New Methodological Developments for the International Comparison Program

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1. Introduction

The final results for the 2005 International Comparison Program (ICP) have been released in February; for a tabulation of the results, see the World Bank (2008). The program compared the level of prices and the quantities or volumes of GDP (and its components) for 146 countries for the year 2005. International price statisticians developed Structured Product Descriptions (SPDs) for approximately 1000 products[2] and the individual countries collected price information on these products for the year 2005. The 1000 products were grouped into 155 Basic Heading (BH) categories. The price information collected in each country was then compared across countries, leading to a matrix of 155 basic heading prices by 146 countries. The precise way in which the individual product prices in each BH category were aggregated into a single country price for each BH heading is the topic which will be investigated in sections 2 and 3 below.

The 2005 ICP differed from previous ICP rounds.[3] In previous rounds, each country attempted to find prices in their country for a common product list. However, it is difficult to find products that are representative for *all* countries in the world and so the decision was made to break up the world into 6 regions and price statisticians developed *separate* product lists for each region. The 6 regions were: (1) Africa with 48 participating countries; (2) South America with 10 countries; (3) Asia Pacific with 23 countries; (4) The Commonwealth of Independent States (CIS) with 10 countries; (5) West Asia with 11 countries and (6) the OECD and other European countries covered by Eurostat plus Israel and Russia adding up to 46 countries in this region. This sums to 148 countries but Egypt appears in both the African and West Asia regions and Russia appears in both the OECD and CIS regions so there are 146 participating countries in all.

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[2] Most of the products referred to are components of individual consumption: "There are about 830 SPDs that cover 100 Basic Headings for individual consumption. Each SPD contains price determining characteristics that will define unique products from any corner of the world." Dennis Trewin (2008; 8). For an overview of the organization and methodology used in the 2005 ICP, see Trewin's paper and the other contributions in the March 2008 issue of the *ICP Bulletin*, which was edited by Yonas Biru.

[3] For an overview of previous ICP rounds and an assessment of the current round, see Heston and Summers (2008).

The fact that the product lists in each region were allowed to be different across regions means that without further information, prices and volumes could not be compared across regions. However, the World Bank, in cooperation with other national and international statistical agencies, developed an additional product list, which was priced out by 18 selected countries across the regions. These 18 countries were called *ring countries*. The prices that were collected by the ring countries using this final product list enabled price comparisons to be made across the 6 regions. We will indicate how this was done at the Basic Heading level in section 3 below and in section 5, we will indicate how comparisons at higher levels of aggregation between regions were made.

There was another methodological innovation made in this current ICP round in addition to having regional product lists: the price parities or Purchasing Power Parities (PPPs) and relative volumes for each country were determined using information on prices and GDP expenditure shares that pertained only to countries within the given region and these parities and relative volumes were preserved in the world comparison. Thus each region was independently allowed to determine its country PPPs and volume shares and the final linking of the regional results into a global world comparison left these regional relative parities undisturbed.[4]

The final results from the 2005 International Comparison Program for the 146 participating countries are available on the World Bank website; see the World Bank (2008) for these results and explanations for various difficulties that were encountered. This publication explained the basic framework for the provision of the data as follows:

"The purchasing power parities and the derived indicators in this report are the product of a joint effort by national statistical offices, regional coordinators, and the ICP global office. PPPs cannot be computed in isolation by a single country. However, each country was responsible for submitting official estimates of 2005 gross domestic product and its components, population counts, and average exchange rates. The regional coordinators worked with the national statistical offices to review the national accounts data to ensure that they conformed to the standards of the 1993 System of National Accounts. Similar reviews were conducted for population and exchange rate data." The World Bank (2008; 2)

The World Bank noted that the data provided by China were not quite complete and that the Tables broke China into 4 separate regions:

"China submitted prices for 11 administrative areas and the urban and rural components. The World Bank and the Asian Development Bank extrapolated these 11 city prices to the national level. The China data do not include Hong Kong, Macao, and Taiwan, China." The World Bank (2008; 2).

The World Bank publication also explained how the ICP dealt with the fact that Egypt appeared in two regions (and priced out the product lists for both regions):[5]

[4] Egypt is an exception to this statement as will be explained below.
[5] The paragraph below explains how Egypt's overall PPP and share of world product can be calculated at higher levels of aggregation (when GDP expenditure information is available). Presumably, at the basic heading level, when linking the regions, the ring country methodology to be explained in section 3 below can be used without modification but there is additional information available due to the fact that Egypt priced the regional baskets for two regions. It is not clear whether this additional information was used at

"Egypt participated in both the Africa and West Asia ICP programs by providing prices for the products included in each comparison. Therefore, it was possible to compute PPPs for Egypt separately for Africa and West Asia. Both regions included Egypt results in their regional reports. Egypt appears in the global report in both regions. The results for Egypt from each region were averaged by taking the geometric mean of the PPPs, allowing Egypt to be shown in each region with the same ranking in the world comparison." The World Bank (2008; 2).

Finally, the World Bank explained how the CIS regional results were obtained:

"Russia participated in the price collection for both the CIS and OECD comparisons. As with Egypt, PPPs for Russia were computed separately for the OECD and CIS comparisons. However, the CIS region did not participate in the Ring. Therefore, following past practices the CIS region was linked to Eurostat-OECD using Russia as a link. For comparison purposes, Russia is shown in both regions in the report." The World Bank (2008; 2).

Thus since Russia is the only country that belongs to both the OECD region and the CIS region, linking the two regions at both the Basic Heading level and higher levels of aggregation can be done though Russia. The same linking strategy could have been used to link the Africa and West Asia regions using Egypt as the linking country (or bridge country using ICP parlance) but a decision was made not to do this.[6]

Our task in the present paper is to present some of the methodological details of the methods that were used to:

- Construct Basic Heading PPPs for the countries within a region (see section 2 below);
- Link the Basic Heading PPPs across the regions (section 3);
- Construct aggregate price and volume comparisons across countries within a region (section 4) and
- Link the price levels and volumes for each country within a region across the regions in a way that preserves the regional relative price and volume measures (section 5).

Thus sections 2 and 3 deal with the problems associated with the aggregation of price information at the lowest level of aggregation where information on expenditures or quantities is not available. Sections 4 and 5 deal with aggregation problems at higher levels of aggregation where expenditure information by category and country is available. It should be noted that the material to be covered in sections 2-5 below overlaps substantially with the material in the *ICP 2003-2006 Handbook*; see Hill (2007a) (2007b)

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the Basic Heading level. Presumably, this will be explained when the World Bank provides a more complete methodology paper on its website.

[6] The problems in the case of Egypt are more complicated than in the case of Russia since there were more than one ring countries in Egypt and in West Asia. Hill (2007c; 13) listed the 18 ring countries as Brazil, Cameroon, Chile, Egypt, Estonia, Hong Kong, Japan, Jordan, Kenya, Malaysia, Oman, Philippines, Senegal, Slovenia, South Africa, Sri Lanka, United Kingdom and Zambia. Thus Cameroon, Jordan, Kenya, Oman, Senegal, South Africa and Zambia join Egypt as ring countries that are present in either the African or West Asian regions.

(2007c) (2007d) (2007e). Also the material in sections 2 and 3 overlaps with Hill (2008) and the material in sections 3 and 5 overlaps substantially with Diewert (2004b).

Section 6 lists some of the methodological problems that require additional research before the next round of the ICP program, which is scheduled to take place in 2011.

Section 7 concludes.

The Appendix develops some of the axiomatic properties of the Dikhanov (1997) Iklé (1972) method for making international comparisons. This additive method was used to make comparisons between the countries in the African region but its axiomatic properties have not been widely explored.

