) / PPP(**) / PPP(-*) can be more efficient.

How could the situation be managed if some of these three PPPs are not available? Table 1 contains the review of all possible variants of calculation.

Table 1

Calculation of binary PPP by modified and traditional methods

<table>
<tr><th colspan="3">Availability of PPPs for different sets of items</th><th colspan="2">Method of calculation (obtaining) of the final binary PPP for respective basic heading</th></tr>
<tr><th>PPP(**)</th><th>PPP(*-)</th><th>PPP(-*)</th><th>Modified method</th><th>Traditional method</th></tr>
<tr><td>Yes</td><td>Yes</td><td>Yes</td><td>Geometric mean (GM) from all three PPPs with weights (DM) –</td><td>Geometric mean of Las-peyres & Paasche PPPs (with possible examination of L/P ratio)</td></tr>
<tr><td></td><td></td><td></td><td colspan="2">If n10 = n01 => the results are equal</td></tr>
<tr><td>Yes</td><td>Yes</td><td>No</td><td>PPP (**) only</td><td>- “ – (GM of L & P)</td></tr>
<tr><td>Yes</td><td>No</td><td>Yes</td><td>PPP (**) only</td><td>- “ - (GM of L & P)</td></tr>
<tr><td>Yes</td><td>No</td><td>No</td><td>PPP (**)</td><td>PPP (**)</td></tr>
<tr><td>No</td><td>Yes</td><td>Yes</td><td>Simple geometric mean from PPP(*-) and PPP(-*)</td><td>Geometric mean of Las-peyres & Paasche PPPs (with possible examination of L/P ratio)</td></tr>
<tr><td>No</td><td>Yes</td><td>No</td><td>Missing value</td><td>Missing value</td></tr>
<tr><td>No</td><td>No</td><td>Yes</td><td>Missing value</td><td>Missing value</td></tr>
<tr><td>No</td><td>No</td><td>No</td><td>Missing value</td><td>Missing value</td></tr>
</table>

*) The versions where both methods produce the same results are highlighted

To illustrate the practical application of the modifications done several actual examples from different Eurostat Surveys are given below.

So, Table 2 contains an example from the Eurostat “Guidelines for conducting price Surveys...” (see Appendix D, page 36). This example was borrowed from a 1988 Survey (see, Eurostat 1991, page 6) where the obtained Laspeyres / Paasche ratio of 1,659 is relatively high (> 1.5, which is very often assumed as crucial value). Indeed, the composition of priced items between countries is very disproportionate: there are three items representative of both countries and eight items representative of France only. The share of set with items non-representative of Greece is about 40% and there is no balancing opposite influence of items representative of Greece only. Therefore the general PPP „DRA/FF“ of Fisher-type seems to be overestimated Probably, the best theoretical solution in this case would be to select the PPP(**) calculated on the basis of three items representative for both countries as the general PPP for this basic heading (in any case from a theoretical point of view).

It is clear from the formulas (6)-(7) and (11)-(12) that if n10 = n01 or if n11= 0 then the results by both methods (traditional and modified) are equal. Table 3 contains one actual example from Survey E95-1 “Food, etc.” The ratio L/P PPPs „DRA / ATS“ is relatively high = 1,520 but the calculation by the modified method shows clear that there is a reciprocal compensation of non-representativity and therefore the BH-PPP seems to be correct.

The check of reliability of obtained results by help of some formal procedures (similar with L/P ratio) can be done also by the modified method. The examination can be carried out in the following ways:

a) Geometric mean from two biased PPP – PPP(*-) and PPP(*-) - must not be too far from PPP obtained on the basis of data representative of both countries PPP(**).

b) If PPP(**) are not available then the ratio PPP(-*)/PPP(*-) must not exceed a certain (conventional) fixed value (for example: 2.0).

So, geometric mean from two biased PPP(PPP^{*-} and PPP^{-*}) from the Table 3 is not be too far from PPP obtained on the basis of data representative of both countries: the ratio is very close to 1 = 1.062. This value indicates indirectly that there is a reciprocal compensation of non-representativity.

Table 2

Binary results “Greece-France” for BH 110441 „Cheese“ (Eurostat: E88-1)

		Greece (DRA)		France (FF)		PPP
		Aster.	Price	Aster.	Price	"DRA/FF"
11441A	Camembert		2093	*	38.9	53.80
11441B	Brie		1356	*	42.0	32.29
11441E	Gouda - Holland		706	*	43.0	16.42
11441F	Gouda type	*	665	*	41.3	16.10
11441G	Edam - Holland	*	671	*	38.4	17.47
11441K	Emmenthal		1342	*	60.5	22.18
11441N	Cheddar type		1137	*	65.1	17.47
11441O	Grated Parmesan		1804	*	77.1	23.40
11441Q	Feta	*	462	*	69.5	6.65
11441S	Mozzarella		1245	*	57.2	21.77
11441V	Processed cheese		987	*	39.8	24.80
				Geometric mean =		**20.45**

	No. of items
N11 (**)	3
N10 (-*)	8
N01 (*-)	0
Total	**11**

Traditional method

			Weights (%) for item sets	
PPP - P "DRA/FF"	=	12.32	63.64	**
PPP - L "DRA/FF"	=	20.45	36.36	-*
			0.00	*-
PPP - F "DRA/FF"	**=**	**15.87**	100.00	
L/P ratio	**=**	**1.659**		

Modified method

				Weights (%) assigned for item sets	
Index 1	PPP ** "DRA/FF"	=	12.32	42.86	**
Index 2	PPP -* "DRA/FF"	=	24.72	28.57	-*
Index 3	PPP *- "DRA/FF"	=	Not exist	28.57	*-
				100	
	PPP "DRA/FF"	**=**	**12.32**		

Table 3

Binary results “Greece-Austria” for BH 110441 „Cheese“ (Eurostat: E95-1)

		Greece (DRA)		Austria (ATS)		PPP
		Aster.	Price	Aster.	Price	"DRA/ATS"
110441a1	Camembert type		1958	*	103.8	18.86
110441e	Gouda - Holland	*	1251		134.2	9.32
110441f	Gouda type		1155	*	105.3	10.97
110441g	Edam - Holland	*	1235		141.6	8.72
110441j	Danablue	*	1235		193.0	6.40
110441l	Emmenthal type		2188	*	106.6	20.53
110441p	Feta	*	1953		129.2	15.12
110441q	Feta II	*	1602	*	143.9	11.14
110441s	Mozzarella		2456	*	132.3	18.56
110441v	Processed cheese	*	2181	*	133.4	16.35
110441w	Processed cheese	*	2152		121.1	17.76
110441x	Cottage cheese		1888	*	56.8	33.22
				Geometric mean =		**14.19**

	No. of items
N11 (**)	2
N10 (-*)	5
N01 (*-)	5
Total	**12**

Traditional method

				Weights (%) for item sets	
PPP - P "DRA/ATS"	=	11.43		28.57	**
PPP - L "DRA/ATS"	=	17.37		35.71	-*
				35.71	*-
PPP - F "DRA/ATS"	**=**	$\boxed{\mathbf{14.09}}$		100	
L/P ratio	**=**	**1.520**			

Modified method

				Weights (%) assigned for item sets	
Index 1	PPP ** "DRA/ATS"	=	13.49	28.57	**
Index 2	PPP -* "DRA/ATS"	=	19.21	35.71	-*
Index 3	PPP *- "DRA/ATS"	=	10.69	35.71	*-
				100	
	PPP "DRA/ATS"	**=**	$\boxed{\mathbf{14.09}}$		

The ratio for the examination by modified method:
$(PPP^{*-} \times PPP^{-*})^{0.5}/PPP^{**} =$ 1.062

Results of experimental calculations

Several countries and experts confirmed after the Eurostat PPP Working Party in June 2003 that the modified method has the theoretical advantages relatively the traditional method. However it was necessary to carry out a broad set of numerical experiments on the basis of actual data from Surveys to check the possible impact of the proposed modifications on the ECP results.

