Constant-utility indexes for groups in Canadian society

An application of a consumer demand system

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Micro-economic theory of consumer behaviour

A utility function	$U = f(Q_i)$	Where consumer can derive utility differently from respective quantities acquired of each consumption category s.t. some basic constraints (non-satiation, completeness, transitivity, monotonicity).
Indirect utility function	$V = f(P_i, M)$	Derived from the utility function, but depending on the prices of each consumption category and assuming maximization of the U function s.t. income.
Cost function	$C = f(\bar{U}, P_i)$	Derived from above, gives the minimum cost that must be incurred to achieve a certain level of U, given the price regime in effect and the utility function from which we started.
Budget share equation	$w_i = f(\bar{U}, P_i)$	Obtained by applying Roy's identity to cost function.

Model Restrictions and Functional forms

- Slutsky identity – constraints on income and price substitution effects
- *Integrability* tells us that a demand or cost function must have an underlying well-behaved utility function (Antonelli, 1886, Uzawa, 1971).
- Rotterdam model - Barten (1969), Theil (1965)
- Generalized Cobb-Douglas, Generalized Leontief - Berndt, Darrough and Diewert (1977)
- Translog model - Jorgensen, Stoker and Lau (1982)
- Linear Almost Ideal Demand System - Deaton and Muellbauer (1980)
- Quadratic Almost Ideal Demand System - Banks, Blundell and Lewbel (1997)
- Normalized quadratic expenditure function with linear or quadratic spline – Diewert and Wales (1993)

We want to carry out this exercise with all of these functional forms, but we began with the LAIDS and QAIDS to assess the feasibility of such an exercise.

Model estimation

QAIDS budget share equations:

$$w_i = \alpha_i + \sum_{i=1}^{N} \gamma_{ij} \ln p_j + \beta_i \ln\left[\frac{m}{a(p)}\right] + \frac{\lambda_i}{b(p)}\left\{\ln\left[\frac{m}{a(p)}\right]\right\}^2, \quad \forall i = 1,\ldots,N$$

$$b(p) = \prod_{i=1}^{N} p_i^{\beta_i}$$

LAIDS budget share equations:

$$w_i = \alpha_i + \sum_{i=1}^{N} \gamma_{ij} \ln p_j + \beta_i \ln\left[\frac{m}{a(p)}\right], \quad \forall i = 1,\ldots,N$$

$a(p)$ is the translog price index

Simplifications had to be made. b(p) in QAIDS had to be set to 1. Likewise, a(p) in LAIDS, which normally is the Translog price index, had to be linearized to the Stone index.

Otherwise, no convergence can be attained via the S.U.R. procedure, after 100 iterations (approx. 1 to 2 weeks)

Demand system estimation

The regression is a system of multiple equations estimated via Seemingly Unrelated Regressions procedure because of shared terms between the equations for each expenditure category.

Additivity restrictions also must be imposed:

$$\sum_{i=1}^{N} \alpha_i = 1, \quad \sum_{i=1}^{N} \beta_i = 0, \quad \sum_{i=1}^{N} \lambda_i = 0$$

$$\sum_{i=1}^{N} \gamma_{ij} = 0, \quad \forall j = 1, \ldots, N$$

Slutsky symmetry also must be imposed:

$$\gamma_{ij} = \gamma_{ji}, \quad \forall i \neq j = 1, \ldots, N.$$

Data source and variables

In order to specify the parameters influencing household purchasing behaviour, we have:

- Value E of purchases, or expenditure share weight *w*, for all relevant consumption categories.
- Either P or Q since $E = P \times Q$ – so we use CPI price indexes at the national, provincial and city level. We have prince indexes and comparative level of P across geography.
- Socio-demographic characteristics that also determine consumption behaviour (i.e. age, geographic location)

Data source: Survey of Household Spending Diary, 2010 to 2012, monthly data source. 20551 households, t = 0 to 35 corresponding to 36 months. Annualized expenditures. Scope: All consumer expenditure in-scope for the Canadian CPI, at the level of 175 basic classes (ideally).