2. The Comparison of Prices Across Countries Within a Region at the BH Level

We will discuss the three methods of linking prices across countries within a region at the Basic Heading level that were actually used by the regions in the 2005 ICP. The three methods used were:

- The Country Product Dummy (CPD) method (used by the African, Asian Pacific and West Asian regions);
- The Extended Country Product Dummy (CPRD) method (used by South America) and
- The EKS* method used by the OECD/Eurostat region.

2.1 The Country Product Dummy Method

The most widely used statistical approach to the multilateral aggregation problem at the first stage of aggregation is the *Country Product Dummy* (CPD) method for making international comparisons of prices, proposed by Robert Summers (1973). This method for making international comparisons of prices can be viewed as a very simple type of hedonic regression model where the only characteristic of the commodity is the commodity itself. The CPD method can also be viewed as an example of the stochastic approach[7] to index numbers. In this section, we will review the algebra of this method assuming that we are attempting to make an international comparison of prices between C countries over a reasonably homogeneous group of say N items.[8] In this section, we also assume that no expenditure weights are available for the price comparisons and for the sake of simplicity, we assume that exactly K outlets are sampled for each of the N

[7] See Selvanathan and Rao (1994) for examples of the stochastic approach to index number theory. A main advantage of the CPD method for comparing prices across countries over traditional index number methods is that we can obtain *standard errors* for the country price levels. This advantage of the stochastic approach to index number theory was stressed by Summers (1973) and more recently by Selvanathan and Rao (1994).

[8] Using the language of the International Comparison of Prices (ICP) project, we are making a comparison of prices at the basic heading level. In ICP 2005 project, there are 155 basic headings. Thus each region using this method would have to run 155 regressions of the type described here.

items in each of the C countries.[9] Thus there are CNK price quotes collected across all of the countries. These assumptions are not very realistic but it is useful to present this model as an introduction to more complex models.[10]

Let p_{cnk} denote the price of item n in outlet k in country c for c = 1,…,C; n = 1,…,N; k = 1,…,K. Each item n must be measured in the same quantity units across countries but the prices are collected in local currency units. The basic statistical model that is assumed is the following one:

$$(1)\ p_{cnk} = a_c b_n u_{cnk}\ ; \qquad c = 1,\ldots,C;\ n = 1,\ldots,N;\ k = 1,\ldots,K$$

where the a_c and b_n are unknown parameters to be estimated and the u_{cnk} are independently distributed error terms with means 1 and constant variances. The parameter a_c is to be interpreted as the *average level of prices* (over all items in this group of items) in country c relative to other countries and the parameter b_n is to be interpreted as the *average* (over all countries) *multiplicative premium* that item n is worth relative to an average item in this grouping of items. Thus the a_c are the basic heading country price levels that we want to determine while the b_n are item or individual product effects. The basic hypothesis is that the price of item n in country c is equal to a country price level a_c times an item commodity adjustment factor b_n times a random error that fluctuates around 1. Taking logarithms of both sides of (1) leads to the following model:

$$(2)\ y_{cnk} = \alpha_c + \beta_n + \varepsilon_{cnk}\ ; \qquad c = 1,\ldots,C;\ n = 1,\ldots,N;\ k = 1,\ldots,K$$

where $y_{cnk} \equiv \ln p_{cnk}$, $\alpha_c \equiv \ln a_c$, $\beta_n \equiv \ln b_n$ and $\varepsilon_{cnk} \equiv \ln u_{cnk}$.

The model defined by (2) is obviously a linear regression model where the independent variables are dummy variables. The least squares estimators for the α_c and β_n can be obtained by solving the following minimization problem:[11]

$$(3)\ \min_{\alpha_c,\beta_n} \{\textstyle\sum_{c=1}^{C} \sum_{n=1}^{N} \sum_{k=1}^{K} [y_{cnk} - \alpha_c - \beta_n]^2\}.$$

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[9] The case of unequal observations in each country for each commodity is discussed in Diewert (2004b) and Hill (2007a).

[10] A special case of the present model can be obtained by setting K equal to 1 and the price p_{cn1} can be set equal to the geometric mean of all of the outlet prices collected for product n in country c. The geometric mean is chosen over other methods for aggregating the outlet prices because, in the absence of weights, it seems to have the best axiomatic properties; e.g., see Diewert (2004a). (Note however, that when aggregating using geometric means, the micro prices should not approach zero). This is the "traditional" CPD model and it is discussed by Hill (2007a) and Rao (2004) in some detail. The problem with this model is that it neglects of the variability of the outlet prices *within* a country c, product n, cell. The advantage of the traditional CPD model is that the associated algebra is much simpler and hence, much easier to understand.

[11] Weighted (by expenditure shares) versions of the CPD model were considered by Prasada Rao (1990), (1995) (2001) (2002) (2004), Heston, Summers and Aten (2001), Sergueev (2001) (2003), Diewert (2004b) (2005) and Hill (2007a; 23-24).

However, it can be seen that the solution for the minimization problem (3) cannot be unique: if α_c^* for c = 1,…,C and β_n^* for n = 1,…,N solve (3), then so does $\alpha_c^* + \gamma$ for c = 1,…,C and $\beta_n^* - \gamma$ for n = 1,…,N, for any arbitrary number γ. Thus it will be necessary to impose an additional restriction or normalization on the parameters α_c and β_n in order to obtain a unique solution to the least squares minimization problem (3). Two possible normalizations are (4) or (5) below:

(4) $\alpha_1 = 0$ or $a_1 = 1$;
(5) $\sum_{c=1}^{C} \alpha_c = 0$ or $\prod_{c=1}^{C} a_c = 1$.

The normalization (4) means that country 1 is chosen as the numeraire country and the parameter a_c for c = 2,…,C is the PPP (Purchasing Power Parity) of country c relative to country 1 for the class of commodity prices that are being compared across the C countries. On the other hand, the normalization (5) treats all countries in a symmetric manner: the geometric mean of the PPP's a_c is set equal to 1.[12] In this section, we will choose to work with the normalization (5).[13]

Initially, we ignore the constraint (5) and we differentiate (3) with respect to α_c and β_n for c = 1,…,C and n = 1,…,N and set the resulting partial derivatives equal to 0. The resulting C + N equations simplify to the following equations:

(6) $\sum_{n=1}^{N} \sum_{k=1}^{K} y_{cnk} = NK\,\alpha_c + K \sum_{n=1}^{N} \beta_n$; c = 1,…,C;
(7) $\sum_{c=1}^{C} \sum_{k=1}^{K} y_{cnk} = K \sum_{c=1}^{C} \alpha_c + CK\,\beta_n$; n = 1,…,N.

If we tentatively set $\sum_{c=1}^{C} \alpha_c = 0$, then equations (7) imply the following least squares solutions for the β_n:

(8) $\beta_n^* \equiv \sum_{c=1}^{C} \sum_{k=1}^{K} y_{cnk}/CK$; n = 1,…,N.

Thus β_n^* is simply the arithmetic average of all of the log prices $y_{cnk} \equiv \ln p_{cnk}$ of item n over all countries and all outlets. Now substitute equations (8) into (6) and we obtain the following least squares solutions for the α_c:

(9) $\alpha_c^* \equiv \sum_{n=1}^{N} \sum_{k=1}^{K} y_{cnk}/NK - \sum_{n=1}^{N} \beta_n^*/N$; c = 1,…,C
$= \sum_{n=1}^{N} \sum_{k=1}^{K} y_{cnk}/NK - \sum_{c=1}^{C} \sum_{n=1}^{N} \sum_{k=1}^{K} y_{cnk}/CNK$.