This section presents the Summary of experimental calculations by the modified method for the BH-PPP calculation and a comparative numerical analysis of the differences with the results obtained by the traditional method for all 6 consumer Surveys (done after the ECP Reform) and one Survey from the “Gross Fixed Capital Formation” (“Construction 2001”).

Table 4 contains a Summary of comparative results at the Survey-Total level. The Summary shows that the numerical differences at the Survey Level for consumer Items are not significant - not more than **±** 2-3%, i.e. they are well in usual margins for errors for international comparisons. Obviously the differences at the detailed BH level are some higher but, in principle, there are not some drastical differences.

The next section contains some considerations about the use of the traditional EKS method as well the modified EKS-S method in the concrete situations.

Table 4

Differences between the PPPs by the traditional EKS and modified EKS-S methods (as % difference from the results by the traditional method) at the Survey-Total level

	Furniture, etc.	Transport / Oth. Prod.	HH Durables	Clothing & footwear	FOBETO	SERVICES	Construc-tion
	E99-1	E99-2	E00-1	E00-2	E01-1	E01-2	2001
OS	-0.74	1.00	0.35	-0.04	0.30	0.90	-0.06
BE	-0.84	-0.94	-0.21	-1.23	-0.89	-0.64	-0.16
CH	-0.73	-1.44	-0.73	-2.09	-1.51	0.11	-0.09
CZE	2.92	0.57	-0.30	-0.32	-0.01	0.52	1.61
DE	0.26	0.33	-0.45	0.24	-0.44	0.85	-0.27
HUN	0.32	-0.15	0.96	0.78	1.54	1.54	0.56
LUX	-0.62	-1.38	-0.13	-2.61	-1.64	-1.12	-0.26
NL *)	0.30	No data	No data	No data	-1.28	-1.17	-0.05
PL	-0.80	-0.18	0.53	0.51	1.09	-0.22	0.17
SVK	1.93	1.29	0.00	1.87	1.04	-0.62	2.03
SVN	0.90	0.62	0.24	0.33	0.75	1.09	0.25
FIN	-0.61	-0.83	-0.28	-0.08	-0.82	-1.00	-0.73
DK	0.34	-1.78	-0.78	-0.22	-0.55	-0.64	-0.35
EST	-0.11	-0.36	0.19	-0.33	0.28	0.00	0.25
ICE	0.03	-0.25	-0.57	-1.20	-0.51	-0.86	0.10
IR	-1.98	-1.51	0.34	-0.94	-0.15	-0.93	-0.21
LVA	-0.19	1.39	-0.14	0.97	0.38	1.32	-0.18
LTU	1.55	0.56	0.46	-0.11	1.31	0.57	0.59
NOR	-0.08	-0.10	0.23	0.46	-0.65	0.03	0.00
SW	0.23	-0.05	-0.30	0.69	0.06	-1.48	-0.16
UK	-1.46	0.09	-1.08	-0.25	-1.45	-0.99	-0.29
IT	-0.95	-0.71	0.16	-0.90	-1.09	-0.51	-0.16
BGR	-1.03	-0.36	-0.10	2.33	1.62	2.12	0.19
CYP	-1.08	0.47	-0.15	0.80	0.01	-1.03	0.53
FR	-1.57	-0.58	-0.15	-0.61	-0.18	-0.45	1.21
GR	0.75	0.25	0.64	-0.28	-0.14	-0.45	0.45
MLT **)	No data	0.77	-0.55	-0.17	-0.86	0.92	-2.09
PRT	1.21	0.70	-0.01	-0.22	-0.67	-0.23	0.79
ROM	1.56	3.10	0.07	2.22	2.00	0.42	0.80
SPA	0.31	-0.66	0.21	-0.35	0.26	0.98	0.24
TUR ***)	No data	No data	1.51	0.59	2.05	0.82	-0.01
MAX	2.92	3.10	1.51	2.33	2.05	2.12	2.03
MIN	-1.98	-1.78	-1.08	-2.61	-1.64	-1.48	-2.09
MAX-MIN	4.90	4.88	2.59	4.95	3.68	3.59	4.12

*) NL did not participate in the Surveys E99-2, E00-1, E00-2
**) MLT joined to the current Group "S" work from the Survey E99-2
***) TUR joined to the current Gr."S" work from the Survey E00-1

Reservations and recommendations

The Eurostat PPP Working Parties (June and November 2002) agreed that the proposed modification is an methodological improvement, which ensures the application of an unbiased method. This is an advantage from the theoretical point of view. Additionally, as it was demonstrated in the previous section, the introduction of the modified method does not change in many cases significantly the numerical results obtained by the traditional approach. However it does not mean that the methods with some theoretical advantages have automatically the advantages in the practice in all cases. The real life is more complicated and many features should be taken into account (one interesting example on this point is given in the box below).

Obtaining of multilateral unbiased PPPs from biased bilateral indices

This box with an example from page 5 (3 products / 3 countries) demonstrates that the use of strickt methods to a set of data which is theoretically insufficient is fruitless but a simple method applied to the same set can bring reliable results.

	Country A	**Country B**	**Country C**
Product 1	Price A1 *	Price B1	----
Product 2	Price A2	----	Price C2 *
Product 3	----	Price B3 *	Price C3

Bilateral Fisher-PPP can’t be calculated for any pair of the countries due to missing counterparts (either Laspeyres or Paasche type indices). However, although the EKS results for this example can’t be produced by the present approach (matrix of F-PPPs is fully empty) but nevertheless the establishing of bilateral PPPs on the basis of available country price relations can lead to correct EKS-PPPs by some general features / hypotheses which are immanent also for the standard approach.

Bilateral PPPs (Row-Country to Column-Country)

	Country A	**Country B**	**Country C**
Country A	1	PPP’(A/B)**x**C↓	PPP’(A/C)**x**C↑
Country B	PPP’(B/A)**x**C↑	1	PPP’(B/C)xC↓
Country C	PPP’(C/A)**x**C↓	PPP’(C/B)xC↑	1

PPP’ means imaginary unbiased bilateral PPP, (PPP’xC) means PPPs calculated on the basis of actual prices
C↓ - a downward coefficient for a bias for PPP in the situation where representative products is compared with non-representative product,
C↑ - a upward coefficient for a bias for PPP in the situation where non-representative products is compared with representative product (it is assumed that C↑ = 1 **/** C↓).

If we simply use available prices then each country has one overestimated PPP and one underestimated PPP. In this case, an EKS-PPP is a combination from overestimated PPPs and underestimated PPPs and, in effect, if the biases are approximately the same for each involved PPPs then an unbiased result is obtained from systematically biased input data:

$$\text{EKS-PPP}(A/B) = \{(\text{PPP}'(A/B) \times C\downarrow)^2 \times [(\text{PPP}'(A/C) \times C\uparrow) \times (\text{PPP}'(C/B) \times C\uparrow)]\}^{1/3} =$$

$$= \{\text{PPP}'(A/B)^2 \times [\text{PPP}'(A/C) \times \text{PPP}'(C/B)] \times (C\downarrow^2 \times C\uparrow \times C\uparrow)\}^{1/3} =$$

$$= \{\text{PPP}'(A/B)^2 \times [\text{PPP}'(A/C) \times \text{PPP}'(C/B)]\}^{1/3}$$

It is necessary to keep in mind, that the efficiency of the proposed modifications depend directly on the quality of input data, first of all, on the attribution of the

asterisks by the countries. The method of asterisks is based on the assumptions that the parities for the item set (**) are “true”, the parities for the item set (*-) – underestimated, the parities for the item set (-*) - overestimated and the individual parities itself within these sets differ not very significant. Therefore a correct attribution of the asterisks is especially important for the basic headings where there is large variation in the price ratios between countries for different products.