We ended up with only 140 expenditure categories, excluding Shelter. 152 including Shelter. Not enough detail to reach 175 basic class level.

We have to assume a two stage budgeting framework:
1. Shelter. 2. Everything else.

LAIDS and QAIDS – setting fixed utility level

QAIDS Cost function:

$$\ln c(u,p) = \ln a(p) + \frac{(\ln u) b(p)}{1-(\ln u)\lambda(p)}$$

$$\ln c(u,p) = \alpha_0 + \sum_{i=1}^{N} \alpha_i \ln p_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \gamma_{ij} \ln p_i \ln p_j + \frac{(\ln u)\prod_{i=1}^{N} p_i^{\beta_i}}{1-(\ln u)\sum_{i=1}^{N} \lambda_i \ln p_i}$$

Isolate the utility level in QAIDS so that we can maintain it fixed for a constant-utility index.

$$\ln(u) = \frac{\ln\left[m/a(p)\right]}{b(p)+\lambda(p)\ln\left[m/a(p)\right]}$$

LAIDS Cost function:

$$\ln c(u,p) == \ln a(p) + u\beta_0 \prod_{i=1}^{N} p_i^{\beta_i}$$

$$\ln c(\mathrm{p},\mathrm{u}) = \alpha_0 + \sum_{i=1}^{N} \alpha_i \ln p_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \gamma_{ij} \ln p_i \ln p_j + u\beta_0 \prod_{i=1}^{N} p_i^{\beta_i}$$

We noticed that for LAIDS, all we can have is the product of the two unknowns:

$$u\beta_0 = \frac{\ln\left[m/a(p)\right]}{b(p)}$$

Model estimation and elasticities

- Estimating socio-demographic and income effects is rather straightforward, regardless of how equation system is set up.
- Precise and reliable estimates of price substitution effects is rather tricky, and requires a lot of observed price and consumption variation (likely need to add more years of data, like 2013 and 2014).
- Compositional differences (with 95% C.I.s) and income elasticity estimates provided in Appendix.
- The estimated socio-demographic and seasonal effects appear rather good and verifiable (i.e. with other data sources).
- There are, for each population group and for each model, 75x140=10500 fixed/sociodemographic parameter estimates to report, as well as 9870 price interactions and 140 income effect parameter estimates.

Estimated parameters – example of sociodemographic effects

Sociodemographic variables' Q elasticity for MOTOR VEHICLE FUEL

Explanatory variables	Q Elasticity
per65over	-0.16
per25to64	NS
per15to24	NS
per4to14	NS
per0to3	EXCL
renters	-0.06
Mortgage-free homeowners	NS
Mortgage-paying homeowners	EXCL
Cars density	0.58
Females as % of household	-0.29
jan	NS
feb	0.10
mar	NS
apr	0.11
may	0.10
jun	NS
jul	0.15
aug	0.13
sep	0.11
oct	0.11
nov	NS
dec	EXCL

Variable	Q Elasticity
TOR Suburb	0.21
Red Deer	0.28
Mtl Suburb	0.29
ON Small	0.29
QC Small	0.30
Winnipeg	0.31
BC Island	0.32
St Johns	0.33
Nanaimo	0.34
Charlottetown	0.34
BC Interior	0.35
STJOHN NB	0.39
QC Rural	0.42
Cape Breton	0.43
ON Rural	0.43
Moncton	0.43
NF Small	0.43
Kamloops	0.45
SK Other	0.46
NS Small	0.48
Brandon	0.51
NB Rural	0.54
Summerside	0.62
NB Small	0.63
NF Rural	0.67
Mb Other	0.69
NS Rural	0.73
PEI Rural	0.86