[12] Note that $\prod_{c=1}^{C} a_c = 1$ is equivalent to $\prod_{c=1}^{C} a_c^{1/C} = 1$.
[13] However, if we obtain a solution to the least squares minimization problem (3) subject to the normalization (5), say $\alpha_1^*, \alpha_2^*, \ldots, \alpha_C^*, \beta_1^*, \beta_2^*, \ldots, \beta_N^*$, then the solution to (3) subject to the normalization (4) is $\alpha_1^* = 0, \alpha_2^* - \alpha_1^*, \ldots, \alpha_C^* - \alpha_1^*, \beta_1^* + \alpha_1^*, \beta_2^* + \alpha_1^*, \ldots, \beta_N^* + \alpha_1^*$. Rao (2004) works with the normalizations (4) for the special case of our model where K=1, whereas Hill (2007a) introduces an additional parameter to represent the overall logarithmic mean of the prices and then imposes the extra two normalizations $\alpha_1 = 1$ and $\beta_1 = 1$. With these extra normalizations, the overall mean price parameter becomes the mean logarithmic price for product 1 in country 1. All three methods of normalization will lead to the same relative purchasing power parities but the resulting confidence intervals for the PPP's in the three models will be somewhat different. For computing confidence intervals, the normalization (5) is the most appropriate one for ICP purposes.

Thus each α_c^* is equal to the arithmetic average of the logarithms of all item prices in country c less the global arithmetic average of the logarithms of all item prices over all countries.

We need to check that the α_c^* defined by (9) satisfy the restrictions (5):

$$(10)\ \sum_{c=1}^{C} \alpha_c^* = \sum_{c=1}^{C} \{\sum_{n=1}^{N} \sum_{k=1}^{K} y_{cnk}/NK - \sum_{d=1}^{C} \sum_{n=1}^{N} \sum_{k=1}^{K} y_{dnk}/CNK\}$$
$$= \sum_{c=1}^{C} \sum_{n=1}^{N} \sum_{k=1}^{K} y_{cnk}/NK - C \sum_{n=1}^{N} \sum_{k=1}^{K} y_{dnk}/CNK$$
$$= 0.$$

Thus (8) and (9) give the unique solution to the least squares minimization problem (3) subject to the normalization (5). Note in particular that this solution can be calculated simply by calculating various averages of log prices without having to do any complicated matrix inversions.[14]

It is of some interest to calculate the difference between any two of the log parities between say countries c and d:

$$(11)\ \alpha_c^* - \alpha_d^* = \sum_{n=1}^{N} \sum_{k=1}^{K} y_{cnk}/NK - \sum_{i=1}^{C} \sum_{n=1}^{N} \sum_{k=1}^{K} y_{ink}/CNK$$
$$- \{\sum_{n=1}^{N} \sum_{k=1}^{K} y_{dnk}/NK - \sum_{i=1}^{C} \sum_{n=1}^{N} \sum_{k=1}^{K} y_{ink}/CNK\} \quad \text{using (9) twice}$$
$$= \sum_{n=1}^{N} \sum_{k=1}^{K} y_{cnk}/NK - \sum_{n=1}^{N} \sum_{k=1}^{K} y_{dnk}/NK.$$

Using (11) and the definitions $y_{cnk} \equiv \ln p_{cnk}$, we can calculate the PPP parity between countries c and d as follows:

$$(12)\ a_c/a_d = \exp[\alpha_c^* - \alpha_d^*]$$
$$= \prod_{n=1}^{N} \prod_{k=1}^{K} p_{cnk}^{1/NK} / \prod_{n=1}^{N} \prod_{k=1}^{K} p_{dnk}^{1/NK}.$$

Thus the PPP between countries c and d can be calculated as the geometric mean of all of the country c prices divided by the geometric mean of all of the country d prices. Hence the PPP's are *transitive* in this equal sample size case so that $[a_c/a_d]\ [a_d/a_e] = [a_c/a_e]$ for any 3 countries, c, d and e.[15] Note also if we dropped some countries from the comparison, then as long as the sample of prices in the remaining countries was not altered, the PPP's in the remaining countries would remain invariant in the ratio form given by (12). This is a very useful property.

Once the least squares estimators β_n^* and α_c^* have been determined by (8) and (9) above, the sample residuals e_{cnk} can be calculated as follows:

[14] This solution is well known in the analysis of variance literature; e.g., see Rao (1965; 209-211). For additional references to the statistics literature on this type of model, see Hill (2007a).
[15] This result was obtained by Triplett and McDonald (1977) in the context of a hedonic regression model. For the case where K = 1, Ferrari, Gozzi and Riani (1996), Hill (2007a) and Rao (2004) obtained this result. Hill (2007a; 8) calls (12) a *Jevons index* following CPI Manual practice; see ILO/IMF/OECD/UNECE /Eurostat/The World Bank (2004).

(13) $e_{cnk} \equiv y_{cnk} - \alpha_c^* - \beta_n^*$; $\quad$ c = 1,…,C; n = 1,…,N; k = 1,…,K.

Standard least squares regression theory tells us that these residuals may be used in order to calculate the following unbiased estimator for the variance σ^2 of the true error terms ε_{cnk}:

(14) $\sigma^{*2} \equiv \sum_{c=1}^{C} \sum_{n=1}^{N} \sum_{k=1}^{K} e_{cnk}^2/[CNK - (C - 1 + N)]$.

Note that if all of the sample residuals e_{cnk} happen to equal 0, then the international sample of prices satisfy the following equations:

(15) $p_{cnk} = a_c^* b_n^*$; $\quad$ c = 1,…,C; n = 1,…,N; k = 1,…,K

where $a_c^* \equiv \exp[\alpha_c^*]$ for c = 2,…,C and $b_n^* \equiv \exp[\beta_n^*]$ for n = 1,…,N. Thus if all of the sample residuals e_{cnk} equal 0, then the item prices are *proportional* across the C countries in the comparison and a_c^* is the factor of proportionality for country c. In the general case where the sample residuals e_{cnk} are not all equal to 0, then σ^{*2} defined by (14) can serve as a quantitative measure of *the lack of proportionality* of the international sample of prices or as a measure of the relative *dissimilarity* of the prices.[16]

In real life applications of the CPD method for making international comparisons of prices, it is almost never the case that all items from the common list of N items can be priced in all countries in the comparison. In fact, it can happen that an item from the common list is only present in a single country. We now indicate how the above equal sample size model presented can be modified to deal with these difficulties.

We need to introduce some additional notation. For country c and item n, let K(c,n) be the number of item n price quotes that are collected in country c. Define the total number of item n price quotes that are collected across all C countries as K(0,n); i.e.:

(16) $K(0,n) \equiv K(1,n) + K(2,n) + \ldots + K(C,n)$; $\quad$ n = 1,…,N.

Define the total number of price quotes collected in country c over all items and outlets as K(c,0); i.e.:

(17) $K(c,0) \equiv K(c,1) + K(c,2) + \ldots + K(c,N)$; $\quad$ c = 1,…,C.

For any c,n, it can happen that K(c,n) = 0, which means that no item n prices were collected in country c. However, we assume that row and column totals, K(0,n) and K(c,0), are all positive so that the price of item n is collected in at least one country and

[16] If we want to bound the dissimilarity measure between 0 (minimum dissimilarity) and 1 (maximum dissimilarity), then we could use the measure $\sigma^{*2}/[1 + \sigma^{*2}]$. Diewert (2002) took an axiomatic approach to measures of relative price dissimilarity but considered only the case of two countries. For the case C = 2, Allen and Diewert (1981) suggested the sum of squared sample residuals (which is (14) times a constant) as a measure of nonproportionality of two price vectors.

each country collects at least one item price. The total number of item prices collected over all countries is K and this total can be obtained by summing the K(0,n) over n or the K(c,0) over c; i.e., we have:

(18) $K \equiv \sum_{c=1}^{C} \sum_{n=1}^{N} K(c,n) = \sum_{c=1}^{C} K(c,0) = \sum_{n=1}^{N} K(0,n)$.

