Application of Age-transformation Approaches to Mortality Projection for Japan

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Introduction

For projecting future mortality in "Population Projection for Japan: 2006-2055" (NIPSSR 2007), a new "age-shifting model", which incorporates age-shifting as well as age-scaling of mortality, has been developed and used (Ishii 2008). These kinds of operations could be incorporated into a more general framework, i.e. an age-transformation approach.

This paper serves to examine and propose a novel method for the mortality projection of Japan that is an application of the age-transformation approach.

1 Two Representations of the Log Mortality Surface

In this section, we discuss two representations of the log mortality surface and define certain functions to describe the log mortality and its inverse functions.

Let $X = [0, +\infty)$ be the space of age and $T = (-\infty, +\infty)$ be the space of time. In the following discussion for modeling mortality, we will use $\mu_{x,t}$, the hazard function for exact age $x \in X$ at time $t \in T$. In this paper, we express the log hazard function of mortality as $y = \lambda_{x,t} = \log \mu_{x,t}$, where $y \in Y = (-\infty, +\infty)$ is the value of the function. Then, the set $S = \{(x, t, y) | y = \lambda_{x,t}\}$ determines a surface in $\mathbb{R}^3$, called the *log mortality surface*. This is a conventional representation of the log mortality surface. In this representation, $y = \lambda_{x,t}$ would be considered as the height from the X-T plane in $\mathbb{R}^3$.

Here, we consider another representation of the log mortality surface under a set of assumptions.

We assume that $\lambda_{x,t}$ is a smooth continuous function with respect to x and t defined on $X_0 \times T_0 = [0, \omega] \times [t_0, t_1] \subset X \times T$, where $\omega < +\infty$ is the finite maximum age for mortality models.

For the purpose of modeling *adult* mortality, we can further assume that $\lambda_{x,t}$ exhibits a strictly monotonic increase with respect to x for each t and $x > x_0(t)$. Here, $x_0(t)$ represents the lower bound of x above which $\lambda_{x,t}$ exhibits a strictly monotonic increase for each t. Then, for each t, the function $\lambda_t(x)$ defined by

$$\lambda_t : \tilde{X}_t \to Y, \quad \lambda_t(x) \underset{\text{def}}{=} \lambda_{x,t}$$

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is the injective (one to one) function of x, where $\tilde{X}_t = [x_0(t), \omega]$. Let $\tilde{Y}_t = \lambda_t(\tilde{X}_t)$, then $\lambda_t(x) : \tilde{X}_t \to \tilde{Y}_t$ has an inverse function $\nu_t(y) : \tilde{Y}_t \to \tilde{X}_t$ defined on $\tilde{Y}_t$ for each t.

Let us define Y_0 as follows:

$$Y_0 \underset{\text{def}}{=} [y_0, y_1] \quad \text{where} \quad y_0 = \sup_{t \in T_0} \min \tilde{Y}_t, \;\; y_1 = \inf_{t \in T_0} \max \tilde{Y}_t,$$

Then, we can define $\nu_{y,t} : Y_0 \times T_0 \to X_0$ by

$$\nu_{y,t} \underset{\text{def}}{=} \nu_t(y)$$

$\nu_{y,t}$ gives the *age* x at which the value of the log hazard function is equivalent to a value y at time t.

Moreover, we define the following two differential functions by time t: (1) $\rho_{x,t}$: the mortality improvement rate and (2) $\tau_{y,t}$: the force of age increase.

$$\rho_{x,t} \underset{\text{def}}{=} -\frac{\partial \lambda_{x,t}}{\partial t} = -\frac{\partial \log \mu_{x,t}}{\partial t}$$

$$\tau_{y,t} \underset{\text{def}}{=} \frac{\partial \nu_{y,t}}{\partial t}$$

2 Age-transformation

Next, we introduce an age-transformation in mortality analysis. In this paper, we define the age-transformation as follows.

Def 1. *Let $x, z \in [0, \infty)$ be coordinates for age. If we have a transformation $f_t : z \to x$, which is continuous and monotonically increasing, we call f_t as an age-transformation from x to z at time t.*

Let us consider graphical representations of the age-transformation. We use the following two representations, the graph of $x = f_t(z)$ and an "iso transformed-age map".

Here, we look at these graphs with an example of shifting age-transformation, which is defined by the following equation.

$$x = f_t(z) \underset{\text{def}}{=} max(5t + z, 0) \qquad (t = -2, -1, 0, 1, 2)$$

The relationship among x, z and t is expressed in three-dimensional space as shown in Figure 1.