Variable	Q Elasticity
cpi_510201_Calgary	NS
cpi_510201_Edmonton	NS
cpi_510201_Gatineau	NS
cpi_510201_GrandePrairie	NS
cpi_510201_Halifax	NS
cpi_510201_Hamilton	NS
cpi_510201_Kelowna	NS
cpi_510201_lethbridge	NS
cpi_510201_London	NS
cpi_510201_MedicineHat	NS
cpi_510201_Montreal	NS
cpi_510201_Moosejaw	NS
cpi_510201_QUEBECVL	NS
cpi_510201_Regina	NS
cpi_510201_Saskatoon	NS
cpi_510201_Thunder	NS
cpi_510201_Vancouver	NS
cpi_510201_Victoria	NS
cpi_510201_Windsor	NS
cpi_510201_Ottawa	NS
Toronto	EXCL

Results – sociodemographic effects

% difference in Q purchased in response to an additional 1% proportion of female adults in the household

Fresh or frozen beef	-0.17%	Women's footwear (excl athletic)	1.53%
Other fresh/frozen meat (excl poultry)	-0.24%	Men's footwear (excl athletic)	-1.69%
Ham and bacon	-0.18%	Athletic footwear	-0.22%
Other processed meat	-0.23%	Clothing accessories	0.31%
Fresh or frozen fish	-0.15%	Watches	-0.39%
Fresh milk	-0.09%	Jewellery	0.67%
Cheese	0.13%	Clothing material and notions	1.39%
Other dairy products	0.33%	Purchase leasing of passenger vehicles	-0.29%
Bread (including rolls and buns)	-0.09%	Rental of passenger vehicles	-0.47%
Breakfast cereal and other grains	0.11%	Motor vehicle fuel	-0.30%
Apples	0.14%	Passenger vehicle parts, accessories and supplies	-0.48%
Oranges	0.22%	Passenger vehicle registration fees	-0.08%
Other fresh fruit	0.23%	Drivers' licences	-0.12%
Fruit juices	-0.09%	Parking fees	0.16%
Tomatoes	0.17%	Rail bus and other intercity transport	0.32%
Lettuce	0.14%	Non-prescribed medicines	0.30%
Other fresh vegetables	0.20%	Eye care goods	0.33%
Canned vegetables/other vegetable	0.11%	Eye care services	0.30%
Confectionery	0.26%	Personal soap	0.33%
Tea	0.39%	Toiletry items and cosmetics	0.75%
Soup	-0.20%	Oral-hygiene products	0.21%
Pre-cooked frozen food preparations	-0.13%	Other personal care supplies/equip	0.28%
Non-alcoholic beverages	-0.21%	Personal care services	0.86%
Food in restaurants	-0.21%	Sporting and athletic equipment	-0.57%
Telephone services	0.14%	Toys, games (excluding video games) and hobby supplies	0.24%
Postal and other communication services	0.58%	Recreational services	-0.24%
Detergents and soaps - household	0.14%	Purchase of recreational vehicles	-1.09%
Other household cleaning products	0.27%	Fuel, parts and accessories for recreational vehicles	-1.44%
Paper supplies	0.44%	Insurance, licences and other services for rec vehicles	-0.93%
Plastic and foil supplies	0.28%	Audio equipment	-0.68%
Pet food and supplies	0.48%	Video equipment	-0.35%
Seeds, plants and cut flowers	0.42%	Traveller accommodation	-0.14%
Other household services	0.86%	Travel tours	0.24%
Financial services	-0.16%	Cablevision and satellite services (including pay per view)	0.13%
Furniture	0.14%	Use of recreational facilities and services	-0.22%
Household textiles	0.32%	School textbooks and supplies	0.41%
Non-electrical kitchen utensils, table+cookware	0.29%	Newspapers	0.65%
Household tools (including lawn, garden, snow)	-0.49%	Magazines and periodicals	0.37%
Other household equipment	0.28%	Alcohol consumed outside of the home	-0.87%
Women's clothing	1.51%	Alcohol consumed at home	-0.63%
Men's clothing	-1.86%	Tobacco	-0.41%
		Electronics	-0.22%