The experience from the former Surveys showed clearly that the countries follow in many cases different rules: some countries attribute asterisks mostly to the cheapest market leaders – in effect, they have few asterisked items only; another countries attribute asterisks to all products that are broadly available – in effect, they have practically all Items with asterisks. In these situations all improvements of the methods are mostly useless for the practical calculations. “Statistics Austria” prepared a short notice on this point, which was presented and discussed in all three Groups during the Group E01-1 meetings. The situation in the meantime was slightly improved but nevertheless the present paper is an additional occasion to draw the attention of the countries to this important topic.

An other problematic point is number of prices (and asterisked Items) involved in the calculation of the binary PPPs (e.g., this is the case for “Construction”). It is clear that if there are only few priced Items (or only few asterisked Items) within a BH then the results are more sensitive and volatile to the choice of the method (see Annex 1 for more detail). Each change of data involved in the calculation (even for one item) can have in this case a significant impact on the results. For example, the deletion of one asterisk * can lead to the situation that the whole chain of binary PPPs will be broken and the set of the EKS-PPPs will have missing values. For example, there was a very interesting and even ambiguous situation during the first calculations for some BHs from the 2000 Survey “Equipment goods”. The EU15 results should be fixed but the multilateral EKS-PPP for EU15 could not be obtained because EU15 countries have not enough no. of direct PPPs due to low no. of asterisked Items. The PPPs for EU15 existed within the overall EUR31 comparison but not within the EU15 comparison. The situation that the BHs have few priced Items and few asterisked Items leads sometimes to the decision that the BH-PPPs are calculated without taking into account the asterisks at all (this was, for example, the case within the ECP'96 / Group II for the "Equipment goods"). From this point of view, the traditional EKS method has some preference because generally it includes more items in the calculations and if input data are not enough exact then the probability of the reduction of the accidential deviations is higher in this case.

So, it is necessary to keep in mind all reservations going from the imperfectness of input data. If it is known in advance that input data is low quality then the use of the complicated (even theoretically more correct) methods is practically useless[5]. The simplest methods should be used in this case. However, obviously that the theoretical improvements should be not rejected due to imperfect quality of input data. Both processes should go parallel: the quality of input data as well as the applied methods should be improved[6].

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[5] If input data is „funny“ then even a theoretically perfect method brings also „funny“ results.

[6] Therefore PPP software (elaborated by Statistics Austria) using within the Eurostat comparison includes many possible options for the calculations in the different situation: with * and without *, with the examination of the Laspeyres/Paasche ratios and without this step, by the traditional EKS method as well as by the modified EKS-S method. Finally, the user can decide what is the most appropriate method for a concrete situation.

It was recommended by the recent Eurostat PPP Working Party (Nov'02) to include both methods (the traditional EKS method as well as the modified EKS-S method) in the PPP Regulation. It leads to the necessity of some indications for the users: in which cases the traditional EKS method should be applied and in which cases - the modified EKS-S method.

In the opinion of the author of this paper, if two methods are compared then, first of all, their theoretical basis should be checked. The present traditional EKS method is a biased method (as it has been indicated already by several experts). If one method has a weaker theoretical basis relatively an other method then the use of a weaker method can be justified from a practical point of view only by one reason - the respective weakness of input data. There is not a lot of sense to use a sophisticated method if input data is low quality or not sufficient. On other side, it would be very strange if the Eurostat PPP Regulation recommends to use generally a biased method with an argument of the weakness of input data. Therefore the introduction of both methods in the PPP Regulation seems to be reasonable. A general recommendation is the following: the traditional EKS method is preferable for the use in the situations with few no. of items in BHs or where the allocation of asterisks * is problematic (although, these features are rather in favour of simple geometric mean without taking into account asterisks * at all). The EKS-S method should be advised in other cases.

It is inevitable that we should have only one official set of the results and therefore a method should be selected for the calculation of the official results. However it is very desirable the use of both methods (the traditional EKS and EKS-S methods) for each Survey as an efficient validation tool for a check of reliability of input price data.

The author analysed the differences between the results of former ECP Surveys calculated by the traditional EKS method and the EKS-S method. A special attention was taken to the sets of data where there were very significant differences. The analysis showed that these significant differences occurred exclusively in the cases with specific structure of reported country data (strange allocation of asterisks *, some irrational relations between prices - like Brandless items were more expensive than similar item with Specified Brands or simple non-detected rough mistakes in a price data).

First of all, the example "Greece – France" for basic heading "Cheese" from the Eurostat Survey E88-1 (see the Table 2) can be analysed from this point of view. The difference between the bilateral PPPs obtained by the traditional and modified methods is significant. This is the muster example from the Eurostat Guidelines and it is difficult to believe that there were some inaccuracies in input data. Nevertheless some elements are questionable. First of all, it is not very reliable that all items for France are representative. Of course, one can believe all kinds of cheese were broadly available in France but the representativity in our case should refer to the share in expenditure but to the availability. From this point of view, it seems that two very expensive kind of non-domestic cheese (Parmesan and Feta) can be regarded as non-representative (without asterisks). Secondly, it seems that very expensive cheese "Camembert" (this item had very high PPP "DRA-FF") was available in Greece but the share in expenditure should be minimal and this product can be omitted for Greece – it means the GR price for this item can be omitted. If we introduced these changes then the difference between the results is significantly

smaller – see Table 2A. However, it seems, the most problematic point is the item “Feta” – the PPP “DRA-FF” is very low (very cheap in GR and very expensive in FR – in effect, this PPP is, at least, 2.5 times lower than each other PPP). It is not desirable to eliminate this item from the considerations because “Feta” is the main representative cheese for GR. Probably an incorrect reference Quantity was indicated by GR or by FR. Unfortunately it was impossible to check exactly these old data from 1988 due to absence of electronic documentation. It is very likely that some corrections for this item should be done and the differences in this case would be much smaller.

This was example from old historical data where an exact examination of the reasons of significant differences was impossible. However the examples from the recent Surveys of the reformed ECP can be checked in detail.

So, Country **X** had in Survey E99-1 a significant difference for the BH “Kitchen furniture. The analysis shows that Country X has not very high no. of priced items and no. of asterisked items was low. Some of few asterisked items belong to the splittings – in effect, these items had a very poor overlap with other countries. Additionally it was found that there were several very strange irrational internal price relations within Country **X**. There was a possibility to compare the price data for the same items from the similar Survey E02-1. It was detected that E02-1 data is in accordance with logic. One can believe that there were some mistakes in input E99-1 data for Country **X** non-detected during the validation. If these mistakes were eliminated then the differences would be not very substantial.

The analysis for former example was done for old historical data where there was no possibility to correct something. However the analysis of the differences EKS / EKS-S can be used efficiently for the current treatment during the validation. An example from the recent Survey E02-1 "Furniture, etc." demonstrate this. The highest difference (EKS vs. EKS-S) in the overall E02-1 referred to Country **Y** for BH 05.1.1.1 "Kitchen furniture". An investigation of detailed data detected that the allocation of asterisks (*) for Country **Y** was not very usual. Country **Y** has 12 priced items in this BH and 3 items with *. Surprisingly all 3 asterisked items belong to the set "Specified Brand / IKEA" but all Well Known Brand (WKB) and Brandless (BL) items were without asterisks. Usually WKB items (in any case, domestic) are representative items (=> therefore they are well known in a country) and, additionally, BL items are usually more representative in the less developing countries relatively international Brands. It is interesting that Country **Y** was a unique country from 31 participants, which had no representative products for the Well Known Brand and Brandless items. A respective message was sent to Country **Y** with a request to clarify the situation. Country **Y** thanked for this indication and corrected significantly the allocation of asterisks *. This corrections led to more plausible BH-PPP and to the situation that the high difference EKS / EKS-S was eliminated.