One way to project this relationship onto two-dimensional space is by plotting the graph of $x = f_t(z)$ for each t on the X-Z plane. Figure 2 illustrates this graph. From this, we are able to read which age in the original coordinate (x) corresponds to the transformed one (z).

Another way to project onto two-dimensional space is to consider which ages in the original coordinate are identified by this transformation. We can express this by showing a plot $f_t(y)$ for $y = 0, 1, \cdots, 110$. We call it "iso transformed-age map". Figure 3 is the iso transformed-age map for this shifting age-transformation. The red lines shows the age 0, 10, ..., 110.

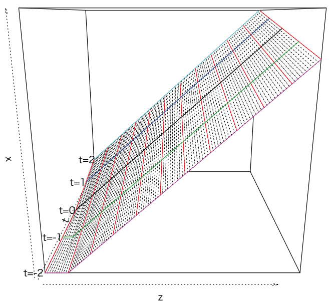


Figure 1 Relationship among x, z and t (shifting)

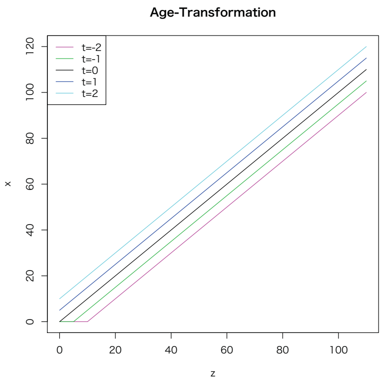


Figure 2 Age-transformation Function (shifting)

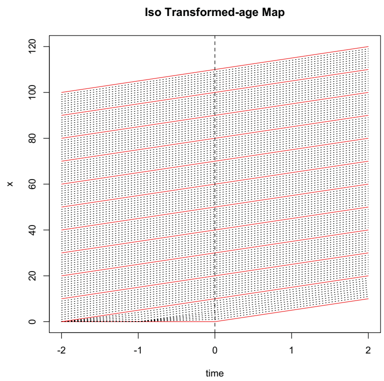


Figure 3 Iso Transformed-age Map (shifting)

3 Lee-Carter model and Age-transformation Approach

In Section 2, we introduced an age-transformation approach for mortality analysis. In this section, we review our preceding work for Japanese mortality projection that combined the Lee-Carter model with age-transformation (Ishii 2008).

The Lee-Carter model (abbreviated as LC) is expressed by the following formula (Lee and Carter 1992).

$$\lambda_{x,t} = \log \mu_{x,t} = a_x + k_t b_x$$

where a_x is a standard age pattern of mortality.

Taking a partial derivative by time t, we obtain the following relationship.

$$\rho_{x,t} = -\frac{dk_t}{dt} b_x = -k'_t b_x$$

This equation shows that the age distribution of $\rho_{x,t}$ is constant in the LC model. If we further assume that k_t is linear over time, $\rho_{x,t}$ is constant over time. Therefore, the LC model works well when the age-specific rate of mortality improvement is considered to be constant over time, that is, the mortality improvement is considered as *decline*.

Then, when does the LC model fail to express mortality improvement? To observe this point, we examine the following stylized examples.

Here, we consider two piecewise linear log mortality functions. At $t = 0$, both functions are identical: $\lambda_{x,t} = -2$ for age 0, -8 for age 25, -6 for age 50, -3 for age 75 and -1 for age 100. In Example 1, age-specific rates of improvement are constant over time. The annual rate of decline is 0.12 for age 0, 0.06 for age 25, 0.06 for age 50, 0.07 for age 75 and 0.04 for age 100.

In Example 2, age-specific rates of improvement for ages under 25 are constant and the same as in Example 1. However, for ages above 50, the mortality curve shifts to the right 3/5 years annually.

Figure 4 shows $\lambda_{x,t}$ (top figure) and $\rho_{x,t}$ (bottom figure) for Example 1. From the bottom figure, we can observe that the rates of mortality improvement are constant over time.

Figure 5 shows the same figures for Example 2. From the bottom figure, we can observe that the peak of the rates of mortality improvement is shifting to the right over time. Such mortality improvement could not be expressed by the LC model. The black line shows the rate of mortality improvement, which is equal to the b_x function under the LC model. We can observe that this line exhibits an average rates of mortality improvement for the entire period, even though no actual $\rho_{x,t}$ shows such rates of mortality improvement.