The following linear regression model is a counterpart to the equal sample size model (2) presented in the previous section:

(19) $y_{cnk} = \alpha_c + \beta_n + \varepsilon_{cnk}$; $\qquad c = 1,\ldots,C;\ n = 1,\ldots,N;\ k = 1,\ldots,K(c,n)$

where $y_{cnk} \equiv \ln p_{cnk}$ as before, the α_c and β_n are parameters to be estimated and the ε_{cnk} are independently distributed error terms with means 0 and variances σ^2. If for any c and n, K(c,n) = 0 so that there are no item n prices collected in country c, then the corresponding equations in (19) are dropped.

The least squares estimators for the α_c and β_n can be obtained by solving the following minimization problem:

(20) $\min_{\alpha_c,\beta_n} \{\sum_{c=1}^{C} \sum_{n=1}^{N} \sum_{k=1}^{K(c,n)} [y_{cnk} - \alpha_c - \beta_n]^2\}$.

As in the previous section, the parameters α_c and β_n cannot be uniquely identified so we will choose to set the purchasing power parity of country 1, $a_1 \equiv \exp[\alpha_1]$, equal to 1, which implies the following normalization on the parameters appearing in (23):

(21) $\alpha_1 = 0$.

After substituting (21) into (20), we can differentiate (20) with respect to $\alpha_2, \alpha_3, \ldots, \alpha_C$ and set the resulting partial derivatives equal to 0. The resulting C − 1 equations simplify to the following equations:[17]

(22) $K(c,0)\alpha_c + \sum_{n=1}^{N} K(c,n)\beta_n = \sum_{n=1}^{N} \sum_{k=1}^{K(c,n)} y_{cnk}$; $\qquad c = 2,3,\ldots,C$.

Now differentiate (20) with respect to $\beta_1,\ldots,\beta_N$ and set the resulting partial derivatives equal to 0. The resulting N equations simplify to the following equations:[18]

(23) $\sum_{c=2}^{C} K(c,n)\alpha_c + K(0,n)\beta_n = \sum_{c=1}^{C} \sum_{k=1}^{K(c,n)} y_{cnk}$; $\qquad n = 1,\ldots,N$.

The country PPPs (relative to country 1), $\alpha_2,\alpha_3,\ldots,\alpha_C$, and the product premium factors, $\beta_1.\beta_2,\ldots,\beta_N$, are the solution to equations (22) and (23). For additional analysis of this unequal sample size model, see Diewert (2004b) and Hill (2007a) (2008).

[17] If K(c,n) = 0, then the corresponding y_{cnk} terms on the right hand side of (22) are omitted.
[18] If K(c,n) = 0, then the corresponding y_{cnk} terms on the right hand side of (23) are omitted.

We turn now to an analysis of the method used by South America to form the Basic Heading PPPs between the countries in their region.

2.2 The Extended Country Product Dummy Method

Cuthbert and Cuthbert (1988; 57) introduced an interesting generalization of the Country Product Dummy method that can be used if information on *representativity* of the prices is collected by the countries in the comparison project along with the prices themselves. Hill (2007a) (2008) explains this method in some detail and he called the method the *extended CPD Method* or *CPDR Method* and he justified the method as follows:

"The reason for distinguishing between representative and unrepresentative products is that the relative prices of representative products in a country may be expected to be low compared with relative prices of the same products in countries in which they are not representative. Conversely, of course, the relative prices of unrepresentative products will tend to be high. This will tend to happen as result of normal substitution effects. Products will tend to be purchased in relatively large (small) quantities precisely because their relative prices are low (high). This conclusion is not merely a theoretical deduction, as there is ample empirical evidence of the substitution effect at work in both inter-temporal and inter-national comparisons." Peter Hill (2007a; 3).

"The expected price depends on the interaction of three factors: the country, the product and its representativity. Given that the coefficient of a representative product is fixed at unity, the coefficient of an unrepresentative product may be expected to be greater than unity. The price of product is expected to be higher relatively to the reference product 1 in a country in which it is unrepresentative than in a country in which it is representative. The improvement over the traditional CPD method comes from the partial relaxation of the unrealistic assumption that the pattern of relative prices is the same in all countries. ...
The addition of the new variable, representativity, does not simply add another parameter to be estimated. It adds another dimension to the analysis. As there are three types of explanatory variables in the regression -- country, product and representativity -- the extended regression will described as the CPRD method to distinguish it from the traditional CPD method." Peter Hill (2007a; 26).

The basic idea is that representative products in a country should tend to be lower in price (and hence they should be more popular) compared to unrepresentative products; thus representativity becomes a price determining characteristic of the commodity.

The CPRD method generalizes the model (19) above as follows. Define $y_{cnkr} = \ln p_{cnkr}$ where p_{cnkr} is the logarithm of the kth outlet price collected in country c for product n and r is an index that denotes whether the collected price is representative (in which case r = 1) or unrepresentative (in which case r = 2). The basic (unweighted) statistical model that is assumed is the following one:

(24) $y_{cnkr} = \alpha_c + \beta_n + \delta_r + \varepsilon_{cnkr}$; $\quad c = 1,\ldots,C;\ n = 1,\ldots,N;\ k = 1,\ldots,K(c,n)\ ;\ r = 1,2$

where the α_c are the log country PPP's, the β_n are the log product price effects and the δ_r are the two log representativity effects and the ε_{cnkr} are independently distributed random variables with mean zero and constant variances. In order to identify the parameters, we impose the following normalizations:

(25) $\alpha_1 = 0$; $\delta_1 = 0$.

Thus the present model is much the same as the model presented in section 2 except that we have an analysis of variance model that has 3 classifications instead of 2. For additional discussion of this model, the reader is referred to Cuthbert and Cuthbert (1988), Diewert (2004b) and Hill (2007a) (2008).

We agree with Hill in endorsing the method in theory. However, in practice, it seems it was difficult for national price statisticians to agree on a workable definition of representativity that was uniform across countries. Thus in the end, it appears that only the South American region used this method to construct its 155 by 10 matrix of PPP's by Basic Heading and country.

We now turn to a discussion of the OECD/Eurostat method used to compare country prices at the lowest level of commodity aggregation for countries within a region.

2.3 The EKS* Method

This method is explained by Hill as follows:

"Eurostat abandoned EKS 1 in 1982 and replaced it by the method described in the present section, which will be called the asterisk method or EKS*. A detailed exposition of EKS* and its properties is given by Sergey Sergeev (2003). The EKS* method is so called because it makes use of the distinction between representative and unrepresentative products, the representative products being identified in the product lists by an *. The EKS* method recognizes, and exploits, the fact that, as already explained, the prices of representative products are likely to be relatively low, whereas the prices of unrepresentative products are likely to be relatively high. The method proceeds by calculating two separate Jevons indices for each pair of countries. One Jevons index covers products that are representative in the first country, treated here as the base country. The other covers products that are representative in the second country. Of course, some products may be representative in both countries and included in both indices. The two indices may be described as *Jevons 1* and *Jevons 2* respectively." Peter Hill (2007a; 9).