Table 2A

Analysis of binary results “Greece-France” for BH 110441 „Cheese“ (E88-1)

		Greece (DRA)		France (FF)		PPP
		Aster.	Price	Aster.	Price	"DRA/FF"
11441A	Camembert			*	38.9	
11441B	Brie		1356	*	42.0	32.29
11441E	Gouda - Holland		706	*	43.0	16.42
11441F	Gouda type	*	665	*	41.3	16.10
11441G	Edam - Holland	*	671	*	38.4	17.47
11441K	Emmenthal		1342	*	60.5	22.18
11441N	Cheddar type		1137	*	65.1	17.47
11441O	Grated Parmesan		1804		77.1	23.40
11441Q	Feta	*	462		69.5	6.65
11441S	Mozzarella		1245	*	57.2	21.77
11441V	Processed cheese		987	*	39.8	24.80
				Geometric mean =>		**18.56**

	No. of items
N11 (**)	2
N10 (-*)	6
N01 (*-)	1
Total	**9**

Traditional method

				Weights (%) for item sets	
PPP - P	"DRA/FF"	=	12.32	45.83	**
PPP - L	"DRA/FF"	=	20.50	37.50	-*
				16.67	*-
PPP - F	**"DRA/FF"**	**=**	**15.89**	100.00	
L/P ratio		**=**	**1.664**		

Modified method

					Weights (%) assig-ned for item sets	
Index 1	PPP **	"DRA/FF"	=	16.77	36.36	**
Index 2	PPP -*	"DRA/FF"	=	21.92	31.82	-*
Index 3	PPP *-	"DRA/FF"	=	6.65	31.82	*-
					100	
	PPP "DRA/FF"		**=**	**13.60**		

items were without asterisks. Usually WKB items (in any case, domestic) are representative items (=> therefore they are well known in a country) and, additionally, BL items are usually more representative in the less developing countries relatively international Brands. It is interesting that Country **Y** was a unique country from 31 participants, which had no representative products for the Well Known Brand and Brandless items. A respective message was sent to Country **Y** with a request to clarify the situation. Country **Y** thanked for this indication and corrected significantly the allocation of asterisks *. This corrections led to more plausible BH-PPP and to the situation that the high difference EKS / EKS-S was eliminated.

This analysis leads to a conclusion / proposal that, although the choice between the EKS and the EKS-S depend on a concrete Survey, but, in any case, **it is desirable to carry out the calculations by both methods for each Survey as a standard validation procedure.** The results with significant differences should be examined especially carefully. Obviously the calculation by both methods can't automatically improve input data itself but, as the experience showed, this brings additional analytical possibilities to detect problematic points especially concerning the allocation of asterisks during the validation of input data. There is no technical problem with this because both methods have built up in the computer program elaborated by the author for the calculation of the BH-PPPs within the Eurostat comparison and this is an option for the users.

Conclusions

The ECP reform and the participation of the CC in the Eurostat comparison led to substantial changes, which have an impact on the structure and the content of input data. Respectively, the numerical procedures applied should be adjusted („fine tuning") to the new situation. Additionally a critical review of the methods is necessary taking into account the forthcoming ICP 2004 round. The methods applying presently within the ECP should be cleaned from weaknesses to be recommended (and to be competitive with other methods like the CPD) for other ICP regions.

Several participants of the recent World Bank ICP Conference (Washington, March 2002) mentioned that the traditional EKS method for the calculation of the BH-PPPs can produce biased results by some circumstances. These arguments are reasonable and some modifications are desirable. To eliminate the drawbacks the author of this paper elaborated a modified method, which was presented during the Eurostat Working Party on Purchasing Power Parities (LUX, 12 - 13 June 2002). The main ideas of the proposed modifications can be summarized as the following:

- to calculate three explicit separate initial PPPs (instead of two PPPs - Laspeyres and Paasche types - as it is done in the present traditional method):

(*) (*) **-** set of Items with asterisks in both countries
(*) (-) **-** set of Items with asterisks in the 1st country only
(-) (*) **-** set of Items with asterisks in the 2nd country only

- to calculate geometric mean (GM) from these three PPPs with some weights where PPP for the set **(*) (*)** receives some higher weight than other PPPs.

The Items non-representative for both countries **(-) (-)** are ignored as earlier.

The discussions confirmed that the proposed modification has theoretical preferences relatively the present approach. To check the possible impact of the modifications on the ECP results, a broad set of numerical experiments was done on the basis of actual data from the Eurostat Surveys (E99-1, E99-2, E00-1, E00-2, E01-1, E01-2). The experiments on the basis of data from numerous consumer Surveys showed that the differences between the PPPs obtained by the modified and the traditional methods are not very substantial in a general case.

So, the proposed modification ensures the application of an unbiased method and, additionally, the introduction of the modified method does not change in many cases significantly the numerical results obtained by the traditional approach. However many features should be taken into account. If one method has a weaker theoretical basis relatively an other method then the use of a weaker method can be justified from a practical point of view only by one reason - the respective weakness of input data. There is no guarantee that the methods with some theoretical advantages have automatically the advantages in the practice. The efficiency of the proposed modifications depend directly on the quality of input data: the accuracy of prices, the content of the set of priced Items and, first of all, the attribution of the asterisks by the countries. There is not a lot of sense to use a sophisticated method if input data is low quality or not sufficient (limited no. of priced items or asterisked items). On other side, it would be very strange if the Eurostat PPP Regulation recommends to use generally a biased method with an argument of the weakness of input data. Therefore it was recommended by the recent Eurostat PPP Working Party (November 2002) to include both methods (the traditional EKS method as well as the modified EKS-S method) in the PPP Regulation.

It leads to the necessity of some indications for the users: in which cases the traditional EKS method should be applied and in which cases - the modified EKS-S method. A general recommendation can be the following: **the traditional EKS method is preferable for the use in the situations with few no. of items in BHs or where the allocation of asterisks * is problematic** like “Construction” (although, these features are rather in favour of simple geometric mean without taking into account asterisks * at all). The EKS-S method should be advised in other cases.

It is inevitable that we should have only one official set of the results and therefore a method should be selected for the calculation of the official results. However both methods can be used parallel for analytical purposes. The author investigated the differences between the results of former ECP Surveys calculated by the traditional EKS method and the EKS-S method. A special attention was taken to the sets of data where there were very significant differences. The analysis showed that these significant differences occurred exclusively in the cases with specific structure of reported country data (like strange allocation of asterisks *, etc.). This investigation leads to a conclusion / proposal that, although the choice between the EKS and the EKS-S depend on a concrete Survey, but, in any case, **it is desirable to carry out the calculations by both methods for each Survey as a standard validation procedure.** This is especially desirable if the price matrix for a basic heading contains substantial number of holes or the number of items marked with asterisks * is very different among participating countries (a frequent case within the reformed ECP). The results with significant differences should be examined especially carefully. Obviously the calculation by both methods can't automatically improve input

data itself but this brings additional analytical possibilities to detect problematic points especially concerning the allocation of asterisks during the validation of input data.

So, it is necessary to keep in mind all reservations going from the imperfectness of input data. If it is known in advance that input data is low quality then the use of the complicated (even theoretically correct) methods is practically useless. The simplest methods should be used in this case. However, obviously that the theoretical improvements should be not rejected due to imperfect quality of input data. Both processes should go parallel: the quality of input data as well as the applied methods should be improved.