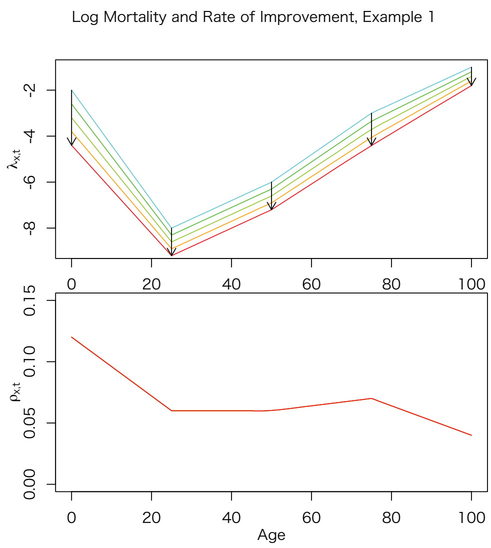


Figure 4 $\lambda_{x,t}$ and $\rho_{x,t}$, Example 1

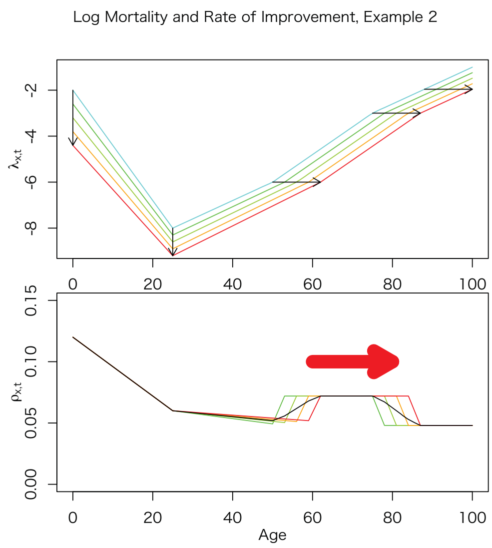


Figure 5 $\lambda_{x,t}$ and $\rho_{x,t}$, Example 2

Following these observations, we could say that use of the LC model may not be considered appropriate if the mortality improvement is considered as *shifting*. We proposed age-transformation approaches for projecting Japanese mortality rates since we observed the recent mortality improvement in Japan could be considered as *shifting*, though this point is reconsidered later.

The age-transformation approach works as follows. Let us denote the LC modeling and projecting procedure as $\mathcal{L}$; then the modeled and projected mortality $\hat{\mu}_{x,t}$ by the LC procedure would be obtained as $\mathcal{L}(\mu_{x,t})$. We proposed performing the Lee-Carter procedure after some age-transformation, and modeling and projecting the rates by inverse age-transformation, i.e., $\mathcal{A}^{-1}\mathcal{L}\mathcal{A}(\mu_{x,t})$.

[Lee-Carter model]

$$\begin{array}{c} \mu_{x,t} \\ \Big\downarrow \mathcal{L} \\ \hat{\mu}_{x,t} \end{array}$$

[Lee-Carter model with Age-transformation]

$$\begin{array}{ccc} \mu_{x,t} & \xrightarrow{\mathcal{A}} & \tilde{\mu}_{z,t} \\ & & \Big\downarrow \mathcal{L} \\ \hat{\mu}_{x,t} & \xleftarrow{\mathcal{A}^{-1}} & \hat{\tilde{\mu}}_{z,t} \end{array}$$

Here, we illustrate how the age-transformation approach will work in Example 2. Let us consider the following age-transformation: shifting mortality curves to the left $3/5t$ years for the group aged 50 and over as in the top figure in Figure 6. Then the transformed mortality rates are in the bottom figure. Figure 7 shows the age-transformed $\lambda_{x,t}$ and the rates of mortality improvement $\rho_{x,t}$. We can see that the $\rho_{x,t}$ function for the age-transformed mortality is constant over time, and thus the LC model provides a perfect fit for the age-transformed mortality rates. Therefore, we can model Example 2 using the LC model with age-transformation. This is a core structure of this approach.

