

**Proposal for Supplement 5 to the 02 series of amendments
to UN Regulation No. 117 amending document
ECE/TRANS/WP.29/GRB/2013/10**

Note: The text reproduced below was prepared by the experts from the Russian Federation as a result from the ad-hoc working group meeting of experts of NAMI (the Russian Federation) and UTAC (France), with the participation of ETRTO organized in accordance with the recommendation of the 58th GRB session (ECE/TRANS/WP.29/GRB/56, para. 15), which endorsed to test the proposed "Deceleration Calculator" tool. This proposal amends document ECE/TRANS/WP.29/GRB/2013/10 distributed at the 58th GRB session. The physic-mathematic principle in this document is not changed relatively to the previous one, but the text of explanations is considerably modified. The modifications to the existing text of the UN Regulation are marked in bold for new or strikethrough for deleted characters.

I. Proposal

Annex 6,

Paragraph 3.5., amend to read:

"3.5. Duration and speed.

When the deceleration method is selected, the following requirements apply:

- (a) The deceleration j shall be determined in **differential** ~~exact~~ $d\omega/dt$ or **discrete** ~~approximate~~ $\Delta\omega/\Delta t$ form, where ω is angular velocity, t – time;

If the differential form $d\omega/dt$ is used, then the recommendations of Appendix 4 to this Annex are to be applied.

- (b) ..."

Annex 6, insert a new Appendix 4, to read:

"Annex 6 – Appendix 4

Deceleration method: Measurements and data processing for deceleration value obtaining in differential form $d\omega/dt$.

1. Record dependency "distance-time" of rotating body decelerated from peripheral with a speed range such as 82 to 78 km/h or 62 to 58 km/h dependently of PC or CV tyre in a discrete form (figure 1) for rotating body:

$$z=f(t_z)$$

where:

z is a number of body revolution during deceleration;

t_z is time of end of revolution number z in seconds recorded with 6 digits after point.

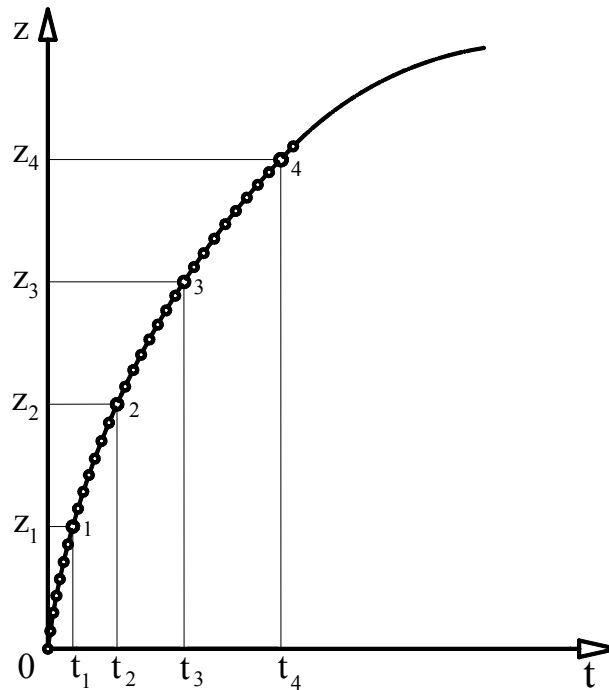


Figure 1.

Note: Lower speed of recording range may be reduced down to 60 (40) km/h.

2. Approximate recorded dependency by continuous, monotonic, differentiable function:

- 2.1. Choose the nearest to the maximum value of z dividable by 4 and divide it on 4 equal parts with bounds: $0, z_1(t_1), z_2(t_2), z_3(t_3), z_4(t_4)$.

- 2.2. Work out the system for 4 equations each of form

$$z_m = A \ln \frac{\cos B(T_z - t_m)}{\cos B T_z}$$

where unknowns:

A is dimensionless constant,
 B is constant in revolutions per second,
 T_{Σ} is constant in seconds,
 m is number of bounds shown in figure 1.
 Insert in these 4 equations coordinates of 4-th bound above.

- 2.3. Take constants A,B, T_{Σ} as solution of equation system described above in paragraph 2.2 using iteration process and approximate measured data by formulae:

$$z(t) = A \ln \frac{\cos B(T_{\Sigma} - t)}{\cos B T_{\Sigma}}$$

where:

$z(t)$ is current continuous angular distance in number of revolutions (not only integer values);
 t is time in seconds.

Note 2: Other approximating functions $z=f(t_z)$ may be used if its adequacy will be proven.

3. Calculate the deceleration j in revolutions per second squared (s^{-2}) by formula:

$$j = AB^2 + \frac{\omega^2}{A}$$

where:

ω is angular speed in revolutions per second (s^{-1}).
 For the case $U_n = 80$ km/h; $\omega = 22.222/R_r$ (or R).
 For the case $U_n = 60$ km/h; $\omega = 16.666/R_r$ (or R).