Annex 1

Some additional considerations about the improvement of quality of BH-PPPs

The quality of BH-PPPs depends on many factors.
So, still derived binary PPPs have equal importance in the multilateral EKS procedure (traditional as well as modified). Obviously, that parities obtained on the basis of few prices (sometimes on the basis of one price only) can be accidental and not very reliable. There is several ways to introduce some checking procedures here:

1) One of simple way is the indication of minimum number of prices involved in the calculation of binary PPPs as a parameter. If actual no. of involved prices is lower than this minimum then the binary parity should be estimated indirectly as it is done in the case of missing PPPs or in the case with the use of the limits for the L/P ratios (a particular case of missing PPP = the lack of reliable PPP)

2) An other more sophisticated way was indicated in several papers presented during the recent World Bank ICP meetings (February 2001 and March 2002): to use No. of matched Items as a measure of the reliability of bilateral comparisons: binary PPP for a given pair of countries is considered to be more reliable if it is based on more matches (No. of common Items) – see for example, a paper by D.S.Prasada Rao (http://www.oecd.org/pdf/M00019000/M00019163.pdf) or a paper by A.Heston and B.Aten (http://www.worldbank.org/data/icp/documents/aheston.doc)

In general, the second approach seems as promised but there is an additional feature, which should be taken into account here - the variation of individual PPPs within a set of common Items. For example, let us have the following different situations for two given pairs of countries:

Situation 1

	PPP
Item 1	2.00
Item 2	2.10
Item 3	2.20
Item 4	2.30
Item 5	2.40

Situation 2

	PPP
Item 1	2.00
Item 2	3.00
Item 3	4.00
Item 4	5.00
Item 5	6.00

Each pair has 5 common Items but obviously the 1st situation can be recognized as more reliable because a high variation of individual PPPs within a given basic heading (which consists, as rule, a set of more or less homogeneous products) indicates indirectly that the comparing products were, probably, not fully comparable.

To include this representative in the consideration we can use the indicator some analogue of "Relative standard error of a random sample with replacement (in % to average value)“:

$$d_{jk} = \frac{(\text{Sigma / Av.PPP})}{\text{Sqr (n-1)}}, \qquad (1)$$

where:

Sigma - standard deviation of individual PPPs for matched Items (for a given pair of countries j and k),

Av.PPP - an average PPP from individual PPPs,

n - no. of Items that are common (matched Items).

The samples using in international comparisons are neither random nor with replacement. Nevertheless this indicator can be used as an empirical tool.

The weights should be inversely proportional to the distance function:

w_{jk} = 1 / d_{jk} = Sqr (n-1) / [(Sigma / Av.PPP)] (2)

If only one Item was matched, the weight is assigned a value of zero.

The weights (2) reflect the no. of common Items as well as variation of individual PPPs: a situation with a higher no. of matched Items and with a lower variation of individual PPPs produces more reliable aggregated PPP (F-PPP in usual case).

The formula (2) is valid for the direct binary PPPs. The EKS procedure utilizes direct as well indirect binary PPPs. The weights for indirect parities can be obtained in the following way (for example, for indirect PPP for countries i and j via a 3rd country k):

W^k_{ij} = Min (W_{ik}, W_{jk}) (3)

where

W_{ik},, W_{jk} – weights for direct binary PPPs for pairs of countries (i,k) and (j,k)

W^k_{ij} - weight for indirect binary PPPs for a pair of countries (i,j) via country k is calculated as minimum of two weights involved ("quality of a chain depends on quality of the weakest link").

Obviously some other approaches for the estimations of the reliability of PPPs can be proposed.

Annex 2

The use of weights within the CPD and EKS methods

The discussions about the advantages and drawbacks of the CPD and EKS methods continue approx. 30 years. The experience showed that, although significant technical computational differences exist, but both methods can use the same collected price data. The numerical differences between the BH-PPPs obtained by both methods are usually not very significant (excl. some specific situations). Therefore the decision done for the forthcoming ICP 2004 that "***It will be for the regions to decide whether they wish to apply CPD or EKS, but product lists are to be established to accommodate both***" seems to be optimal.

The "Research Proposal Related to 2004 ICP Round" (A. Heston, WB, 27.08.02) contains an indication on "Estimation of heading parities": "*The expert group has proposed introducing weights into parity estimation, even if only qualitative information for an item is available such as **very**, **somewhat**; and **not of importance** in national markets. This information can be introduced into the EKS or CPD procedure with notional weights like 2, 1, 0 for the above 3 responses".*

It seems that the CPD method allows to introduce new set of weights without big additional problems. However it is not very easy to introduce this weighting system in the traditional EKS method because it is necessary to manage the set of numerous situations. However some possible versions are considered in this notice.

Presentation of the CPD method as an index number method

The original version of Country-Product-Dummy (CPD) method proposed by R.Summers is based on the multidimensional regression procedure (see, for example, the paper by R.Summers, „International Comparison with Incomplete Data", *Review of Income and Wealth*, March 1973):

(1) $$Ln(P_{ij}) = \beta_1 {*} X_{i1} + \beta_2 {*} X_{i2} + ... + \beta_{N-1} {*} X_{i,N-1} + \gamma_1 {*} Y_{1j} + \gamma_2 {*} Y_{2j} + ... + \gamma_M {*} Y_{Mj} + \nu_{ij};$$

$i = 1,2,...,M;$
$j = 1,2,..,N-1$

where

P_{ij} is price of ith item in the jth country (expressed in the units of national currency);

$Ln(P_{ij})$ is natural logarithm of P_{ij} ;

X_{ij} and Y_{ij} - two sets of dummy variables ($i = 1,2,...,M$; $j = 1,2, ..., N-1$);

β_j is interpreted here as the natural log of the PPP of country j's currency relative to the base country (in our case country N; $\beta_N = 0$);

γ_1 is interpreted here as the natural log of the price of the ith item in the currency of the base (numeraire) country (in our case country N);

ν_{ij} is a normally distributed variable with mean zero and variation σ^2;

N - number of comparing countries;

M - number of items in given basic heading.

If **P-**matrix is complete (no missing values), no difference exists between unweighted geometric mean and the CPD-results (this is valid also for the results obtained for the EKS without taking into account the asterisks).

It is possible to obtain by the regression equation (1) not only PPPs but also to estimate the accuracy of the obtained parameters in stochastic terms. Additionally, as it was demonstrated by Kim Ziemchang, Alan Heston, Prasada Rao, a.o., it is possible to combine the CPD method with different hedonics. Nevertheless, the CPD in the regression form has also some difficulties and disadvantages:

- economic sense of the equation (1) is hidden and this looks for many users as a pure mathematical exercise;

- examination of stochastic assumptions for the regression procedure (lognormal distributed random variable, etc.) is not very realistic in the practice when no. of items in the basic heading is small (it is usual case).

- number of parameters in the equation (1) can be very high. For example, some basic headings within the Eurostat comparison with 31 participants have sometimes till 500-600 items (e.g., “Pharmaceutical products”). So, total number of variables in the equation (1) should be more than 500 (M+N-1) in this case. The modern computers have very powerful statistical packages. However it seems that the calculation the BH-PPP in the form (1) will need very much time.

The original presentation of the CPD method as a regression procedure introduces many complementary possibilities bit it hinders the comparative analysis with other index number methods. P. Hill indicated many years ago: “*The CPD treats the calculation of the basic PPPs as an estimation problem rather than an index problem. ...The difficulty is whether or not it is legitimate to by-pass index number problem in this way by falling back on the somewhat unfashionable concept of price level, even at the very detailed level of disaggregation of a basic heading”* (P.Hill “Multilateral measurements of purchasing power and real GDP”, Eurostat, 1982, p.40). Therefore if there is no an intention to combine the CPD method with hedonics then it is better to use the presentation of the CPD method in a form of an index number method.