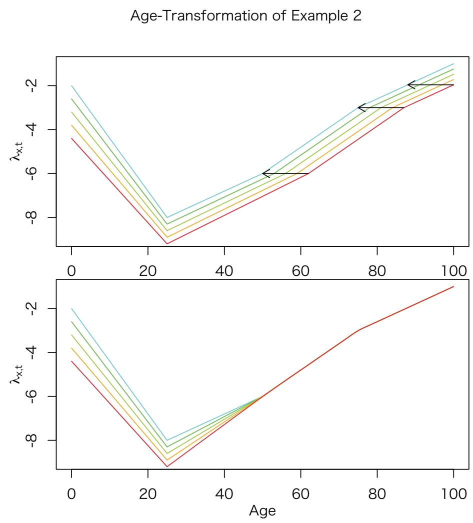


Figure 6 Age-transformation for Example 2

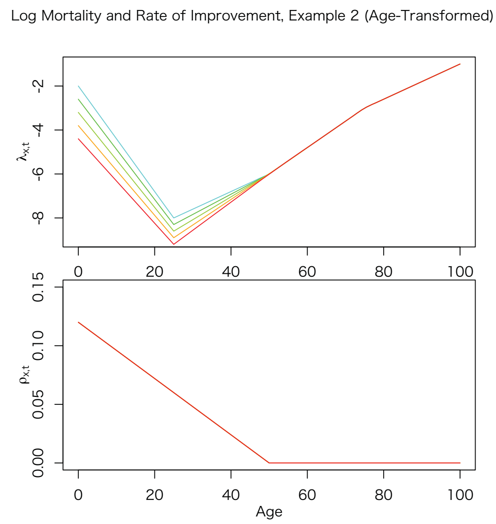


Figure 7 $\lambda_{x,t}$ and $\rho_{x,t}$, Example 2 (with Age-transformation)

In Ishii (2008), we proposed the following age-transformation $\mathcal{A}$ for entire age to express the mortality improvement as a *decline* in younger age and a *shift* in older age in order to apply the Lee-Carter procedure.

First, we fit the three parameter logistic curve

$$\mu_{x,t} = \frac{\alpha_t \exp(\beta_t x)}{1 + \alpha_t \exp(\beta_t x)} + \gamma_t$$

to the actual mortality rates. Then, we obtain the parameter $S_t = -\frac{\ln(\alpha_t)}{\beta_t}$, which is used to express the shift amount in the shifting logistic model (Bongaarts 2005), and another parameter β_t, which expresses the slope of the curve.

Next, let x be the original age and z be the transformed one, and define the relation $x = f_t(z)$ as follows.

$$f_t(z) \underset{\text{def}}{=} \begin{cases} z & (z \leq B_1) \\ \left\{ \frac{\beta_{t_0}}{\beta_t} (B_2 - S_{t_0}) + S_t - B_1 \right\} \frac{z - B_1}{B_2 - B_1} + B_1 & (B_1 \leq z \leq B_2) \\ \frac{\beta_{t_0}}{\beta_t} (z - S_{t_0}) + S_t & (B_2 \leq z) \end{cases}$$

Then set $\tilde{\mu}_{z,t} \underset{\text{def}}{=} \mu_{f_t(z),t}$.

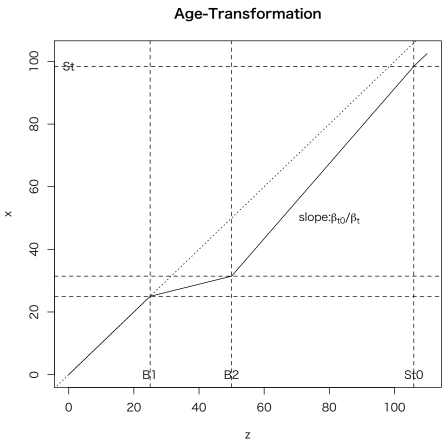


Figure 8　Age-transformation Function

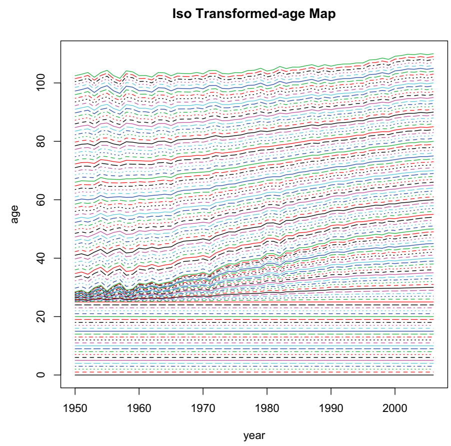


Figure 9　Iso Transformed-age Map

Figure 8 shows an example of age-transformation function, and Figure 9 shows the iso transformed-age map. Using the age-transformation $\mathcal{A}$, modeling and projecting mortality rates are performed as $\mathcal{A}^{-1}\mathcal{L}\mathcal{A}(\mu_{x,t})$.