Thus two bilateral Jevons type indexes are calculated for any two countries. Jevons 1 (2) compares only the price relatives of products that are representative in country 1 (2). The final bilateral index of prices between the two countries under consideration is a geometric mean of the two Jevons indexes.[19] Once all of these bilateral parities have been constructed over each pair of countries in the region, they can be harmonized by using the EKS procedure.[20] Thus let $P_{j\,/\,i}$ denote the geometric mean of the two Jevons indexes of the prices of country j relative to the prices of country i and suppose that there are C countries in the region. The final EKS parity between country j relative to country k, $P_{EKS\,j\,/\,k}$, is defined as follows:

[19] Note that prices which are not representative in both countries but are collected in both countries do not appear in the final bilateral index of prices between the two countries. This means that the EKS* procedure is not fully efficient in a statistical sense, whereas the CPRD procedure is fully efficient.

[20] The EKS method is explained in more detail by Balk (1996), Diewert (1999) and Hill (2007a) (2008). The method is due to Gini (1924) (1931) and independently rediscovered by Eltetö and Köves (1964) and Szulc(1964).

(26) $P_{EKS\, j/k} \equiv \prod_{i=1}^{C} [P_{j/i} / P_{k/i}]^{1/C}$; j = 1,...,C ; k = 1,...,C.

Hill sums up the properties of the method as follows:

"Provided the direct PPPs satisfy the country reversal test, the EKS PPP can be interpreted as the geometric mean of the direct PPP between *j* and *k* and all *C-2* indirect PPPs, the direct PPP carrying twice the weight of the indirect PPPs. The EKS formula may be derived by minimizing the sum of the squares of the logarithmic differences between the original intransitive PPPs and the transformed transitive PPPs. The EKS PPPs constitute the set of transitive PPPs that are closest to the original intransitive PPPs. The formula has been widely used and is extensively discussed in the literature." Peter Hill (2007a; 6).

A majority of the members of the Technical Advisory Group who provided advice to ICP 2005 favoured the CPRD method described in the previous section over the EKS* method described in this section for two reasons:

- The CPRD method used all of the available price information whereas EKS* did not and
- The CPRD method gave straightforward measures of the statistical precision of the estimated parities.

However, it appears that Eurostat price statisticians are locked into the EKS* method by legislation and thus the OECD/Eurostat region stuck by its EKS* method in the current ECP round. More research is required in order to determine how much difference there would be between CPRD and EKS*.[21]

Having described the methods used to construct PPPs for the 155 basic headings for each country in a region, we now consider how to link these PPPs across regions.

3. The Comparison of Prices Across Regions at the Basic Heading Level

As noted in the introduction, a group of *ring countries* collected prices from a common list and this price information was used to link the regional basic heading prices across the 6 regions. However, since the CIS region was locked into the OECD/Eurostat region, in practice, there were only 5 regions to link, with the CIS, OECD and Eurostat countries forming a single region.

The methodology used to link basic heading prices across regions was developed by Diewert (2004b; 36-39) and we review that methodology here.[22] The model is basically an adaptation of the unweighted CPD model presented in section 2.1.

In order to set the stage for what was actually done in linking the regions, we first generalize the CPD model presented in section 2.1 to allow for a reorganization of the list of C countries into 5 regions and C(r) ring countries in each region r. Thus C(r) is not

[21] There is also the related research question as to how reliable or consistent are the representativity designations across countries.

[22] The basic methodology is also described in Hill (2007d). However, Hill uses somewhat different normalizations than (28) and (29).

the total number of countries in region r; it is only the number of ring countries in each region because only the ring countries collected data on prices from a common international product list. With these changes, the basic model becomes:

(27) $p_{rcnk} \approx a_r b_{rc} c_n$; $r = 1,\ldots,5$; $c = 1,\ldots,C(r)$; $n = 1,\ldots,N$; $k = K(r,c,n)$;
(28) $a_1 = 1$;
(29) $b_{r1} = 1$; $r = 1,\ldots,5$.

The normalization (28) means that we have to choose a numeraire region. The normalizations (29) mean that within each region, we need to choose a numeraire country in order to identify all of the parameters uniquely. Thus the parameters a_r and b_{rc} replace our initial model parameters a_c. Note that the total number of parameters remains unchanged when we group all of the countries in the comparison into regions and countries within the regions.

Taking logarithms of both sides of (27) and then adding error terms ε_{rcnk} (with means 0) leads to the following regression model:

(30) $\ln p_{rcnk} = \ln a_r + \ln b_{rc} + \ln c_n + \varepsilon_{rcnk}$; $r = 1,\ldots,5$; $c = 1,\ldots,C(r)$; $n = 1,\ldots,N$; $k = K(r,c,n)$;
$= \alpha_r + \beta_{rc} + \gamma_n + \varepsilon_{rcnk}$

where we impose the following normalizations on the parameters in order to uniquely identify them:

(31) $\alpha_1 = 0$;
(32) $\beta_{r1} = 0$; $r = 1,\ldots,5$

where $\alpha_r \equiv \ln a_r$, $\beta_{rc} \equiv \ln b_{rc}$, $\gamma_n \equiv \ln c_n$.

If all of the data collected for each regional comparison could be pooled and if there are product overlaps between the regions, then there will be 155 regressions of the form (30) to run, one for each basic heading category. In the above model, the interregional log parities (the α_r) are estimated along with the within region country log parities (the β_{rc}) and the product log price premiums (the γ_n). Call this the *first approach* to estimating the regional parities for each basic heading. It uses all of the available information in making comparisons between all of the countries.

However, the above one big regression approach (for each basic heading) is *not consistent* with approaches that used only the regional data to determine the within region parities, the β_{rc} parameters, holding r fixed. But a principle of the current ICP methodology was that regions should be allowed to determine their own parities, independently of other regions. However, the regression model (30) can be modified to deal with this problem. If the regional log parities β_{rc} are known, then the term β_{rc} (which is equal to $\ln b_{rc}$) can be subtracted from both sides of (30), leading to the following regression model:

(33) $\ln p_{rcnk} - \ln b_{rc} = \ln a_r + \ln c_n + \varepsilon_{rcnk}$; $r = 1,\ldots,5$; $c = 1,\ldots,C(r)$; $n = 1,\ldots,N$; $k = K(r,c,n)$

or

(34) $\ln [p_{rcnk}/b_{rc}] = \alpha_r + \gamma_n + \varepsilon_{rcnk}$;

where the normalization (28) still holds. Thus if the within region parities are known, then prices in each region p_{rcnk} can be divided by the appropriate regional parity for that country in that region b_{rc}, and these regionally adjusted prices can be used as inputs into the usual CPD model that has now only the regional log parities α_r and the commodity adjustment factors γ_n as unknown parameters to be estimated.[23] Call the model defined by (31) and (34) the *second approach* to estimating the regional parities for each basic heading. This second approach respects the within region parities that have been constructed by the regional price administrators. It is this second approach that was used in ICP 2005.[24]

We now turn our attention to the problems associated with aggregating up the basic heading PPP information (along with country expenditure information) in order to form aggregate country price and volume comparisons within a region.

4. Aggregate Price and Volume Comparisons Across Countries Within a Region

Once the 155 BH price parities for each of the K countries in a region have been constructed, aggregate measures of country prices and relative volumes can be constructed using a wide variety of multilateral comparison methods that have been suggested over the years. These aggregate comparisons assume that in addition to BH price parities for each country, national statisticians have provided country expenditures (in their home currencies) for each of the 155 BH categories for the reference year 2005. Then the 155 by K matrices of Basic Heading price parities and country expenditures are used to form average price levels across all commodities and relative volume shares for each country.