This notice proposes **a modification of the original CPD-method as an index number method** without any loss of generality and with more economic clarity. If the method of least squares (MLS) is used for the estimation of the parameters of regression equation (1) then, taking into account the specific structure the equation (1), we can use (instead of regression procedure) a system of linear (in logarithmic terms) equations which is **a particular kind of the G-K method** in logarithmic terms with notional quantities (weights) for products (1; 0)[7].

Let an average „International price“ of the **i**th item (denoted π_i) is calculated as a ‘implicit quantity’-weighted geometric average of the purchased-power-adjusted national prices of the ith item in the N countries:

$$\textbf{(2)} \qquad \pi_i = \Big(\prod_{j=1}^{N} (P_{ij} / f_j)^{q_{ij}}\Big)^{1/n_i}; \qquad i = 1,2,\dots,M$$

[7] This modification was suggested by the author of this notice approx. 20 years ago in his Ph.D. Dissertation: S.Sergeev „Multilateral Methods for International Comparisons“. Central Statistical Committee of Soviet Union, Moscow, 1982 (in Russian). Some other different interpretations of the CPD method in a bilateral case (two countries only) can be found in W.E. Diewert “Weighted Country Product Dummy Variable Regressions and Index Number Formulae”, the University of British Columbia, Vancouver, Canada, 2002 (http://www.econ.ubc.ca/discpapers/dp0215.pdf).

where

P_{ij} is price of ith item in the jth country (expressed in the units of national currency);

q_{ij} is implicit quantity (weight) for ith item in the jth country: $q_{ij} = 1$, if ith item was priced in the jth country and 0 - otherwise. Practically the variables q_{ij} are some equivalents of the dummy variables X_{ij} and Y_{ij} in the (1);

f_j is the purchasing power parity of given basic heading for the jth country (the definition of this variable will be given below);

n_i is number of countries, where there is priced item i.

The purchasing power parity (PPP) for the jth country for given basic heading can be derived as the geometric average (implicit weighted) deviation of its national prices from the international prices:

$$\textbf{(3)} \qquad f_j = \left(\prod_{i=1}^{M} (P_{ij} / \pi_i)^{q_{ij}}\right)^{1/m_j}; \qquad j = 1,2,...,N$$

where

f_j is the purchasing power parity (PPP) for the jth country for given basic heading;

P_{ij} and q_{ij} are defined above in (2);

m_j is number of items, which are priced in the country j.

Combining the equations (2) and (3), we become a system that makes it possible to find the international prices (π_i) and PPPs (f_j) simultaneously. We can rewrite the joint system in the logarithmic terms:

$$n_1{*}\pi'_1 + ... + 0 \quad + q_{11}{*}\, f_1' + q_{12}{*}\, f_2' + ... + q_{1N}{*}\, f_N' = \sum_{j=1}^{N} P'_{1j}{*}q_{1j}\ ;$$

$$0 + n_2{*}\pi'_2 + ... + 0 \quad + q_{21}{*}\, f_1' + q_{22}{*}\, f_2' + ... + q_{2N}{*}\, f_N' = \sum_{j=1}^{N} P'_{2j}{*}q_{2j}\ ;$$

$$..$$

$$0 \ + \ 0 \ + ... + 0 \ + \ ....n_M{*}\pi'_M + \quad q_{M1}{*}\, f_1' + q_{M2}{*}\, f_2' + ... + q_{MN}{*}\, f_N' = \sum_{j=1}^{N} P'_{Mj}{*}q_{Mj}\ ;$$

(4) ---

$$q_{11}{*}\ \pi'_1 + q_{21}{*}\ \pi'_2 + ... + q_{M1}{*}\pi'_M \quad + m_1{*}\ f_1' \ + 0 + \ .... + 0 = \sum_{i=1}^{M} P'_{i1}{*}q_{i1}\ ;$$

$$q_{12}{*}\ \pi'_1 + q_{22}{*}\ \pi'_2 + ... + q_{M2}{*}\pi'_M \quad + 0 + \ m_2{*}\ f_2' ..\ .... + 0 = \sum_{i=1}^{M} P'_{i2}{*}q_{i2}\ ;$$

$$..$$

$$q_{1N}{*}\ `\pi'_1 + q_{2N}{*}\ \pi'_2 + ... + q_{MN}{*}\pi'_M \quad + \ 0 + \quad 0 + ...+m_N{*}\ f_N' = \sum_{i=1}^{M} P'_{iN}{*}q_{iN}\ ;$$

The variables P'_{ij}, f_j', π'_i are the natural logarithms of corresponding variables P_{ij}, f_j and π_i. The system (4) is an analogy of the G-K system but in logarithmic terms with

notional quantities (weights) for products (1; 0)[8]. This consists of (N+M) log-linear equations in (N+M) unknowns, one of them is redundant. The system (4) is homogeneous. By dropping one equation and setting $\mathbf{f}'_N = 1$ a modified system is obtained which is no longer homogeneous because everything is now standardized on the country N. The modified system has (M+N-1) equations and (M+N-1) unknowns. The matrix of left-hand-side coefficients consists of two diagonal sub-matrices along the diagonal. By taking advantage of a theorem about inverse of portioned matrices, it is possible to solve (M+N-1)-equation system with dispatch by engaging in computations no more complicated than various matrix multiplications and the inversion of (N-1)-by-(N-1) matrix. The reduced system has (N-1) unknown variables $\mathbf{f}'_N$ (the Gauss-method with the selection of main elements or an iterative method can be used for solving of this system).

The obtained values f_j', π_i' and their exponentiated forms f_j and π_i **(the international** average prices and the basic heading countries' PPPs) allow to produce in effect the full price matrix, i.e. we can replace the holes in the initial price matrix by their estimations: implicit (missing) price is the combination of two variables - corresponding international price and country PPP for given basic heading.

The modification of the CPD-method proposed above describes the main idea of the CPD method in economic terms rather than in stochastic terms and simplifies the procedure of computation. It is necessary to mention that the CPD method as a specific version of the G-K method uses the notional quantities for items (but not actual quantities in physical terms) and therefore the results do not depend on the prices of big countries (i.e. free from the Gerschenkron effect). The number of items priced by the countries has some impact but this is valid for each method. If real quantities are available for items then such weighted version of the CPD-method can be considered as a particular kind of the Rao-method.

Reflection of representativity of products within the CPD and EKS methods

The classical CPD method does not use explicitly the information about the representativity of priced products in the countries. It means that the item list should be established in such way that the countries have a possibility to price enough many representative items from the product list. However this feature can be included in the CPD framework by different ways.[9]

The EKS method takes explicitly into account the data about the representativity of priced items: representative (*), non-representative (-). It was demonstrated in the main part of this paper that actually the EKS method uses three partial average PPPs calculated on the basis of the following sets of Items:

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[8] It is clear that an arithmetic version of the G-K system with notional quantities is impossible because this would be non-invariant to the measurement units of products.

[9] See, for example, Cuthbert, J.R., M. Cuthbert (1988), „On Aggregation Methods of Purchasing Power Parities", OECD Working Papers.) and Cuthbert, J.R. (1997), „Aggregation of price relatives to basic heading level: Review and comparison". International Statistical Institute Meeting, 18-26 August 1997, Istanbul) who included in the CPD model (1) an additional factor representing the effect of characteristicity of items. It is possible also to use some explicit weights, i.e. the characteristic items receive some higher weight than non-characteristic items. For example, e.g. the weights'2' and '1' can be used. Note: the weights "1" and "0" are applicable for the EKS method (1 = for asterisked items *; 0 – for non-asterisked items) but not for the CPD method because the items with "0-weights" will be eliminated here from the calculations at all.