4　Mortality Improvement: Decline or Shift?

In Section 3, we reviewed the age-transformation approach developed in Ishii (2008). For the modeling of adult mortality, the projection is based on the assumption that the mortality improvement is considered as *shifting*. It is suggested from the trends in $\mu_{x,t}$ and $l_{x,t}$ that the recent improvement in adult mortality in Japan could be better understood when considering it as *shifting*. In this section, we reconsider whether it is more plausible to understand mortality improvement in Japan as *declining* or *shifting*. First, we describe the definitions of the proportional hazard model and the Lee-Carter model, which are *decline*-type models. Then, we introduce the horizontal shifting model and the horizontal Lee-Carter model, which are *shift*-type models corresponding to the two *decline*-type ones. Through this consideration, we propose a new type of adult mortality model and discuss another way to define age-transformation.

4.1　Decline-Type Mortality Models

4.1.1　The Proportional Hazard Model (PH)

The proportional hazard model (abbreviated as PH) is a simple model that expresses mortality improvement as *decline*. In the PH model, $\lambda_{x,t}$: the log hazard rate function at time t is expressed by

$$\lambda_{x,t} = \log \mu_{x,t} = a_x + k_t$$

where a_x: the baseline logged hazard rates.

In the PH model, $\rho_{x,t}$: the rate of mortality improvement

$$\rho_{x,t} = -\frac{dk_t}{dt} = -k_t'$$

is constant with respect to age. This is the differential form for this model.

In this paper, we fit and numerically evaluate the models against the Japanese female mortality. We use

$$m_{x,t_c}, \quad x = x_s(=25), \cdots, x_e(=110) \quad \text{and} \quad t_c = t_s(=1970), \cdots, t_e(=2007)$$

from the HMD (Human Mortality Database), where t_c is a calendar year.

Here, we set a_x as the average log hazard rate in the entire period. Figure 10 shows the actual log hazard rates (λ_{x,t_c}) and the estimated rates with the PH model. We can observe that the estimated rates do not exhibit good fit particularly in the older age groups. Figure 11 shows the difference between the actual and estimated rates. From this graph, we can see that the actual values are higher than those of the model for age around from 60 to 80 in 1970, whereas these values are decreasing over time. However, opposite movement is observed for ages over 90. This is caused by the limitation of the PH model whereby the rate of mortality improvement is constant with respect to age.

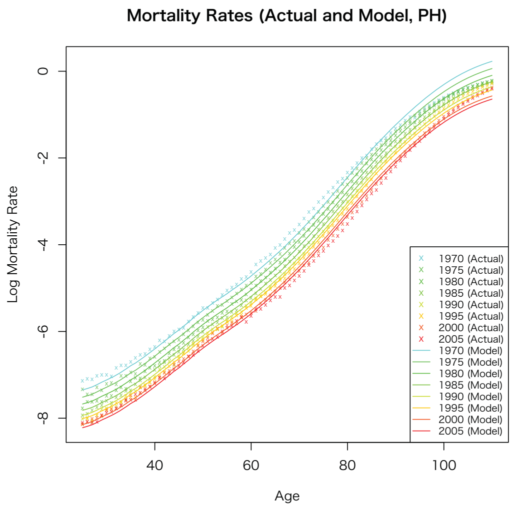


Figure 10 Mortality Rates (Actual and Model, PH)

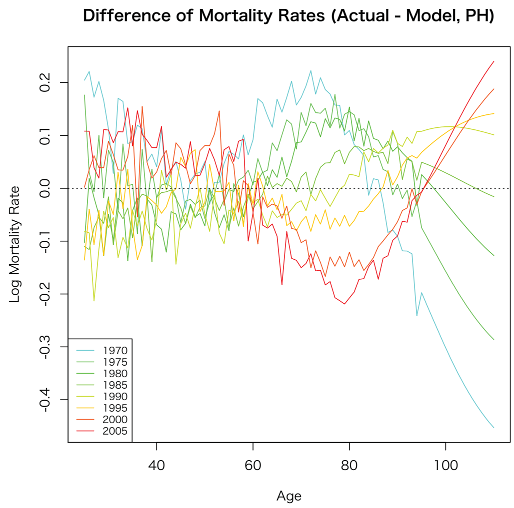


Figure 11 Difference of Mortality Rates (Actual - Model, PH)

4.1.2 The Lee-Carter Model (LC)

The LC model is already defined in Section 3. It expresses mortality improvement as *decline* in a more general manner as compared with the PH model. Here, we set a_x as the average log hazard rate for the entire period. Figure 12 shows the actual log hazard rates (λ_{x,t_c})and the estimated rates by the LC model. This figure illustrates that the fit with the actual values is fairly improved by using the LC model, due to its flexibility which admits different mortality improvement rates by age.

However, we can observe from Figure 13 that the difference between the actual and estimated rates exhibits a trend whereby the actual values are higher in younger age groups and lower in older age groups near the beginning and the end of the entire period, whereas the opposite is true around the middle of the period.

