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INTERPOLATING ANNUAL ESTIMATES OF PURCHASING POWER PARITY BETWEEN TRI-ANNUAL BENCHMARKS

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I Introduction

Estimates of Purchasing Power Parity (PPP) are used, among other things, to convert various national statistics, denominated in national currencies, into a common international currency in order to facilitate international comparisons. For example, international comparisons of GDP per capita, a key indicator in analysing global patterns of income distribution, depend critically on PPPs in order to make such comparisons in a common currency[1].

Early in 2003 new benchmark estimates of PPPs will become available for the year 2000. These estimates are dubbed “benchmarks” because they are based on direct comparisons of relative prices of a common international basket of expenditure items. Earlier, benchmark estimates were compiled for the years 1990, 1993 and 1996[2].

While benchmark estimates are essential, it is also important for some studies to a have annual time series of PPPs. Eurostat compiles annual estimates from tri-annual benchmarks taken for 1/3 of the expenditure basket each year, while for non-EU members the OECD extrapolates annual estimates from tri-annual benchmarks.[3] For the economies of Eastern Europe and the CIS the ECE projected the1996 benchmarks backwards and forwards, at the overall GDP level, using the ratio of the subject country’s implicit GDP deflator to that of the reference country (the US) as the projector[4].

When the benchmarks for 2000 become available the ECE could use a similar procedure to project them backwards and forwards, except that there is no reason to expect the backcast value for 1996 to equal the benchmark value for that same year. It is highly desirable that the annual time series of PPPs be consistent with the benchmarks, both to reduce confusion among users of these data and to fully and efficiently utilise the available information. Accordingly the projection procedure needs to be modified so that the annually projected series will resemble the projector (the ratio of implicit GDP deflators) as much as possible, subject to the constraint that it also equals the benchmark values in 1996 and 2000[5].

This paper proposes a procedure to accomplish this, and uses it to compile annual estimates of PPPs, at the GDP level for example countries from each of the CIS, EU and non-EU OECD regions. The ECE intends to apply this method of projection to all countries in the CIS region, and it is hoped that OECD and Eurostat may also find it useful in their approach to projecting annual PPP estimates as it overcomes the problem of the discontinuity introduced by extrapolating backwards and forwards one year from each benchmark.[6]

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[1] For a variety of reasons PPPs are generally preferable to exchanges rates for making these kinds of comparisons.

[2] United Nations Statistical Commission and Economic Commission for Europe, *International Comparison of Gross Domestic Product in Europe 1996, International Comparison of Gross Domestic Product in Europe 1993.*

[3] OECD Statistics Directorate, Purchasing Power Parities 1999 Benchmark Results, p. 15. www.oecd.org/pdf/M00025000/M00025222.pdf

[4] The 1993 benchmark was not included in this extrapolation because of large methodological changes between the 1993 PPP round and the 1996 round. In 1993 productivity adjustments were made for non-market services, but this type of adjustment was reconsidered and dropped from the 1996 round.

[5] While the OECD procedure of extrapolating backwards and forwards one year from each benchmark could be adapted to a four year span between benchmarks, it still introduces a discontinuity between the forward extrapolations from 1996 and the backward extrapolations from 2000. More will be said about this later.

[6] A visual basic/EXCEL macro to implement this procedure is available on request.

II Use of the Ratio of Implicit GDP Deflators as Projector

The benchmark PPP for year *t* can be thought of as the ratio of the costs of the same comprehensive, representative basket of goods priced in each of two countries A and B.

$$PPP^{AB}{}_{t} = \frac{\sum_i q^A{}_{it} p^A{}_{it}}{\sum_i q^A{}_{it} p^B{}_{it}}.$$

where the same quantities $q^A{}_{it}$ are valued at prices $p^A{}_{it}$ and $p^B{}_{it}$ in A and B respectively.