Results – Price effects

Price elasticities in the food sector, QAIDS and LAIDS

Price elasticity between	QAIDS-All Households	LAIDS Lowest quintile	LAIDS Seniors only	LAIDS Not seniors only
BEEF - BREAD (INCL ROLLS AND BUNS)	-1.45	-2.1	-1.81	-1.21
Beef – Potatoes	-0.43	N/S	-0.53	-0.44

So far, approx. 1.5% of price effects are significant at the 1% level, 6% at the 5% level and just over 10% at the 10% level.
We suspect that we still have some spurious effects in the gamma matrix, which may disappear by adding more years of data to the estimation.

Results – Price effects

Price elasticities in the transportation sector, QAIDS and LAIDS

Price elasticity between	QAIDS-All Households	LAIDS Lowest quintile	LAIDS Seniors only	LAIDS Not seniors only
PURCHASE LEASING OF PASSENGER VEHICLES / PARKING FEES	0.24			0.27
PURCHASE LEASING OF PASSENGER VEHICLES / AIR TRANSPORTATION	0.39			0.52
RENTAL OF PASSENGER VEHICLES / AIR TRANSPORTATION	-3.51			-6.18
MOTOR VEHICLE FUEL – MOTOR VEHICLE FUEL	-0.31	-0.11		-0.12
MOTOR VEHICLE FUEL – VEHICLE REGISTRATION FEES	N/S	-0.31		
MOTOR VEHICLE FUEL - PASSENGER VEHICLE INSURANCE PREMIUMS	0.24			0.29
MOTOR VEHICLE FUEL TRAVELLER ACCOMMODATION	-0.21			-0.22

Applying the LAIDS Cost function

The constant utility index is a ratio of the exp of the LAIDS cost function, under different price regimes and keeping utility constant. We separate the function into 3 terms and imagine baseline scenarios.

$$\ln c(\mathrm{p},\mathrm{u}) = \alpha_0 + \sum_{i=1}^{N} \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \ln p_i \ln p_j + u\beta_0 \prod_{i=1}^{N} p_i^{\beta_i}$$

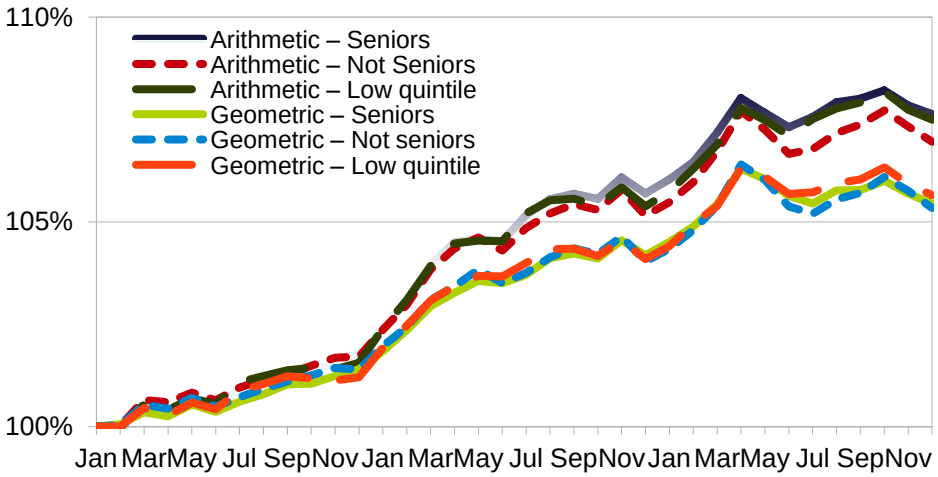
Index comparisons 1

Formula and compositional effects

$$\ln c(p,u) = \alpha_0 + \sum_{i=1}^{N} \alpha_i \ln p_i$$

Chart 5A – Arithmetic Fixed basket and geometric (fixed-w) indexes

Newfoundland price regimes, Jan 2010 to Dec 2012



If there is only perfect own-price elasticity, and no demographic or income effects, the LAIDS (Translog) Constant-Utility index is just a geometric average.