The reason why this trend for the error components is observed is ascribed to the change in the age-specific mortality improvement rates over time. Therefore, we will next examine the $\rho_{x,t}$ functions for these two models.

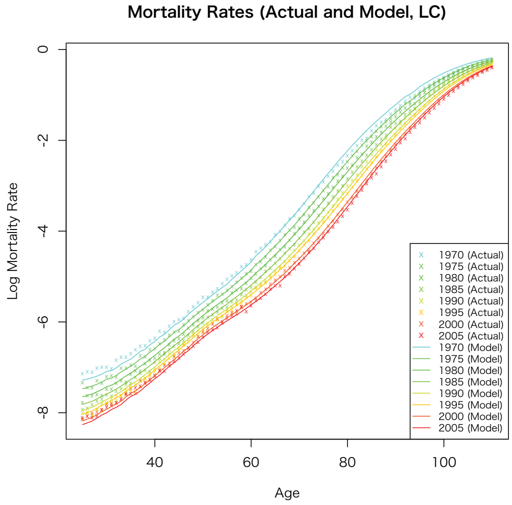


Figure 12 Mortality Rates (Actual and Model, LC)

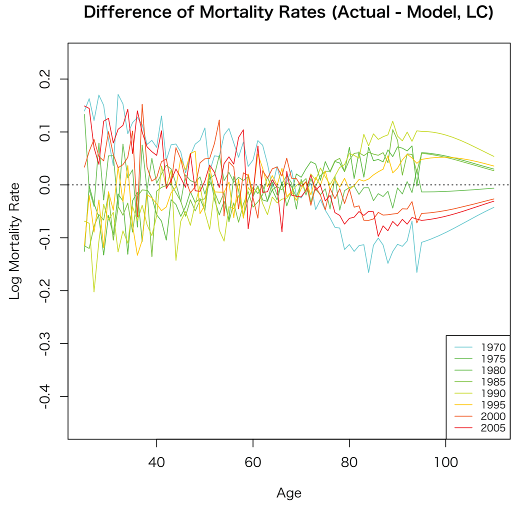


Figure 13 Difference of Mortality Rates (Actual - Model, LC)

Figures 14 and 15 show the ρ_{x,t_c} functions for the actual values and the estimated values for each of the two models. The blue lines show the ρ_{x,t_c} by the actual mortality rates. We can observe that most of the mortality improvement rates have mountain-shaped curves with peaks. In contrast, the mortality improvement rates under the PH model, expressed by the pink line, are horizontal. This difference in shape would be viewed as a cause that the estimates by the PH model are not well-fitted, as we observed before.

The mortality improvement rate by the LC model, indicated with the green curves, has a peak like that of the actual value, and this improves the fit as we have seen before. However, the age distribution of the rates is fixed in the LC model, whereas it changes dynamically in the actual values.

Thus, the actual age distribution of mortality improvement rates change over time and are not constant as in the LC model, and caused the propensity for the error in the LC model observed in Figure 13. We could see this result as a limitation when the mortality improvement is considered as *decline*.

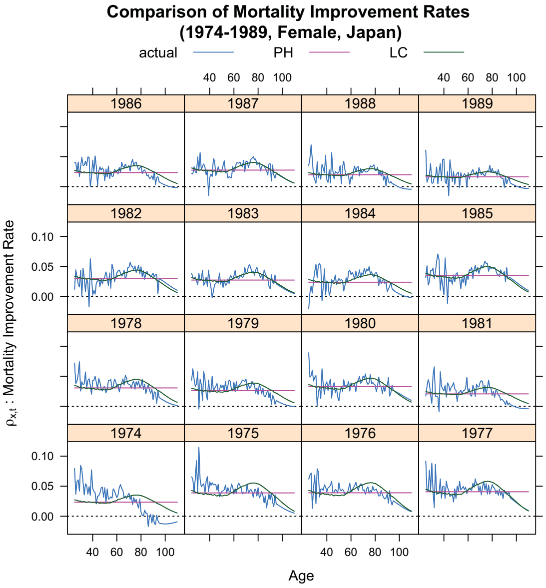


Figure 14 Comparison of Mortality Improvement Rates (1974-1989)