A new benchmark compiled for year t + τ updates this formula both for the new prices that prevail in t + τ as well as for the new quantities. Thus

$$PPP^{AB}{}_{t+\tau} = \frac{\sum_i q^A{}_{it+\tau} p^A{}_{it+\tau}}{\sum_i q^A{}_{it+\tau} p^B{}_{it+\tau}}$$

An ideal projector $K_{t+\tau}$ would project $PPP^{AB}{}_{t}$ forward to time t + τ in such a way that the result would exactly equal $PPP^{AB}{}_{t+\tau}$. In other words

$$\frac{\sum_i q^A{}_{it+\tau} p^A{}_{it+\tau}}{\sum_i q^A{}_{it+\tau} p^B{}_{it+\tau}} = PPP^{AB}{}_{t+\tau} = K_{t+\tau}\ \ PPP^{AB}{}_{t} = K_{t+\tau} \frac{\sum_i q^A{}_{it} p^A{}_{it}}{\sum_i q^A{}_{it} p^B{}_{it}}$$

or

$$K_{t+\tau} = \frac{\sum_i q^A{}_{it+\tau} p^A{}_{it+\tau}}{\sum_i q^A{}_{it+\tau} p^B{}_{it+\tau}} \frac{\sum_i q^A{}_{it} p^B{}_{it}}{\sum_i q^A{}_{it} p^A{}_{it}}$$

In comparison, the projector used to move the 1996 benchmark values forwards to 2002 (and backwards to 1990), and which we are proposing to use to project through the 1996 and 2000 benchmark values in the new PPP round, is given by the ratio of implicit GDP deflators:

$$\frac{IGDP^{At}{}_{t+\tau}}{IGDP^{Bt}{}_{t+\tau}} = \frac{\sum_i q^A{}_{it+\tau} p^A{}_{it+\tau}}{\sum_i q^A{}_{it+\tau} p^A{}_{it}} \frac{\sum_i q^B{}_{it+\tau} p^B{}_{it}}{\sum_i q^B{}_{it+\tau} p^B{}_{it+\tau}} = \frac{\sum_i q^A{}_{it+\tau} p^A{}_{it+\tau}}{\sum_i q^B{}_{it+\tau} p^B{}_{it+\tau}} \frac{\sum_i q^B{}_{it+\tau} p^B{}_{it}}{\sum_i q^A{}_{it+\tau} p^A{}_{it}}$$

This is the same form as the ideal projector except the weights on B's prices $p^B{}_{it}$; $p^B{}_{it+\tau}$ are the quantities in B $q^B{}_{it}$; $q^B{}_{it+\tau}$ rather than those in A $q^A{}_{it}$; $q^A{}_{it+\tau}$

Since the projector differs from the ideal only by the weights used on the same set of prices $\left\{ p^B{}_{it}; p^B{}_{it+\tau} \right\}$, we would expect it to be sensitive to changes in

the price structure but not to changes in the overall price level. Indeed we can see that if $p^{B}{}_{it+\tau} = (1+r_B)\,p^{B}{}_{it}$, that is if inflation uniformly raises all prices in B by $100\,r_B$ percent, and if all prices in A rise uniformly by $100\,r_A$ percent;
$p^{A}{}_{it+\tau} = (1+r_A)\,p^{A}{}_{it}$, then

$$\frac{IGDP^{At}{}_{t+\tau}}{IGDP^{Bt}{}_{t+\tau}} = \frac{\sum_i q^{A}{}_{it+\tau}\,p^{A}{}_{it+\tau}}{\sum_i q^{A}{}_{it+\tau}\,p^{A}{}_{it}}\,\frac{\sum_i q^{B}{}_{it+\tau}\,p^{B}{}_{it}}{\sum_i q^{B}{}_{it+\tau}\,p^{B}{}_{it+\tau}} = \frac{\sum_i q^{A}{}_{it+\tau}(1+r_A)\,p^{A}{}_{it}}{\sum_i q^{A}{}_{it+\tau}\,p^{A}{}_{it}}\,\frac{\sum_i q^{B}{}_{it+\tau}\,p^{B}{}_{it}}{\sum_i q^{B}{}_{it+\tau}(1+r_B)\,p^{B}{}_{it}}$$