This geometric index is 24% to 27% lower than the arithmetic fixed-basket counterpart depending on the provincial price regime.

It is always lower than an arithmetic counterpart due to Jensen's (1906) inequality.

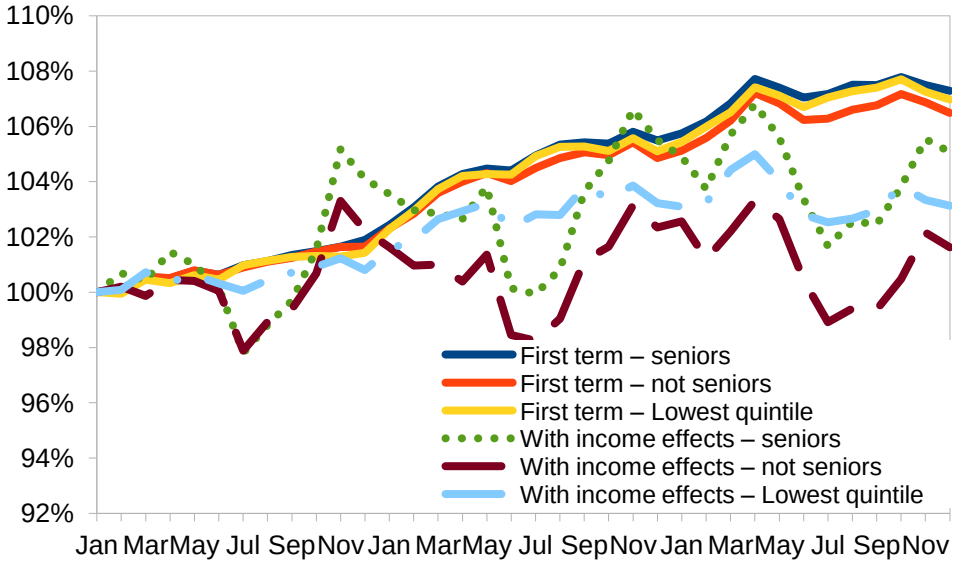
Difference between unweighted geometric and arithmetic indexes is explained by the difference in the variances of logarithm of prices between time periods (Silver and Heravi, 2004, Fenwick and Roe, 2004).

Economic interpretation: This is the difference between a fixed basket index and an index that provides for own-elasticities of -1 for all goods (Translog functional form).

Index comparisons 2 – Income effects

$$\ln c(p,u) = \alpha_0 + \sum_{i=1}^{N} \alpha_i \ln p_i + u\beta_0 \prod_{i=1}^{N} p_i^{\beta_i}$$