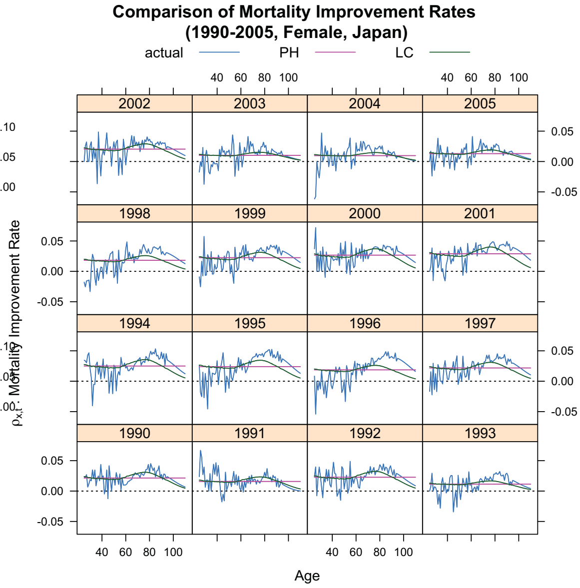


Figure 15 Comparison of Mortality Improvement Rates (1990-2005)

4.2 Shift-Type Mortality Models

4.2.1 The Horizontal Shifting Model (HS)

Next, we discuss models that express mortality improvement through a *shift*. The simplest model for *shifting* would be one whereby the entire log hazard curve moves to the right-hand side. We can restate this model using the inverse function of log hazard mortality $\nu_{y,t}$, that is, the proportional hazard model for $\nu_{y,t}$.

This model that we call the horizontal shifting model (abbreviated as HS) here is formally expressed as follows:

$$\nu_{y,t} = a_y + k_t$$

In the differential form,

$$\tau_{y,t} = \frac{dk_t}{dt} = k'_t$$

Parameter estimation for the HS model is completely identical to the PH models, except for adapting these procedures to ν_{y,t_c} instead of λ_{x,t_c}. Figures 16 and 17 are the actual inverse mortality rates (ν_{y,t_c}) and the estimated rates by the HS model, and the difference between the actual and the estimated.

We can see that the performance of fitting by the HS model is much better than by the PH model, even though both have the same structure. For 1970, indicated with the light blue line, the actual values are higher in younger ages and lower in older ages, though the errors are not as high for other years.

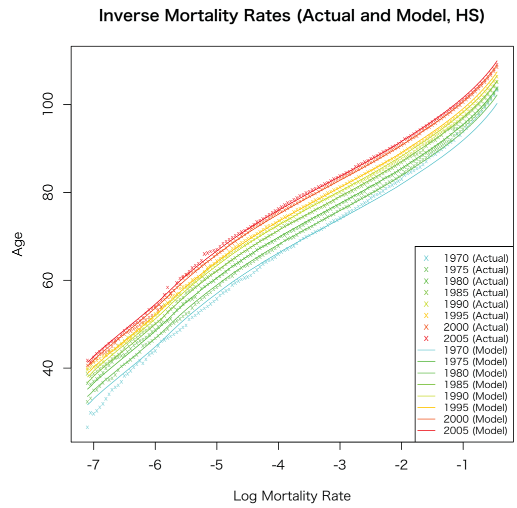


Figure 16 Inverse Mortality Rates (Actual and Model, HS)

Figure 17 Difference of Inverse Mortality Rates (Actual and Model, HS)

4.2.2 The Horizontal Lee-Carter Model (HL)

As we considered the LC model which admits a different amount of decline by age and provides a more general framework compared with the PH model, we can also consider the Lee-Carter model for $\nu_{y,t}$, which in turn supports a more general shifting feature. We call it the horizontal Lee-Carter model (abbreviated as HL).

$$\nu_{y,t} = a_y + k_t b_y$$

In the differential form,

$$\tau_{y,t} = \frac{dk_t}{dt} b_y = -k_t' b_y$$

Figures 16 and 17 are the actual inverse mortality rates (ν_{y,t_c}) and the estimated rates under the HS model, and the difference between the actual and the estimated. We can see that the HL model seems to be improved compared to the HS model. However, it is also observed that the improvement between the *shift* pair is not as large as the *decline* pair. This means that relaxing the limitation, which the force of age increase in the HS model is restricted to the constant function, does not cause significant improvement of fit in the HL model. It could be explained by the difference in the shape of τ_{y,t_c}, the force of age increase.

Figures 20 and 21 show the τ_{y,t_c} functions for the actual values and the estimated values by the two *shifting* models.

We observe that the green curves, which correspond to τ_{y,t_c} by the HL model, are close to a horizontal line, which coincides with the force of age increase by the HS model shown in the pink lines. This fact endorses that the improvement between the *shift* pair is not as large as the *decline* pair.