$$\frac{IGDP^{At}{}_{t+\tau}}{IGDP^{Bt}{}_{t+\tau}} = \frac{(1+r_A)}{(1+r_B)}$$

This is the same result the ideal projector would give under these conditions:

$$K_{t+\tau} = \frac{\sum_i q^{A}{}_{it+\tau}(1+r_A)\,p^{A}{}_{it}}{\sum_i q^{A}{}_{it+\tau}(1+r_B)\,p^{B}{}_{it}}\,\frac{\sum_i q^{A}{}_{it}\,p^{B}{}_{it}}{\sum_i q^{A}{}_{it}\,p^{A}{}_{it}} = \frac{(1+r_A)}{(1+r_B)}$$

In other words, the ratio of implicit deflators will disagree with the ideal projector only if the change in prices involves not only a shift in the overall price levels in the two countries, but also a change in the relative price structure in one country in comparison to that in the other. Even in this case the implicit deflator approach will capture that portion of the relative price changes that reflects changes in the average price levels in the two countries. Changes in relative price structures, however, will not be fully reflected in the implicit deflator approach. The degree to which they will be picked up will depend on how close the new expenditure patterns in B $\{q^{B}{}_{it+\tau}\}$ are to those in A $\{q^{A}{}_{it+\tau}\}$. Generally this effect due to shifts in the relative price structures should be small in comparison to effects due to changes in the overall price levels, and as a result the ratio of implicit price deflators will be an adequate projector for PPPs.

Nevertheless, the existence of new benchmark PPPs for 2000 gives us an opportunity to improve on the "pure" implicit deflator projections from 1996, provided we can constrain the projected series to go through both the 1996 and 2000 benchmarks. In the following section we outline an approach for accomplishing this goal.

III The Modified Extrapolation Procedure

In this section we develop an adaptation of Denton's quadratic minimisation[7], which is widely used by statistical agencies to adjust monthly or quarterly series to independently derived annual benchmarks, and is used in seasonal adjustment programs such as X-12 ARIMA to ensure that adjust seasonally data equal the

[7] Denton, F.T., 1971, "Adjustment of Monthly or Quarterly Series to Annual Totals: An approach Based on Quadratic Minimization", *Journal of the American Statistical Association,* Vol. 66 (March), pp. 92-102.

original annual totals. We will adapt the "proportional" version of this approach, which has been found to be generally preferable in a comparison to other methods[8].

Let $\tilde{I}_t$ be the series of annual PPP estimates we are looking for. We want $\tilde{I}_t$ to look as much as possible like the series of ratios of implicit GDP deflators $IGDP_t = \frac{IGDP^A{}_t}{IGDP^B{}_t}$ while at the same time going through t he benchmark values I_{1996} and I_{2000}

When we say we want $\tilde{I}_t$ to look as much as possible like the series of ratios of implicit GDP deflators $IGDP_t$, we mean we want $\sum_t \left[\left(\frac{\tilde{I}_t}{IGDP_t} - \frac{\tilde{I}_{t-1}}{IGDP_{t-1}} \right) \right]^2$ to be a minimum.

This will ensure that the growth rates of the two series are as close as possible. Additionally we want to minize this expression subject to the constraints $\tilde{I}_{1996} = I_{1996}$ and $\tilde{I}_{2000} = I_{2000}$