4. Estimate a quality of approximation of measured data and its accuracy by parameters:

- 4.1. Standard deviation in percents:

$$\sigma = \sqrt{\frac{1}{n-1} \sum_1^n \left[1 - \frac{z(t)}{z} \right]^2} \times 100\%$$

- 4.2. Coefficient of determination

$$R^2 = 1 - \frac{\sum_1^n [z - z(t)]^2}{\sum_1^n [z - \bar{z}]^2}$$

where:

$$z = \frac{1}{n} \sum_{z=1}^n z = \frac{1}{n} (1+2+\dots+n) = \frac{1+n}{2}$$

Note: The calculation above described for this variant of the deceleration method for tyre rolling resistance measurement can be executed by the computer program “Deceleration Calculator” downloadable from the [WP.29 website] as well as any software which allows to perform nonlinear regression calculation.

II. Justification

1. The proposed principle and its application in the computer program “Deceleration Calculator” is based on the exact relationship:

$$j = \frac{d\omega}{dt} = \frac{d^2z}{dt^2}$$

2. Numerous experiments show that the formula in paragraph 2.3 of proposed Annex 6 - Appendix 6 is very effective for experimental data approximating. This formula of constrain between current time t and current angular distance z is the result of transformation of the dependence between retroreflective distance S and time T [1], [2] (in common case $T = T_{\Sigma} - t$):

$$S = A_m \ln \frac{1}{\cos BT}$$

Retroreflective time T relates to the total deceleration time T_{Σ} by formula $T = T_{\Sigma} - t$ (in local case $T = T_{\Sigma}$, $|S = S_{\Sigma}$).

The second derivative of the function described by formula from paragraph 2.3 of Annex 6 - Appendix 6 is deceleration j in revolution per second squared or s^{-2} :

$$j = \frac{d^2z}{dt^2} = \frac{AB^2}{\cos^2 B(T_{\Sigma} - t)}$$

3. There is no simplification or assumption between this formulae and the formula of z in paragraph 2.3 of Annex 6 - Appendix 6 because correspondent transformation is performed according to the rules of differential calculus of higher mathematics. Thus the need to measure and calculate speed is excluded.

4. The algorithm of determining of parameters A, B, T_{Σ} includes the following steps:

4.1. The measuring time of each revolution of rotating body which give experimental dependency shown on figure 1:

$$z = f(t_z)$$

4.2. Finding the value nearest to the maximum z equals n , dividable by 4, dividing it on 4 equal parts and recording the coordinates of 4 points on the experimental curve (see figure 1).

4.3. Working out the equation system on the base of formula from paragraph 2.3 of Annex 6 - Appendix 6 with substitution of 4 point coordinates as shown on Figure 1:

$$\left. \begin{aligned} z_1 &= A \ln \frac{\cos B(T_\Sigma - t_1)}{\cos B T_\Sigma} \\ \dots\dots\dots \\ z_4 &= A \ln \frac{\cos B(T_\Sigma - t_4)}{\cos B T_\Sigma} \end{aligned} \right\}$$

4.4. Pairwise transformations of the set of equations from paragraph 4.3 above give set of two equations:

$$\left. \begin{aligned} \cos^2 B(T_\Sigma - t_1) &= \cos B T_\Sigma \cos B(T_\Sigma - t_2) \\ \cos^2 B(T_\Sigma - t_3) &= \cos B(T_\Sigma - t_2) \cos B(T_\Sigma - t_4) \end{aligned} \right\}$$

Parameters B and T_Σ are received from this set by iteration process. Then parameter A may be obtained from the fourth equation from set of 4 equations above, multiplied by 2π :

$$A = \frac{2\pi z_4}{\ln \frac{\cos B(T_\Sigma - t_4)}{\cos B T_\Sigma}}$$

Thus formulae $z = f(t_z)$ and $j = d^2z/dt^2$ become with determined parameters and become ready for subsequent application. The first derivative of function $z = f(t)$ from paragraph 2.3 of Annex 6 - Appendix 6 above is the angular speed ω in revolutions per second (s^{-1}):

$$\omega = \frac{dz(t)}{dt} = AB \operatorname{tg} B(T_\Sigma - t)$$

One can see from this that:

$$\operatorname{tg} B(T_\Sigma - t) = \frac{\omega}{AB}$$

The next formula follows from geometry and previous relation:

$$\cos^2 B(T_\Sigma - t) = \frac{1}{1 + \operatorname{tg}^2 B(T_\Sigma - t)} = \frac{1}{1 + (\omega/AB)^2}$$

Substitution of this equality into formula j in paragraph 2 of this Justification yields:

$$j = AB^2 + \frac{\omega^2}{A}$$

This relationship is the main formula for the “Deceleration Calculator”.

5. The proposed derivative mathematical method approach associated to the used deceleration calculator provides for the approximation an evaluated estimate close to 1.