There are a large number of methods that can be used to construct these aggregate Purchasing Power Parities and relative country volumes and Hill (2007b) surveys the main methods that have been used in previous rounds of the ICP and other methods that might be used.[25] Basically, only two multilateral methods have been used in previous rounds:

- The Gini-EKS method based on Fisher (1922) ideal bilateral indexes and
- The Geary (1958) Khamis (1972) method, which is an additive method.

[23] Thus we have saved 144 degrees of freedom in this model compared to our previous example where we had 625 observations and 249 parameters to estimate.
[24] Yuri Dikhanov at the World Bank carried out the computations for the global linking.
[25] For additional methods, see Balk (1996), R.J. Hill (1997) (1999a) (1999b) (2001) (2004) and Diewert (1999).

We will discuss each of these methods in turn as well as a new method due to Dikhanov (1997) who generalized a bilateral index number formula proposed by Iklé (1972).

4.1 The Gini EKS Method

It will be useful to introduce some notation at this point. Let N equal 155 and let K be the number of countries in the regional comparison for the reference year. Denote the regional PPP for country k and commodity category n by $p_n^k > 0$ and the corresponding expenditure (in local currency units) on commodity class n by country k in the reference year by e_n^k for n = 1,...,N and k = 1,...,K. Given this information, we can define *implicit quantity levels* y_n^k for each Basic Heading category n and for each country k as the category expenditure deflated by the corresponding commodity PPP for that country:

(35) $y_n^k \equiv e_n^k/p_n^k$; n = 1,...,N ; k = 1,...,K.

It will be useful to define *country commodity expenditure shares* s_n^k as follows:

(36) $s_n^k \equiv e_n^k/\sum_{i=1}^N e_i^k$; n = 1,...,N ; k = 1,...,K.

Now define *country vectors of BH prices* as $p^k \equiv [p_1^k,...,p_N^k]$, *country vectors of BH quantities* as $y^k \equiv [y_1^k,...,y_N^k]$, *country expenditure vectors* as $e^k \equiv [e_1^k,...,e_N^k]$ and *country expenditure share vectors* as $s^k \equiv [s_1^k,...,s_N^k]$ for k = 1,...,K.

In order to define the EKS parities $P^1, P^2, ... , P^K$, we first need to define the *Fisher* (1922) *ideal bilateral price index* P_F between country j relative to k:[26]

(37) $P_F(p^k,p^j,y^k,y^j) \equiv [p^j\cdot y^j\ p^j\cdot y^k/p^k\cdot y^j\ p^k\cdot y^k]^{1/2}$; j = 1,...,K ; k = 1,...,K.

The *aggregate PPP for country* j, P^j, is defined as follows:

(38) $P^j \equiv \prod_{k=1}^K [P_F(p^k,p^j,y^k,y^j)]^{1/K}$; j = 1,...,K.

Once the EKS P^j's have been defined by (38), the corresponding EKS *country real outputs or volumes* Y^j can be defined as the country expenditures $p^j\cdot y^j$ in the reference year divided by the corresponding EKS purchasing power parity P^j:

(39) $Y^j \equiv p^j\cdot y^j/P^j$; j = 1,...,K.

If we divide all of the P^j defined by (38) by a positive number, α say, then we can multiply all of the Y^j defined by (39) by this same α without materially changing the EKS multilateral method. If country 1 is chosen as the numeraire country in the region, then we set α equal to P^1 defined by (38) for j = 1 and then the price level P^j is interpreted as the number of units of country j's currency it takes to purchase 1 unit of country 1's

[26] Notation: $p\cdot y \equiv \sum_{n=1}^N p_n y_n$ denotes the inner product between the vectors p and y.

currency and get an equivalent amount of utility and the rescaled Y^j is interpreted as the volume of output of country j in the currency units of country 1.

It is also possible to normalize the outputs of each country in common units (the Y^k) by dividing each Y^k by the sum $\sum_{j=1}^K Y^j$ in order to express each country's real output as a fraction or share of total regional output; i.e., we can define the country k's share of regional output, S^k, as follows:[27]

$$(40)\ S^k \equiv Y^k/\sum_{j=1}^K Y^j\ ; \qquad k = 1,...,K.$$

Of course, the country shares of regional real output, the S^k, remain unchanged after rescaling the PPPs by the scalar α.

This completes our brief overview of the Gini EKS method for making multilateral comparisons.[28]

4.2 The Geary Khamis Method

The method was suggested by Geary (1958) and Khamis (1972) showed that the equations that define the method have a positive solution under certain conditions.

The GK system of equations involves K country price levels or PPPs, $P^1,...,P^K$, and N international commodity reference prices, $\pi_1,...,\pi_N$. The equations which determine these unknowns (up to a scalar multiple) are the following ones:

$$(41)\ \pi_n = \sum_{k=1}^K [y_n^k/\sum_{j=1}^K y_n^j][p_n^k/P^k]\ ; \qquad n = 1,...,N\ ;$$
$$(42)\ P^k = p^k\cdot y^k/\pi\cdot y^k\ ; \qquad k = 1,...,K$$

where $\pi \equiv [\pi_1,...,\pi_N]$ is the vector of GK regional average reference prices. It can be seen that if we have a solution to equations (41) and (42), then if we multiply all of the country parities P^k by a positive scalar λ say and divide all of the reference prices π_n by the same λ, then we obtain another solution to (41) and (42). Hence, the π_n and P^k are only determined up to a scalar multiple and we require an additional normalization such as

$$(43)\ P^1 = 1$$

in order to uniquely determine the parities. It can also be shown that only N + K − 1 of the N equations in (41) and (42) are independent. Once the parities P^k have been

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[27] There are several additional ways of expressing the Gini EKS PPP's and relative volumes; see Balk (1996) and Diewert (1999; 34-37).

[28] It should be noted that all of the multilateral methods that are described in this section can be applied to subaggregates of the 155 basic heading categories; i.e., instead of working out aggregate price and volume comparisons across all 155 commodity classifications, we could just choose to include the food categories in our list of N categories and use the multilateral method to compare aggregate food consumption across the countries in the region.

determined, the real output for country k, Y^k, can be defined as country k's *nominal value of output in domestic currency units*, $p^k \cdot y^k$, divided by its PPP, P^k; i.e., we have

(44) $Y^k = p^k \cdot y^k / P^k$; $\quad k = 1,...,K$
$\quad = \pi \cdot y^k$ $\quad$ using (42).

Finally, if we substitute equations (44) into the regional share equations (40), we find that country k's share of regional output is

(45) $S^k = \pi \cdot y^k / \pi \cdot y$ $\quad k = 1,...,K$

where the *region's total output vector* y is defined as the sum of the country output vectors; i.e., we have

(46) $y \equiv \sum_{j=1}^{K} y^j$.

Equations (44) show how convenient it is to have an additive multilateral comparison method: when country outputs are valued at the international reference prices, values are additive across both countries and commodities. However, additive multilateral methods are not really consistent with economic comparisons of utility across countries if the number of countries in the comparison is greater than two; see Diewert (1999; 48-50) on this point.[29] In addition, looking at equations (41), it can be seen that large countries will have a larger contribution to the determination of the international prices π_n and thus these international prices will be much more representative for the largest countries in the comparison as compared to the smaller ones.[30] This leads us to the next method for making multilateral comparisons: an additive method that does not suffer from this problem of big countries having undue influence in the comparison.