To find this constrained minimum it is convenient to restate the problem in matrix notation:

$$Let\ A = \begin{bmatrix} \frac{1}{IGDP_1} & 0 & . \ . \ . & 0 & 0 \\ 0 & \frac{1}{IGDP_2} & . \ . \ . & 0 & 0 \\ . & . & . \ . \ . & . & . \\ . & . & . \ . \ . & . & . \\ . & . & . \ . \ . & . & . \\ 0 & 0 & . \ . \ . & \frac{1}{IGDP_{N-1}} & 0 \\ 0 & 0 & . \ . \ . & 0 & \frac{1}{IGDP_N} \end{bmatrix} \quad \text{and let} \quad D = \begin{bmatrix} -1 & 1 & 0 & . & 0 & 0 & 0 \\ 0 & -1 & 1 & . & 0. & 0 & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & .-1 & 1 & 0 \\ 0 & 0 & 0 & . & 0 & -1 & 1 \end{bmatrix}$$

D is the first difference operator such that if $\tilde{I}$ is a vector of N observations on $\tilde{I}_t$ then $D\tilde{I}$ is the vector of N - 1 observations on $\tilde{I}_t - \tilde{I}_{t-1}$

$$\text{Then } \sum_t \left[\left(\frac{\tilde{I}_t}{IGDP_t} - \frac{\tilde{I}_{t-1}}{IGDP_{t-1}} \right) \right]^2 = \left(DA\tilde{I}\right)'\left(DA\tilde{I}\right) = \tilde{I}'A'D'DA\tilde{I}$$

[8] Adriaan M. Bloem, Robert J. Dippelsman, and Nils O. Maehle, Quarterly National Accounts Manual — Concepts, Data Sources, and Compilation, International Monetary Fund 2001.
http://www.imf.org/external/pubs/ft/qna/2000/Textbook/
http://www.imf.org/external/pubs/ft/qna/2000/Textbook/ch6.pdf pp-83-108

The constraints can be expressed as $\mathrm{MAT}\ \tilde{\mathrm{I}} = \mathrm{I}_C$
where

$$\mathrm{MAT} = \begin{bmatrix} 0 & 0 & . & . & 1 & . & . & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & . & . & 1 & 0 & 0 \end{bmatrix}$$

with the 1s in the positions corresponding to 1996 and 2000 and

$\mathrm{I}_C' = [\mathrm{I}_{1996} \quad \mathrm{I}_{2000}]$

The Lagrangian to be minimised is then

$$L = \tilde{\mathrm{I}}'\mathrm{A}'\mathrm{D}'\mathrm{DA}\tilde{\mathrm{I}} + \lambda'(\mathrm{MAT}\ \tilde{\mathrm{I}} - \mathrm{I}_C)$$

and the first order conditions are:

$$\frac{\partial L}{\partial \tilde{\mathrm{I}}} = 2\mathrm{A}'\mathrm{D}'\mathrm{DA}\tilde{\mathrm{I}} + \mathrm{MAT}'\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = \mathrm{MAT}\ \tilde{\mathrm{I}} - \mathrm{I}_C = 0$$

or

$$\begin{bmatrix} \dfrac{\partial L}{\partial \tilde{\mathrm{I}}} \\ \dfrac{\partial L}{\partial \lambda} \end{bmatrix} = \begin{bmatrix} 2\mathrm{A}'\mathrm{D}'\mathrm{DA} & \mathrm{MAT}' \\ \mathrm{MAT} & 0 \end{bmatrix} \begin{bmatrix} \tilde{\mathrm{I}} \\ \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ \mathrm{I}_C \end{bmatrix}$$