Newfoundland price regimes, 2010 to 2012



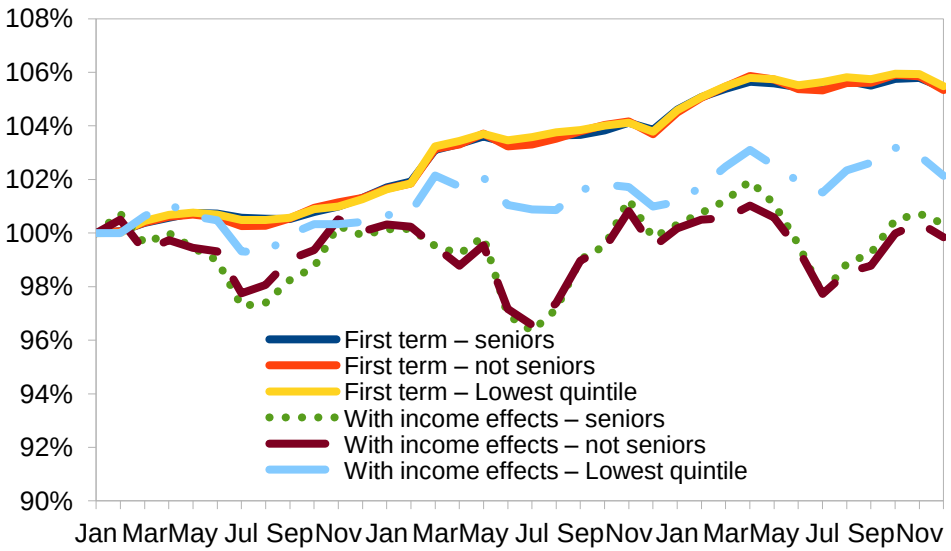
Removing the homotheticity assumption, we apply the β terms estimated, which appear quite reasonable.

We must use a level of $u\beta_0$ that corresponds to the group in question. Seniors and non-seniors get a very similar level, but lowest-quintile households will experience comparatively smaller income effects.

Income effects appear to have a pronounced downward impact on the index, much more so than compositional effects.