4.3 The Dikhanov Iklé Method

Iklé (1972) suggested a bilateral index number formula that was implicitly defined and Dikhanov (1997) generalized the bilateral formula into a multilateral method. Dikhanov's (1997; 6-7) equations that are the counterparts to the GK equations (41) and (42) are the following ones:

[29] "Figure 1.1 also illustrates the Gerschenkron effect: in the consumer theory context, countries whose price vectors are far from the 'international' or world average prices used in an additive method will have quantity shares that are biased upward. ... It can be seen that these biases are simply quantity index counterparts to the usual substitution biases encountered in the theory of the consumer price index. However, the biases will usually be much larger in the multilateral context than in the intertemporal context since relative prices and quantities will be much more variable in the former context. ... The bottom line on the discussion presented above is that the quest for an additive multilateral method with good economic properties (i.e., a lack of substitution bias) is a doomed venture: nonlinear preferences and production functions cannot be adequately approximated by linear functions. Put another way, if technology and preferences were always linear, there would be no index number problem and hundreds of papers and monographs on the subject would be superfluous!" W. Erwin Diewert (1999; 50).
[30] Dikhanov (1997; 5) made this point.

(47) $\pi_n = [\sum_{k=1}^{K} s_n^{\,k} [p_n^{\,k}/P^k]^{-1}/\sum_{j=1}^{K} s_n^{\,j}]^{-1}$ $\quad$ n = 1,...,N
$= [\sum_{j=1}^{K} s_n^{\,j}]/[\sum_{k=1}^{K} s_n^{\,k}\, P^k/p_n^{\,k}]$;
(48) $P^k = [\sum_{n=1}^{N} s_n^{\,k} [p_n^{\,k}/\pi_n]^{-1}]^{-1}$ $\quad$ k = 1,...,K.

As in the GK method, equations (47) and (48) involve the K country price levels or PPPs, $P^1,...,P^K$, and N international commodity reference prices, $\pi_1,...,\pi_N$. Equations (47) tell us that the *nth international price*, π_n, is a *share weighted harmonic mean of the country k prices for commodity n*, $p_n^{\,k}$, deflated by country k's PPP, P^k. The country k share weights for commodity n, $s_n^{\,k}$, do not sum (over countries k) to unity but when we divide $s_n^{\,k}$ by $\sum_{j=1}^{K} s_n^{\,j}$, the resulting normalized shares do sum (over countries k) to unity. Thus (47) is similar to the GK equations (41), except that now a harmonic mean of the deflated commodity n prices, $p_n^{\,k}/P^k$, is used in place of the old arithmetic mean and before country k's share of commodity n in the region, $y_n^{\,k}/\sum_{j=1}^{K} y_n^{\,j}$, was used as a weighting factor (and hence large countries had a large influence in forming these weights) but now the weights involve country expenditure weights and so each country in the region has an equal influence in forming the weighted average. Equations (48) tell us that P^k, the *PPP for country* k, P^k, is equal to a *weighted harmonic mean of the country k commodity prices*, $p_n^{\,k}$, deflated by the international price for commodity n, π_n, where we sum over commodities n instead of over countries k as in equations (47). The share weights in the harmonic means defined by (48), the $s_n^{\,k}$, of course sum to one when we sum over n, so there is no need to normalize these weights as was the case for equations (47).

It can be seen that if we have a solution to equations (47) and (48), then if we multiply all of the country parities P^k by a positive scalar λ say and divide all of the reference prices π_n by the same λ, then we obtain another solution to (47) and (48). Hence, the π_n and P^k are only determined up to a scalar multiple and we require an additional normalization such as (43).

Although the DI equations (48) do not appear to be related very closely to the corresponding GK equations (42), it can be shown that these two sets of equation are actually the same system. To see this, note that the country k expenditure share for commodity n, $s_n^{\,k}$, has the following representation:

(49) $s_n^{\,k} = p_n^{\,k} y_n^{\,k}/p^k \cdot y^k$; $\quad$ n = 1,...,N ; k = 1,...,K.

Now substitute equations (49) into equations (48) to obtain the following equations:[31]

(50) $P^k = 1/\sum_{n=1}^{N} s_n^{\,k} [p_n^{\,k}/\pi_n]^{-1}$ $\quad$ k = 1,...,K
$= 1/\sum_{n=1}^{N} [p_n^{\,k} y_n^{\,k}/p^k \cdot y^k][\pi_n/p_n^{\,k}]$
$= p^k \cdot y^k/\sum_{n=1}^{N} \pi_n y_n^{\,k}$
$= p^k \cdot y^k/\pi \cdot y^k$.

[31] Dikhanov (1997; 7) also obtains equations (50) but his derivation is not easy to follow.

Thus equations (48) are equivalent to equations (42) and the DI system is an additive system; i.e., equations (44)-(46) can be applied to the present method just as they were applied to the GK method for making international comparisons.

Equations (47) and (48) can be rewritten as the following more symmetric equations:

(51) $\sum_{k=1}^{K} s_n^k [p_n^k]^{-1}\pi_n P^k = \sum_{j=1}^{K} s_n^j$; $\quad n = 1,...,N;$
(52) $\sum_{n=1}^{N} s_n^k [p_n^k]^{-1}\pi_n P^k = \sum_{n=1}^{N} s_n^j = 1$; $\quad k = 1,...,K.$

Define the N by K matrix A which has element a_{nk} in row n and column k where

(53) $a_{nk} \equiv s_n^k [p_n^k]^{-1}$; $\quad n = 1,...,N ; k = 1,...,K.$

Define the N by K matrix S which has the country k expenditure share for commodity n, s_n^k in row n and column k. Let 1_N and 1_K be vectors of ones of dimension N and K respectively. Then equations (51) and (52) can be written in matrix form as follows:

(54) $\hat{\pi} AP = S1_K$;
(55) $\pi^T A \hat{P} = 1_N^T S$

where $\pi \equiv [\pi_1,...,\pi_N]$ is the vector of DI international prices, $P \equiv [P^1,...,P^K]$ is the vector of DI country PPPs, $\hat{\pi}$ denotes an N by N diagonal matrix with the elements of the vector π along the main diagonal and $\hat{P}$ denotes an K by K diagonal matrix with the elements of the vector P along the main diagonal. There are N equations in (54) and K equations in (55). However, examining (54) and (55), it is evident that if N+K–1 of these equations are satisfied, then the remaining equation is also satisfied. Equations (54) and (55) are a special case of the biproportional matrix fitting model due to Deming and Stephan (1940) in the statistics context and to Stone (1962) in the economics context (the RAS method). Bacharach (1970; 45) studied this model in great detail and gave rigorous conditions for the existence of a unique π, P solution set to (54), (55) and a normalization such as (43).[32]

In order to find a solution to (47) and (50), start with a P vector equal to 1_K and calculate the vector of international prices $\pi^{(1)}$ using equations (47). Now use equations (50) to calculate a new vector of parities and normalize this vector so that the first parity is one. Call this vector $P^{(1)}$ and calculate a new vector of international prices $\pi^{(2)}$ using equations (47). Now use equations (50) to calculate a new $P^{(2)}$ and so on until the process converges. Evidently, this procedure tends to converge quite rapidly.

As was mentioned in the introduction, the Dikhanov Iklé method was used by the African region in order to construct regional aggregates. Basically, this method appears to be a big improvement over the GK method and so if an additive method is required, DI appears to be "better" than GK.

[32] Dikhanov (1997; 12-13) also derived conditions for the existence and uniqueness of the solution set using a different approach.

We now turn our attention to the problem of linking the regions at higher levels of aggregation.

5. Aggregate Price and Volume Comparisons Across Regions

There are 146 countries in the ICP project and 155 basic headings. At this stage of the aggregation procedure, we assume that we have two 155 by 146 matrices of data: one matrix contains the PPPs, p_n^k, and the other contains country expenditures in each country's currency, e_n^k, so that the notation is basically the same as in the previous section but now k runs over all 146 countries instead of just the countries in a given region. At this stage, we could use any suitable multilateral method to aggregate up these data into a set of 146 country PPP's and volumes, such as the EKS or DI methods explained in the previous section. Call this *Approach 1*. However, the problem with this approach is that the multilateral method to be used would not necessarily respect the regional PPP's unless it was restricted in some manner.