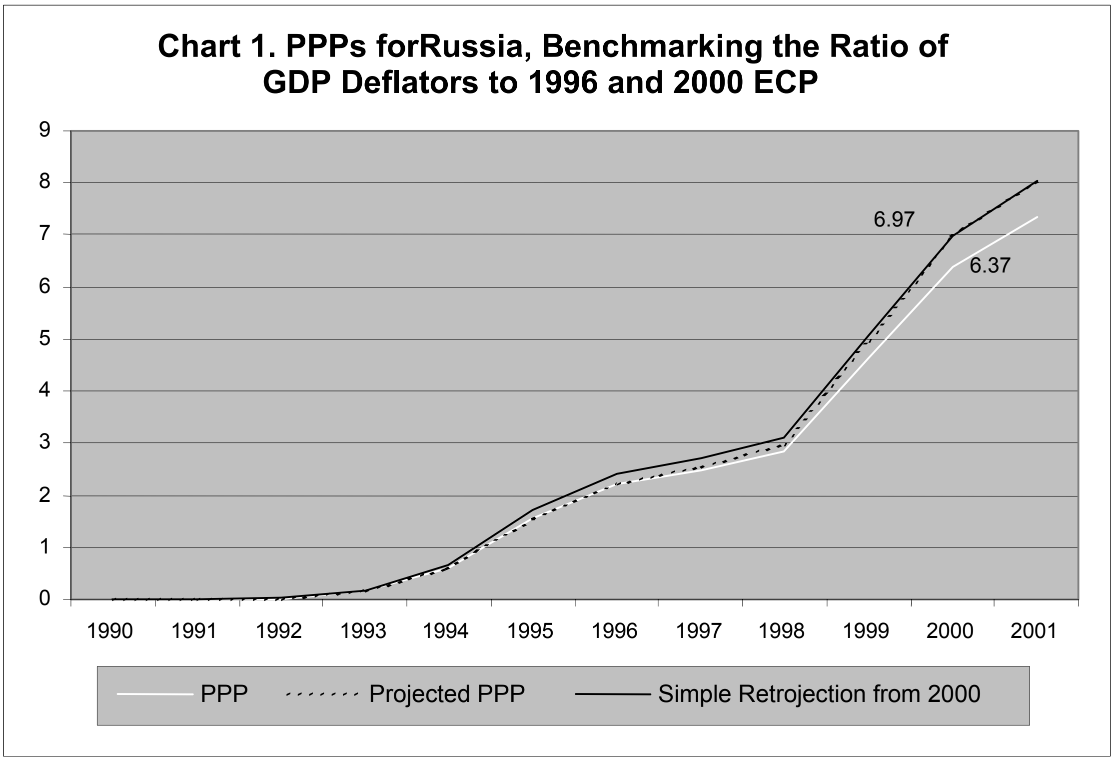
Therefore the time series we are looking for is the first N values of the vector

$$\begin{bmatrix} \tilde{\mathrm{I}} \\ \lambda \end{bmatrix} = \begin{bmatrix} 2\mathrm{A}'\mathrm{D}'\mathrm{DA} & \mathrm{MAT}' \\ \mathrm{MAT} & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \mathrm{I}_C \end{bmatrix}$$

IV <u>Results</u>

This section presents results of applying the procedure to a sample country from each of the CIS, the EU and non-EU OECD regions. The CIS example is based on actual ECP results for 2000, while the EU and OECD examples are mock-ups that will be explained later.