Index comparisons 2 – Income effects

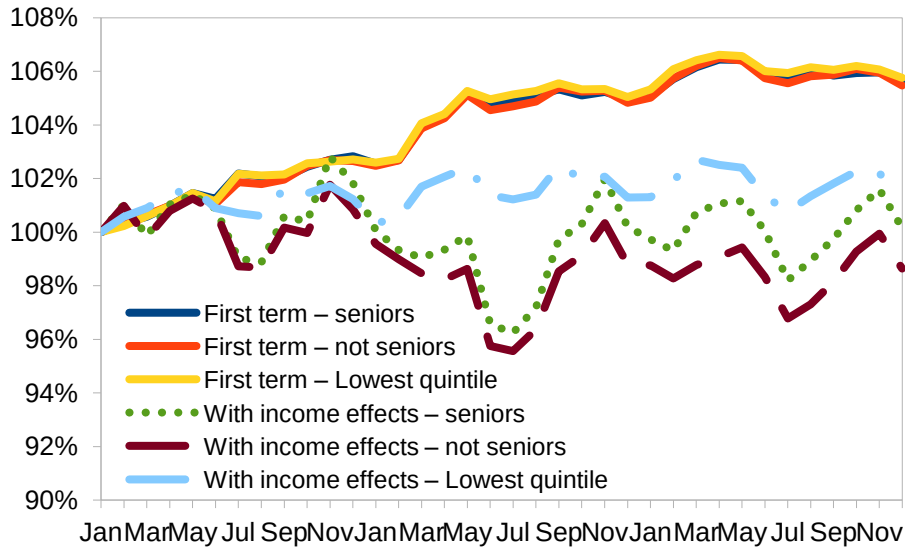
Quebec price regimes, 2010 to 2012



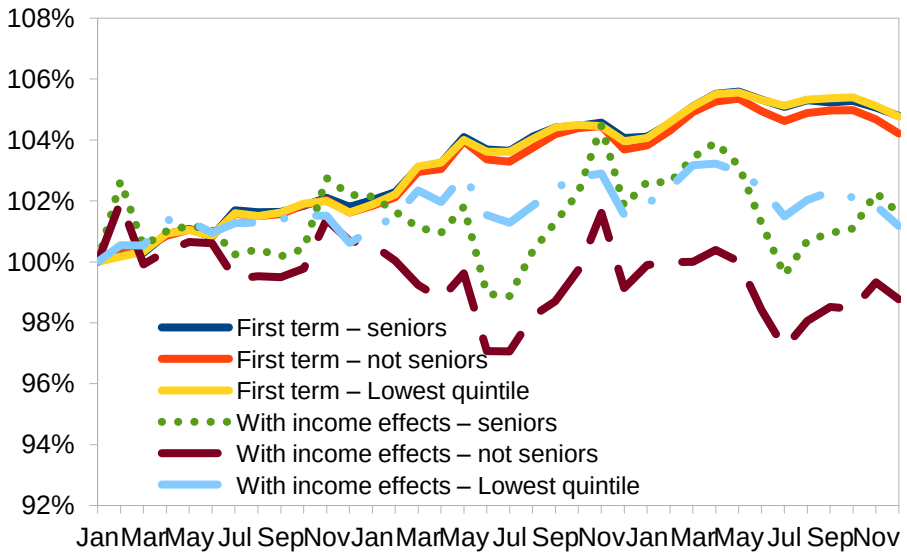
The evolution of prices in Quebec seems equally beneficial for the constant-utility index of all groups in society, but not as much for lowest-income households, since the utility level that their index is evaluated at is lower and thus diminishes income effects.

Index comparisons 2 – Income effects

Ontario price regimes



B.C. Price regimes



Income effects and their impact

The LAIDS functional form provides for an easy interpretation of income effects. The effects of superior expenditure categories are the opposite of the effects of less-than superior categories, depending on the price trajectory (increase or decrease).

Relationship between income effects, price changes and constant-utility index

	Income Elasticity Less than superior good (< 1)	Income Elasticity Superior good (>1)
Price decrease	Higher constant utility index	Lower constant utility index
Price increase	Lower constant utility index	Higher constant utility index

LAIDS cost function (again):

$$\ln c(u,p) = \ln a(p) + u\beta_0 \prod_{i=1}^{N} p_i^{\beta_i}$$

The $u\beta_0$ term is fixed, for each group in society, representing a reference utility level.

Let us envision this cost function in two different price regimes for a bilateral comparison.

Contributors to change – NF Prices

This period is generally condusive to lower constant-utility indexes for all groups in society, but not at all time periods.

It is possible that seniors-only households did not benefit as much as the rest from lower prices of electronics, or (somewhat paradoxically), higher prices of motor vehicle fuel.

Seniors-only, main contributors to income effects

	$p_{i,t=35}^{\beta_i} - p_{i,t=0}^{\beta_i}$	Income elasticity	$p_{i,t=35}$
CABLEVISION AND SATELLITE SERVICES	-0.0019	-0.13	112.8
PURCHASE / LEASING OF PASSENGER VEHICLES	-0.0016	2.87	98.5
MOTOR VEHICLE FUEL	-0.0014	0.77	114.7
PASSENGER VEHICLE INSURANCE PREMIUMS	-0.0014	0.24	103.0
BREAD (INCLUDING ROLLS AND BUNS)	-0.0012	0.16	128.9
HOUSEKEEPING SERVICES	0.0005	3.25	112.2
PASSENGER VEHICLE MAINTENANCE AND REPAIR SERVICES	0.0009	1.78	111.7
FINANCIAL SERVICES	0.0009	1.48	124.3
TRAVEL TOURS	0.0021	1.84	137.4
PRESCRIBED MEDICINES	0.0034	-0.38	76.4

Not seniors-only, main contributors to income effects

	$p_{i,t=35}^{\beta_i} - p_{i,t=0}^{\beta_i}$	Income elasticity	$p_{i,t=35}$
MOTOR VEHICLE FUEL	-0.0023	0.64	114.7
PURCHASE/ LEASING OF PASSENGER VEHICLES	-0.0014	2.53	98.5
PASSENGER VEHICLE INSURANCE PREMIUMS	-0.0012	0.34	103.0
CABLEVISION AND SATELLITE SERVICES	-0.0011	0.32	112.8
Electronics	-0.0010	1.48	77.7
PASSENGER VEHICLE MAINTENANCE AND REPAIR SERVICES	0.0005	1.38	111.7
FOOD IN RESTAURANTS	0.0005	1.13	109.4
RECREATIONAL SERVICES	0.0006	1.26	157.0
TRAVEL TOURS	0.0008	1.34	137.4
PRESCRIBED MEDICINES	0.0015	0.40	76.4