Thus we consider *Approach 2*, which will link the regions, while respecting the within region overall PPP's that the regions deem best for their purposes.[33] The first step is to reorganize the countries into 5 regions (we regard the OECD/Eurostat/CIS countries as forming one region). Consider region r which has C(r) countries in it. Let p_n^{rc} denote the within region PPP for basic heading class n and country c in region r[34] and let e_n^{rc} denote the corresponding expenditure in local currency. The total regional expenditure on commodity group n in currency units of country 1 in each region, E_n^r, is defined as follows:

$$(56)\quad E_n^r \equiv p_n^{r1} \textstyle\sum_{c=1}^{C(r)} e_n^{rc}/p_n^{rc} ; \qquad r = 1,...,5 ; n = 1,...,155.$$

The corresponding regional PPPs by region and commodity, P_n^r, are defined to be the world BH parities for the numeraire country in each region:

$$(57)\quad P_n^r \equiv p_n^r ; \qquad r = 1,...,5 ; n = 1,...,155.$$

Now each region can be treated as if it were a single supercountry with supercountry expenditures and basic heading PPPs defined by (56) and (57) respectively for the 5 supercountries and any of the linking methods described in the previous section can be used to link the regions. Once the interregional price and volumes have been determined, the regional price and volume aggregates can be used to provide world wide price and volume comparisons for each individual country. This method necessarily preserves all regional relative parities. Moreover, Hill (2007e) shows that the overall procedure does not depend on the choice of numeraire countries, either within regions or between

[33] This Approach was proposed by Diewert (2004b; 45-47). It is further described in much more detail by Hill (2007e).

[34] The parities p_n^{rc} are the interregionally consistent PPP's that were linked across regions as described in section 3 above. Assuming that country 1 is the numeraire country in each region, then the p_n^{r1} are the parities that link the numeraire countries in each region.

regions; i.e., the relative country parities will be the same no matter what the choices are for the numeraire countries.

Approach 2 in conjunction with the EKS method was used to link the regions in the current ICP round; i.e., the EKS method was used to link the 5 supercountry regions.

Hill (2007e) discusses other possible methods that could be used to link the regions and these various alternative methods should part of the research agenda for the next round of comparisons.

6. Problem Areas and the Future Research Agenda

There are a number of problem areas associated with making international comparisons that require additional research and discussion before the next round of the ICP takes place:

- If a country experiences hyperinflation during the reference year, the average price concept may not be meaningful. A possible solution to this problem is to use within the year inflation rates to "discount" prices collected throughout the year to a single reference week or day.[35]
- The problem of pricing exports and imports.[36] At present, exchange rates are taken as the price of exports and imports. This is a reasonable approximation in most cases but the question is can we do anything better (that is not too costly)?
- The problem of zero expenditure categories. Looking at the equations for the DI method, it can be seen that the reciprocals of prices appear frequently and so some care must be taken to modify the equations when some prices are zero.
- The problem of negative expenditure categories. This problem arises with the net export category and the net additions to inventory category. Typically, there is not a problem provided that we do not attempt to provide PPPs for a single category that could be positive or negative across countries.[37] If it is necessary to provide PPPs across countries for such a category, the problems can be avoided by providing separate PPPs for exports and imports or for starting and finishing inventory stocks and users can difference the results.
- Inaccurate expenditure weights can cause grave difficulties. In the next ICP round, it would be very desirable to have more accurate information on expenditures by basic heading available from participating countries.
- Methodological difficulties with hard to measure areas of the accounts. There are particular problems with housing, financial services and nonmarket production.[38]

[35] See Hill (1996) for a discussion of the accounting problems when there is high inflation.
[36] See Heston and Summers (2008; 4) for a discussion of this problem.
[37] Index number theory tends to break down if a value aggregate crosses zero or is equal to zero!
[38] See Heston and Summers (2008), Giovannini (2008) and Bevacqua, Fantin, Quintslr and Ruiz (2008) for a discussion of these problems. The fact that current System of National Accounts conventions do not allow an imputed interest charge for capital that is used in the nonmarket sector tends to understate the contribution of this sector and the degree of understatement will not be constant across rich and poor countries.

These are problem areas for regular country accounts as well due to the lack of consensus on an appropriate methodology. Hopefully, international groups and academic economists interested in measurement problems will undertake additional research in these areas before the next ICP round.

- There is a very basic problem that makes international comparisons of prices and volumes very difficult and that is the lack of matching of products. The same problem occurs in the time series context due to the introduction of new products and the disappearance of "old" products but the lack of matching is much worse in the international context due to differences in tastes and big differences in the levels of development across countries, leading to very different consumption patterns. However, Structured Product Descriptions were introduced in the current ICP round and this does open up the possibility for undertaking hedonic regression exercises in the next round in order to improve the matching process. There are many problems to be addressed however, and it would be wise to undertake experimental hedonic studies well in advance of the next round.
- The fact that the ring list of commodities to be priced was almost entirely different from the regional lists means that there is the possibility of anomalies in the final results; i.e., if entirely different products are priced in the ring list, we cannot be sure the relative ring price levels really match up with the relative prices within the regions. Thus in the next ICP round, there should be at least some coordination in the determination of the ring product list with the regional product lists so that within each basic heading level, one or more products are on all of the lists.[39]
- It would be advisable to undertake some studies on alternative methods of aggregation at the higher levels of aggregation. In particular, the program of making comparisons based on the degree of similarity of the price and quantity data being compared that was initiated by Robert Hill (1999a) (1999b) (2001) (2004) seems to be sensible but users have not embraced it, perhaps due to the instability of the method. In any case, the World Bank now has a considerable data set based on the current ICP round that could be used to experiment with alternative methods of aggregation.
- Looking ahead into the more distant future, it would be desirable to integrate the ICP with the EU KLEMS project[40], which is assembling data on the producer side of the economy as opposed to the final demand side, which is the focus of the ICP. Producer data are required in order to calculate relative productivity levels across economies, a topic of great interest to policy makers.

7. Conclusion

[39] Fred Vogel in a personal communication made this point as follows: "Therefore, serious consideration should be given to integrating a core ring list with the regional lists and attempt to have all countries price at least some of the core ring items. I think we made a good decision when we decided to use a global list for equipment, construction, and government compensation because the noise from multiple lists was removed."

[40] See van Ark, Maddison and Timmer (2008) on this topic.

My overall conclusion is that the 2005 ICP round was a big success. The regions liked the idea that they could define their own list of products for international pricing and this improved the quality of the data. The new methodology to link prices across the regions using ring countries also seems to be a clear improvement over previous rounds. Finally, the use of hand held computers and the structured product description methodology led to improvements in the production of national price statistics in many cases.[41]

One issue that has not been entirely satisfactorily resolved is the issue of disclosure of the data; i.e., a great deal of effort has gone in to collecting PPPs for 155 categories for 146 countries but only data on 15 highly aggregated PPPs will be released. Why the reluctance to release the data? Probably because at lower level of aggregation, the results can be quite unreliable. Still one would think that more than 15 categories could be released.[42]

As indicated in the previous section, some challenges remain but hopefully, these problems will be addressed before the next round takes place.

Appendix: The Axiomatic Properties of the Dikhanov Iklé Multilateral System

Forthcoming!

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[41] See Trewin (2008) and Fenwick and Whitestone (2008) on the externalities created by the ICP program.
[42] Heston and Summers (2008; 5) discuss this issue.

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