(*) (*) a set where Items have asterisks in both countries
(*) (-) a set where Items have asterisks in the 1st country only
(-) (*) a set where Items have asterisks in the 2nd country only

The Items non-representative for both countries **(-) (-)** are ignored.

The binary PPP between a pair of the countries is calculated as geometric mean from these three PPPs with some weights. The items which are representative for both countries **(*) (*)** have twice the weights as other items included in the calculation. The author of this paper proposed to assign the equal weights for the PPP of the set of items with **(*) (-)** and for the PPP of the set of items with **(-) (*.).** In this case the bias of one set is compensated by opposite influence of another set[10]. Schematically this can be presented as the following (the situations with a compensated effect, are highlighted):

	Country A	Country B
Set 1	**(*)**	**(*)**
Set 2	**(*)**	**(-)**
Set 3	**(-)**	**(*)**

Items non-representative in both countries
are outside the calculations

Set 4	(-)	(-)

It is not an usual case that all 3 sets of items are present. Therefore the management of the **8** (2^3) different situations is necessary („Yes" means that a given set contain respective data, „No" – a given set contains no data):

	Set 1: **(*) (*)**	Set 2: **(*) (-)**	Set 3: **(-) (*)**
Situation 1	**Yes**	**Yes**	**Yes**
Situation 2	**Yes**	**Yes**	No
Situation 3	Yes	No	Yes
Situation 4	**Yes**	No	No
Situation 5	No	Yes	**Ye**s
Situation 6	No	**Yes**	No
Situation 7	No	No	Yes
Situation 8	No	No	No

The introduction of more complicated weighting systems

How the proposal to use more complicated weighting system (2 = very representative, 1 = moderate representative, 0 = non-representative) within the ICP 2004 can be realized practically by the CPD and the EKS methods?

The CPD method allows to introduce new set of weights in a straightforward simple way. The term (2) for average „International price" of the **i**th item (π_i)can presented as a 'implicit quantity'-weighted geometric average of the PPP-adjusted national prices of the ith item in the N countries:

[10] The recent Eurostat WP PPP (LUX, 18.11.02) named this EKS modification as the EKS-S method.

$$(5) \qquad \pi_i = \left(\prod_{j=1}^{N}(P_{ij}/f_j)^{q_{ij}}\right)^{1/_j\Sigma q_{ij}}; \qquad i = 1,2,\ldots,M$$

where

q_{ij} is implicit quantity (weight) for ith item in the jth country: q_{ij} = **3**, if ith item was indicated as **very representative** in the jth country, q_{ij} = **2** if ith item was indicated as **representative** in the jth country and q_{ij} = **1 otherwise**.

$_j\Sigma q_{ij}$ is the accumulated indicator of representativity of item i among all countries.

The term (3) for the purchasing power parity (PPP) for the jth country (f_j) for given basic heading can be presented as the geometric average (implicit weighted) deviation of its national prices from the international prices:

$$(6) \qquad f_j = \left(\prod_{i=1}^{M}(P_{ij}/\pi_i)^{q_{ij}}\right)^{1/_i\Sigma q_{ij}}; \qquad j = 1,2,\ldots,N$$

where $_i\Sigma q_{ij}$ is the accumulated indicator of representativity of items priced in the country **j.**

<u>Note</u>: The weights "2", "1" and "0" indicated in the ICP Researcher Proposal are applicable for the EKS method (as analogues for ****; *, -**) but not for the CPD method because the items with "0-weights" will be eliminated here from the calculations at all. Therefore it is better to use the following set of weights (notional quantities) - "3", "2", "1" as it has been done, for example, by the ESCAP 1985 ICP.

An other reason of the possible use of the CPD method can be the circumstance that it permits a linking of additional countries into an exercise at a later date on the basis of the ratios with international average CPD prices. Non-official research exercise for Taiwan based on the CPD average of 20 core countries from the 1980 benchmark. This approach was used also in the official comparisons: for example, this was the case with Laos and Malaysia within the ESCAP 1993 comparison. The EKS method has no such simple possibility for a link of the countries-newcomers. For example, Cyprus was linked (catch up program) with the results of the 1997 / 1998 Eurostat exercises for many Surveys via Germany only. Of course, the link via one arbitrary country is less reliable than a link via average international prices calculated for a broad set of the countries.

The situation with the EKS method is more complicated. The number of the possible situations with new weighting system is increased drastically for bilateral comparisons. If we divide 3 types of items [very characteristic (**), characteristic (*), non-characteristic (-)] then **9** sets of items can be obtained within a binary comparison between the countries A and B:

	Country A	Country B
Set 1	**(**)**	**(**)**
Set 2	**(*)**	**(*)**
Set 3	**(**)**	**(*)**
Set 4	**(*)**	**(**)**
Set 5	**(**)**	**(-)**

Set 6	**(-)**	**(**)**
Set 7	**(*)**	**(-)**
Set 8	**(-)**	**(*)**

Items non-representative in both countries
are outside the calculation

Set 9	(-)	(-)

The situations with a different representativity for the countries which should have a compensatory effect like (**)/(*) and (*)/(**) are highlighted (Sets 3 - 4, 5 - 6; 7 – 8).

256 !!! (2^8) different possible situations for each pair of the countries (see the Table below) should be considered by **8** possible sets of items indicated above:

	Set 1 (**) (**)	**Set 2 (*) (*)**	**Set 3 (**) (*)**	**Set 4 (*) (**)**	**Set 5 (**) (-)**	**Set 6 (-) (**)**	**Set 7 (*) (-)**	**Set 8 (-) (*)**
Situation 1	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**
Situation 2	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	No
Situation 3	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	No	Yes
Situation 4	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	No	No
....	....	....	....	....	....	....	....	....
Situation 254	No	No	No	No	No	No	Yes	**No**
Situation 255	No	No	No	No	No	No	No	Yes
Situation 256	No	No	No	No	No	No	No	No

It is very difficult to manage efficiently this set of numerous possible situations. First of all, to obtain the correct binary PPPs, the sets with more representative items should have more impact on the results and the PPPs for the compensatory sets should have the equal weights in the calculation of the combined PPPs. A possible assignation of the weights to the items with different representativity is given below:

	Country A	Country B	Representativity of an item
Set 1	**(**)**	**(**)**	**4 = 2 + 2**
Set 2	**(*)**	**(*)**	**2 = 1 + 1**
Set 3	**(**)**	**(*)**	**3 = 2 + 1**
Set 4	**(*)**	**(**)**	**3 = 1 + 2**
Set 5	**(**)**	**(-)**	**2 = 2 + 0**
Set 6	**(-)**	**(**)**	**2 = 0 + 2**
Set 7	**(*)**	**(-)**	**1 = 1 + 0**
Set 8	**(-)**	**(*)**	**1 = 0 + 1**

Items non-representative in both countries
are outside the calculation (Zero-weights)

Set 9	(-)	(-)	0 = 0 + 0

These weights are based on a simple but enough reasonable idea: <u>each asterisk (*) receives an imaginary weight/quantity = 1</u>. So, Items (**)(**) which are very representative in both countries receive the weight = 4, Items (**)(*) and Items (*)(**) – the weight = 3, etc. The total representativity of the sets of items for a pair of the

countries can be calculated as (4n22+3n21+3n12+2n20+2n02+2*n11+n10+n01), where n22 is no. of items (**)(**) etc. Obviously, this system of weights is arbitrary. However the system of asterisks is arbitrary *per se*. It is impossible to quantify exactly the qualitative indicators (like “very representative”, “representative” and “non-representative”). Each system of the notional quantities (weights) attributed to them will be inevitable a convention only. However the proposed method assigns higher weights for items with higher representativity and assigns the equal weights for the compensatory items. In effect, the desirable premises for the calculation of the reliable non-biased PPPs is obtained. The next step is the assignation of the weights for the different sets of Items taking into account the desirable compensatory effect. A simple (probably, not optimal version) is presented in the table below:

Sets of Items	Country A	Country B	Represen-tativity of Item sets	Shares (weights=w) of Item sets
Set 1 (n22) / PPP(22)	**(**)**	**(**)**	**4*n22**	**w(22) = 4*n22 / Σ(r)**
Set 2 (n11) / PPP(11)	**(*)**	**(*)**	**2*n11**	**w(11) = 2*n11 / Σ(r)**
Set 3 (n21) / PPP(21)	**(**)**	**(*)**	**3*n21**	**If** (n21 > 0) **And** (n12 > 0): **w(21) = w(12) = 0.5*(3*n21+3*n12)/Σ(r)** **If** (n21 = 0) **Or** (n12 = 0): **w(21) = w(12) = 0**
Set 4 (n12) / PPP(12)	**(*)**	**(**)**	**3*n12**	**If** (n21 > 0) **And** (n12 > 0): **w(12) = w(21) = 0.5*(3*n21+3*n12)/Σ(r)** **If** (n21 = 0) **Or** (n12 = 0): **w(12) = w(21) = 0**
Set 5 (n20) / PPP(20)	**(**)**	**(-)**	**2*n20**	**If** (n20 > 0) **And** (n02 > 0): **w(20)** = **w(02) = 0.5*(2*n20+2*n02)/Σ(r)** **If** (n20 = 0) **Or** (n02 = 0): **w(20) = w(02) = 0**
Set 6 (n02) / PPP(02)	**(-)**	**(**)**	**2*n02**	**If** (n20) > 0 **And** (n02 > 0): **w(02)** = **w(20) = 0.5*(2*n20+2*n02)/Σ(r)** **If** (n20 = 0) **Or** (n02 = 0): **w(02) = w(20) = 0**
Set 7 (n10) / PPP(10)	**(*)**	**(-)**	**n10**	**If** (n10 > 0) **And** (n01 > 0): **w(10) = w(01) = 0.5*(n10+ n01)/Σ(r)** **If** (n10 = 0) **Or** (n01 = 0): **w(10) = w(01) = 0**
Set 8 (n01) / PPP(01)	**(-)**	**(*)**	**n01**	**If** (n10 > 0) And (n01 > 0): **w(01) = w(10) = 0.5*(n10+ n01)/Σ(r)** **If** (n10 = 0) **Or** (n01 = 0): **w(01) = w(10) = 0**
TOTAL	**----**	**------**	**Σ(r)**	**Σ(w)**

The proposed scheme is based on the following assumptions:

- sets of items with an equal representativity in the countries – **(**)/(**)** and **(*)/(*)** - produce the unbiased PPPs,

- sets of items with a higher representativity for the country A **– (**)/(*), (*)/(-)** and **(**)/(-)** - produce the underestimated PPPs (relatively “true” values) for the country A (respectively, overestimated PPPs for the country B); the bias for the set **(**)/(-)** is some higher than for the sets **(**)/(*)** and **(*)/(-)**,

- sets of items with a lower representativity for the country A **– (*)/(**), (-)/(*)** and **(-)/(**)** - produce the overestimated PPPs (relatively “true” values) for the country A (respectively, underestimated PPPs for the country B); the bias for the set **(-)/(**)** is some higher than for the sets **(*)/(**)** and **(-)/(*).**

In accordance with this scheme the sets of Items with non-equal representativity - **(**)/(*), (*)/(-), (**)/(-) -** are included in the calculation only if there are respective compensatory counterparts - **(*)/(**), (-)/(*), (-)/(**)**. If a respective counterpart is missing then both sets are excluded. So, the calculation of average weighted (**Σw=1 or 100**) binary PPPs from the PPPs of the different item sets can be done as follows:

(5) **PPP-Av** = **{**PPP(22)^w(22)*PPP(11)^w(11)***[**PPP(21)^w(21)*PPP(12)^w(12)**]** *****
***** **[**PPP(20)^w(20)*PPP(02)^w(02)**]*[**PPP(10)^w(10)*PPP(01)^w(01)**]}**

The presence of all possible sets for a pair of countries is not very realistic in the practice. Some sets will be missing in the most of the cases and, respectively, there will be many situations where the decisions are problematic. For example - Can be regarded the situation like

<u>n12 and n10 are equal to 0</u> but <u>n21>0 and n01 > 0</u>
(or n21 and n01 are equal to 0 but n12 > 0 and n10 > 0

as a situation with the compensatory sets? Following strictly to the proposed scheme, we should exclude all non-compensatory sets from the calculation. However intuitively, one can believe that the Set (21) should have some compensatory effect with the Set(01) or one can believe that the combination of the Set(21) and Set(10) should have a compensatory effect with the Set(02), i.e. we can use simple geometric mean from these three PPPs as an appropriate approximation. Some more complicated cases can occur in the practice – all Items belong to non-compensatory sets; an average PPP can’t be calculated in this case at all if a puristic approach is applied. It means that a direct PPP will not exist and an indirect estimation should be done. However it is very likely that a PPP obtained on the basis of original direct prices with some corrections will be, probably, more plausible than a PPP obtained indirectly via the 3rd countries.

These examples demonstrates clearly that the intention to use the imaginary weights for items (like 2, 1, 0) within the traditional EKS method leads to considerable practical problems. It is not easy to propose some corrections, which should be done in a general case for numerous possible situations for each pair of countries (256 situations with the weights 2, 1, 0 and exponentially much more situations by more diversified weights). However it is possible to propose a general adoption of the traditional EKS method to the more complicated weighting systems.

This can be done by the use of traditional forms of the Laspeyres and Paasche indices (arithmetic and harmonic averages) with further calculation of the Fisher index or by the calculation of the index of the Tornqvist type.

A parity of Laspeyres-type can be obtained as the arithmetic mean of the price ratios with the weights of the denominator country **h**:

$$L(j/h) = \sum_{i=1}^{k} \left(\frac{P_{ij}}{P_{ih}}\right) * w_{ih} \Big/ \sum_{i=1}^{k} w_{ih}$$

where

w_{ih} –weights for item **i** in the denominator country **h** (3, 2, 1, 0 or some other values; these are the same values which are regarded as quantities q_{ih} in the CPD method)

k – no. of items for which exist bilateral PPP(j/h)

A parity of Paasche-type can be obtained as the harmonic mean of the price ratios with the weights of the numerator country **j**

$$P(j/h) = \sum_{i=1}^{k} w_{ij} / \sum_{l=1}^{m} w_{ij} / \left(\frac{P_{ij}}{P_{ih}}\right)$$

where

w_{ij} –weights for item **i** in the numerator country **j** (3, 2, 1, 0 or some other values; these are the same values which are regarded as quantities q_{ij} in the CPD method)

k – no. of items for which exist bilateral PPP(j/h).

The standard Fisher-PPP can be obtained from these two indices.

The Tornqvist type can be also calculated on the basis of the same imaginary weights of countries (w_{ij}, w_{ih}) as it is done by the calculation of the L-, P- indices:

$$T(j/h) = \left[\prod_{i=1}^{k} \left(\frac{P_{ij}}{P_{ih}}\right)^{(w_{ij}+w_{ih})/2}\right]^{1/[\sum_{i=1}^{k+m} (w_{ij}+w_{ih})/2]}$$

Of course, the proposals done above change considerably the original concept of equi-representativity, a possibility to have, in principle, one priced representative item per country, the compensatory effect, etc. However these indices are closer to the aggregated indices where the expenditure are applied. If selected weighting system is reasonable then the same features of aggregated indices should bring the reliable indices with these sophisticated weights. On other side, these notional weights can't of course, play the same role as the actual expenditure and a careful analysis of structure of price sets of the countries should be done in any case.

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