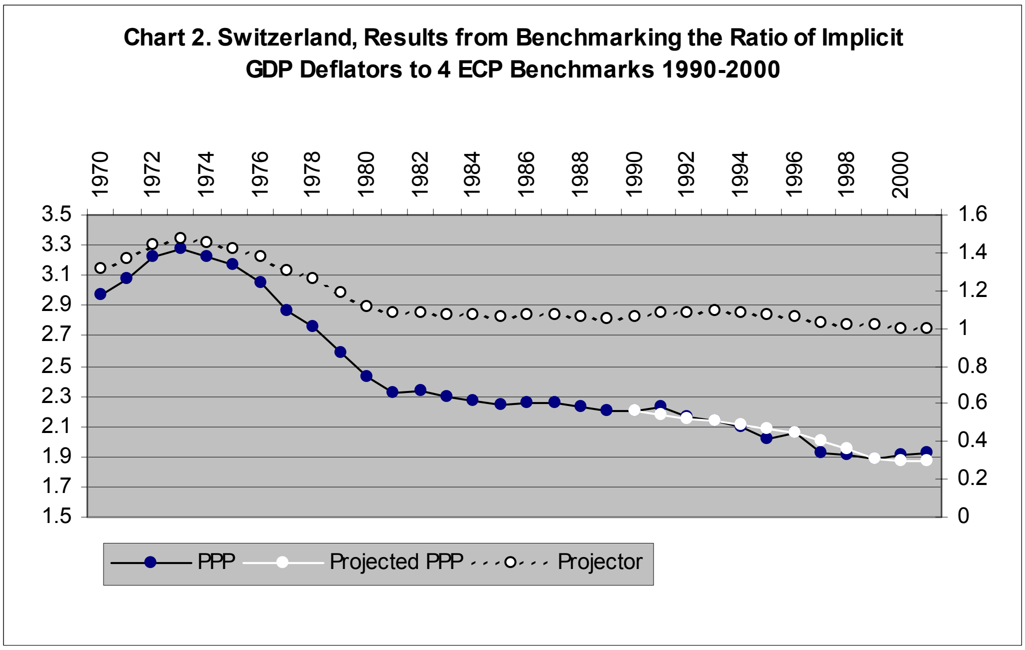
The solid white line in Chart 1 below shows the PPP series for Russia, as previously projected beyond and backwards from the 1996 benchmark value. This series is essentially the ratio of implicit GDP deflators adjusted to the level of the 1996 benchmark. This series projected a value of 6.37 for the year 2000, whereas the ECP for 2000 came in some 9% higher at 6.97. If we were to apply the same projection procedure to this 2000 PPP benchmark, we would simply raise the solid white line (by 9%) until it goes through the 2000 value, but then it would no longer pass through the 1996 value.



It would in fact be 9% above the solid white line in 1996, and indeed for all years. The procedure outlined in Section III above defines a new series indicated by the dotted line, that is benchmarked to both the 1996 and 2000 benchmarks, and whose movement resembles as much as possible that of the original. Note that in addition to being consistent with both benchmark values, the requirement that this series be as consistent as possible with the ratio of implicit deflators also implies minimal revisions to the previously published series, especially in the vicinity of 1996 and earlier. With this approach the 9% revision to 2000 is gradually worked into the projected series over the years from 1996 and 2000.

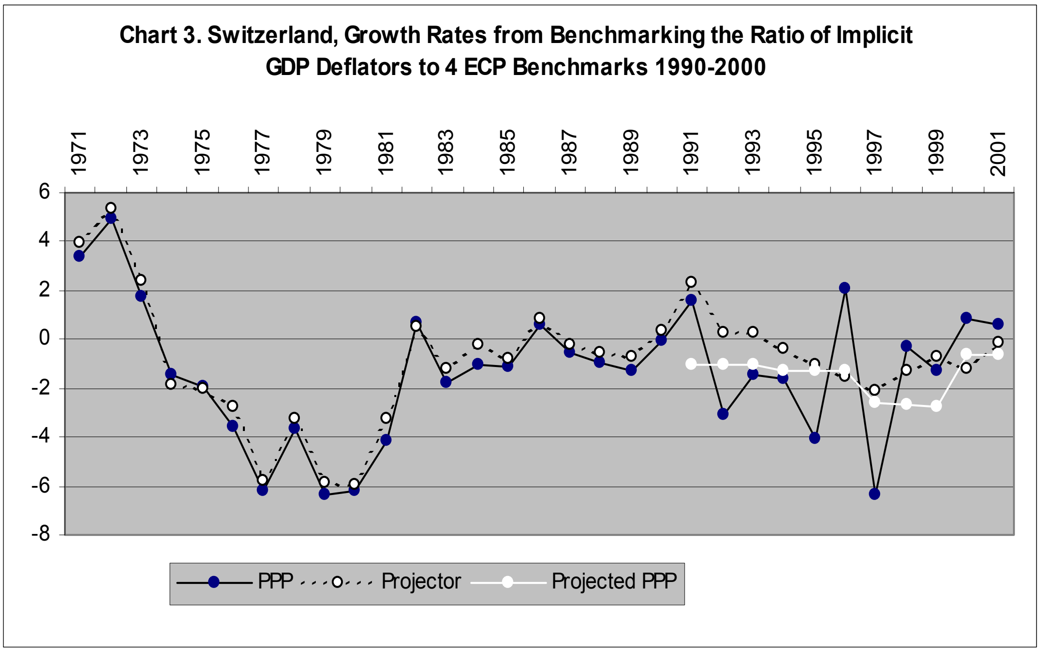
Since the early 1990s the OECD has projected tri-annually available benchmarks backwards one year and forwards one year, at the level of total GDP, using the ratio of implicit GDP deflators as the projector. These three year segments, each centered on a benchmark year, are concatenated to obtain a long time series of PPP estimates. Each segment is internally consistent with the projector movement, but movement between segments is determined by the benchmark results, not by the projector. As a result the projected PPP has a discontinuity every third year relative to the movement in the projector. Chart 2. Shows the situation for Switzerland, a non-EU OECD country. The solid dark line represents PPP as historically projected by the OECD. We treated the years 1990, 1993, 1996, and 2000 from this series as if