Contributors to change in income effects NF Prices

Lowest income quintile, main contributors to income effects

	$p_{i,t=35}^{\beta_i} - p_{i,t=0}^{\beta_i}$	Income elasticity	$p_{i,t=35}$
CABLEVISION AND SATELLITE SERVICES (INCLUDING PAY PER VIEW TELEVISION)	-0.0020	0.24	112.8
Electronics	-0.0019	2.28	77.7
VIDEO EQUIPMENT	-0.0014	1.88	66.8
PASSENGER VEHICLE INSURANCE PREMIUMS	-0.0013	0.40	103.0
TELEPHONE SERVICES	-0.0012	0.34	104.8
AIR TRANSPORTATION	0.0004	1.48	108.5
NON-PRESCRIBED MEDICINES	0.0004	1.44	111.1
RECREATIONAL SERVICES	0.0006	1.61	157.0
TRAVEL TOURS	0.0010	1.76	137.4
FOOD IN RESTAURANTS	0.0014	1.50	109.4

Price term (middle term) of LAIDS Cost Function

Example of a 3x3 matrix of price substitution terms

	i = 1	i = 2	i = 3
i = 1 (w = 0.045)	-0.3	$\gamma_{1,2}$	$\gamma_{1,3}$
i = 2 (w = 0.55)	$\gamma_{1,2}$	$\gamma_{2,2}$	$\gamma_{2,3}$
i = 3 (w = 0.405)	$\gamma_{1,3}$	$\gamma_{2,3}$	$\gamma_{3,3}$

Full matrix is 140x140.

We can either:

- apply the current estimated LAIDS matrix, of which 90% to 98.5% of terms are not significant.
- Take all values as estimated, and Allow them to vary within a given confidence interval in Monte Carlo simulations
- We should be able to assess the potential of price substitution effects to impact the final index results

Price term (middle term) of LAIDS Cost Function

So far, simulations have indicated that price substitution effects for gasoline are 10 to 90 times smaller in magnitude than compositional effects at various levels of own-price elasticity of gasoline. This is compared to the baseline case of perfect own-price elasticity.

It appears that this is inevitable, because of the construction of the formula. Price substitution deviation from the perfectly own-price elastic case will always be rather small compared to the income effects' deviation from the homothetic case.

	Simul 2	Simul 3	Simul 4	Simul 5	Simul 6
Own-price elasticity gasoline	-0.11	-0.33	-0.56	-0.78	-0.89
Own-price elasticity all else	-1.04	-1.03	-1.02	-1.01	-1.01
Difference in exp(income effects) t=35/t=0	-0.00112	-0.00112	-0.00112	-0.00112	-0.00112
Difference in exp(income effects*uB0) t=35/t=1	-0.00649	-0.00649	-0.00649	-0.00649	-0.00649
Difference in exp(price effects) t=35/t=0	0.0001	0.0001	0.00005	0.00003	0.00001
Difference in exp(compositional effects) t=35/t=0	-0.00304	-0.00304	-0.00304	-0.00304	-0.00304

Conclusions, questions, next steps

- Our impression is that demand system estimation is viable for this and other applications. The parameter results appear reliable - although most cross-price elasticities are challenging to estimate based on this time period and models used.

- It appears that by construction, there will be minimal price substitution effects (compared to the baseline case) in the LAIDS cost function, whereas income effects appear much larger than we expected. Could this be true?

- The LAIDS and QAIDS cost functions are not flexible enough in their income effects, therefore we seek to compare these results to results obtained via other functional forms. Next step: Normalized quadratic expenditure function with linear or quadratic spline, which is a fully flexible functional form (Diewert and Wales, 1993).