they were benchmarks, and re-projected the intervening years using the procedure outlined above. The result is shown by the solid white line, which is clearly smoother because the projector it resembles (the dotted line) is smooth. The differences between PPP and projected PPP created by the discontinuities between segments do



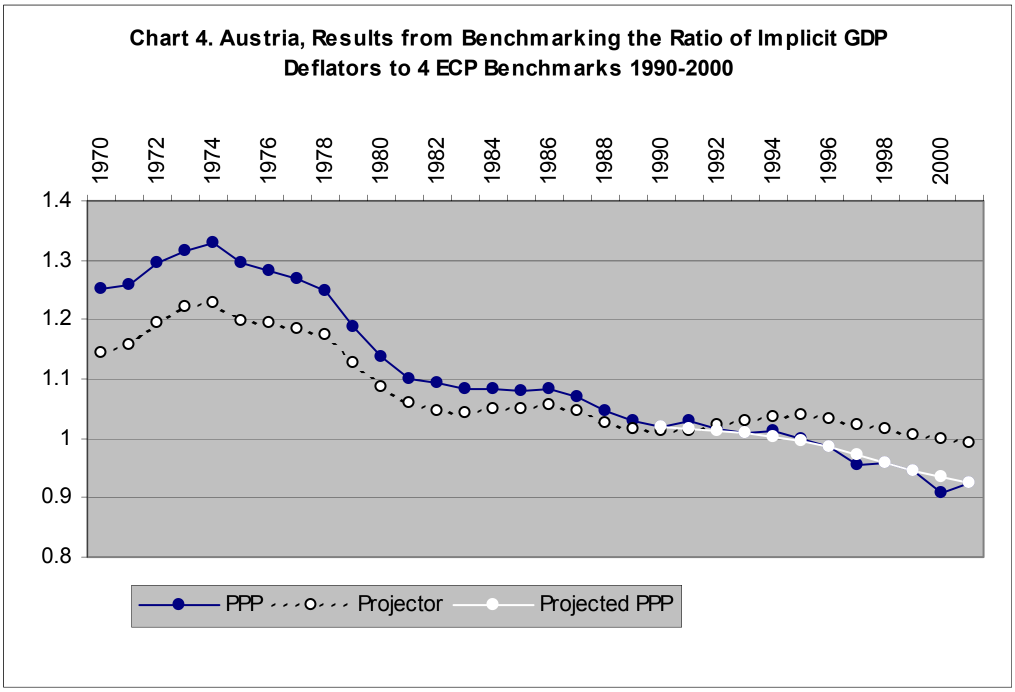
not appear to be large on this chart, but are in the order of 3-4%.

The effect of the discontinuities is more evident if one looks at growth rates of the respective series, as shown in Chart 3 below. This chart is very revealing. It shows a good correspondence between the projector and PPP over the period 1971 to 1991, but over the 1990's, when the tri-annual projection technique was implemented, the correspondence breaks down because of the discontinuities it introduces between segments. While not perfect, the Projected PPP (white line) better reflects the projector movement, and is consistent with the benchmarks (Chart 2).



While Eurostat's projection technique is not the same as OECD's, it is subject to a similar discontinuity problem. Tri-annual benchmarks are available for a different third of the expenditure items each year, and for any given third of the items are available every third year. For each third of the expenditure items, the intervening years are projected in the same way as OECD, and this introduces a discontinuity every third year into the time series representing each third. However, the placement in time of these discontinuities is staggered by one year for each third, and as a result discontinuities are introduced every year instead of every third year. The projections are also made at a lower level of aggregation than total GDP. Discontinuities are likely to be more important at this lower level.

Charts 4 and 5 show the situation for Austria, an EU county handled by Eurostat. While we did not have access to the different thirds of the expenditure items Eurostat works with, and not at the lower level of disaggregation Eurostat uses, these charts show that Eurstat's projection procedure still produces a more erratic aggregate PPP than the projector.



While the projection procedure outlined above will help the ECE to publish PPPs for CIS countries that are consistent over time, it may also be of interest to OECD and Eurostat to improve temporal consistency of annual PPP estimates for their countries.

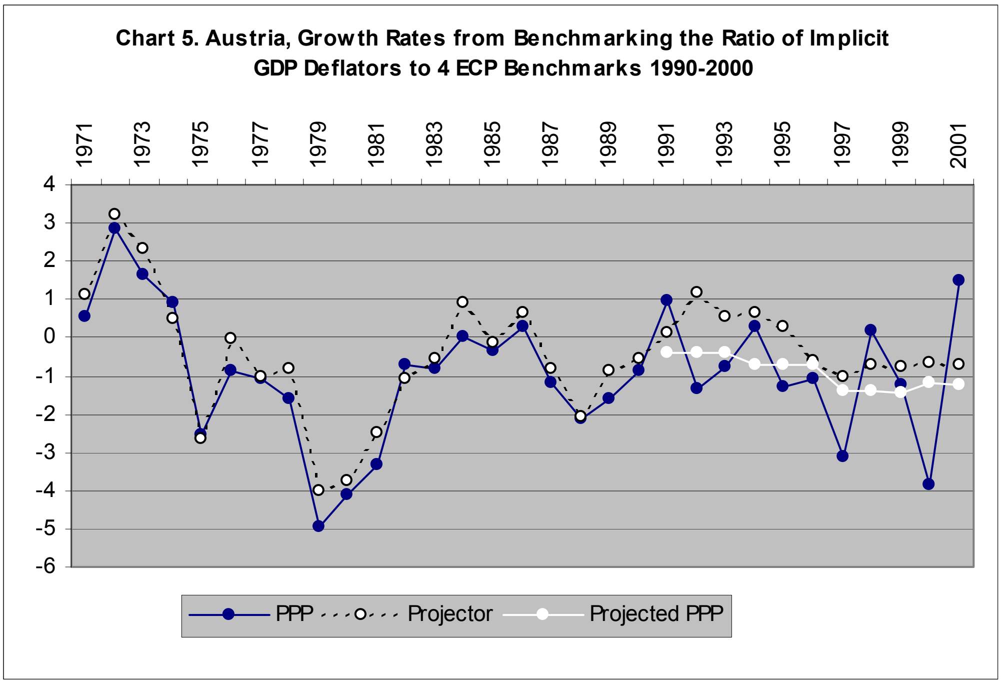


Chart 5. Austria, Growth Rates from Benchmarking the Ratio of Implicit GDP Deflators to 4 ECP Benchmarks 1990-2000
1971
1973
1975
1977
1979
1981
1983
1985
1987
1989
1991
1993
1995
1997
1999
2001
4
3
2
1
0
-1
-2
-3
-4
-5
-6
PPP
Projector
Projected PPP