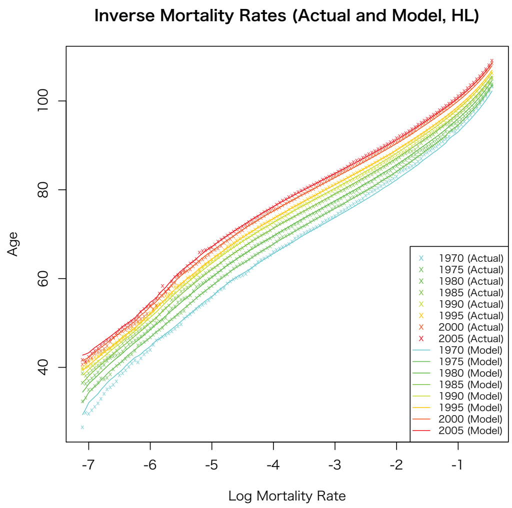
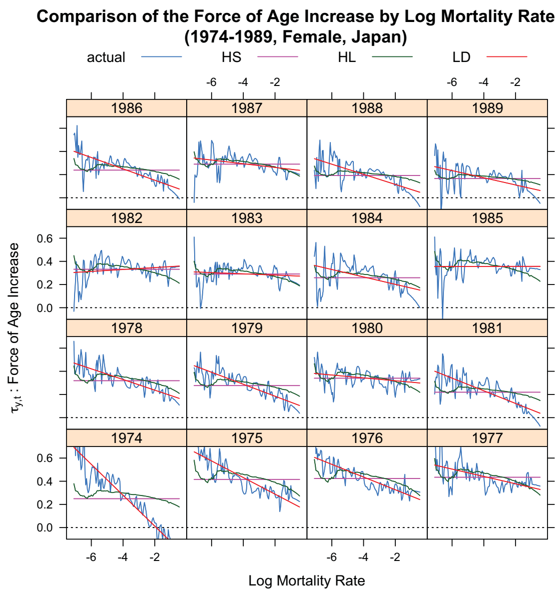


Figure 18 Inverse Mortality Rates (Actual and Model, HL)

Figure 19 Difference of Inverse Mortality Rates (Actual and Model, HL)

However, from the observation of these figures, we have noticed that the blue lines for the actual τ_{y,t_c} for each year could be more well-modeled by a linear function of y, which has led us to the development of a new model: the linear difference model. We will define and examine this new model in the next section.



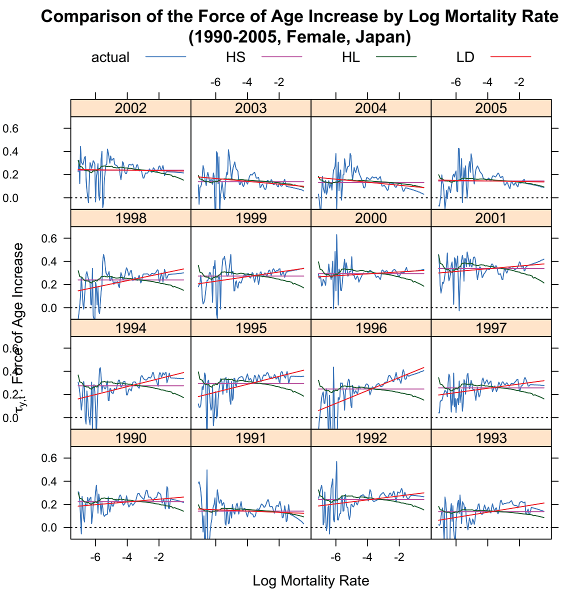


Figure 20 Comparison of the Force of Age Increase by Log Mortality Rate (1974-1989)

Figure 21 Comparison of the Force of Age Increase by Log Mortality Rate (1990-2005)

4.3 The Linear Difference Model (LD)

First, we describe the linear difference model (abbreviated as LD) in the continuous form as we did in other models. In the LD model, we assume that $\tau_{y,t}$ is a linear function of y for each t.

$$\tau_{y,t} = k'_t + c'_t y$$

This is the differential form. By integrating both sides with t, we obtain

$$\nu_{y,t} = k_t + c_t y + a_y$$

where a_y denotes a standard pattern of inverse log hazard rates.

Figures 22 and 23 are the actual inverse mortality rates and the estimated rates by the LD model, and the difference between the actual and the estimated. From these figures, we can observe that the LD model fits quite well with the actual values.

This is also confirmed from the observation of τ_{y,t_c} functions in Figures 20 and 21. We can observe that the linear assumption for τ_{y,t_c} in the LD model works better than in the other two models.

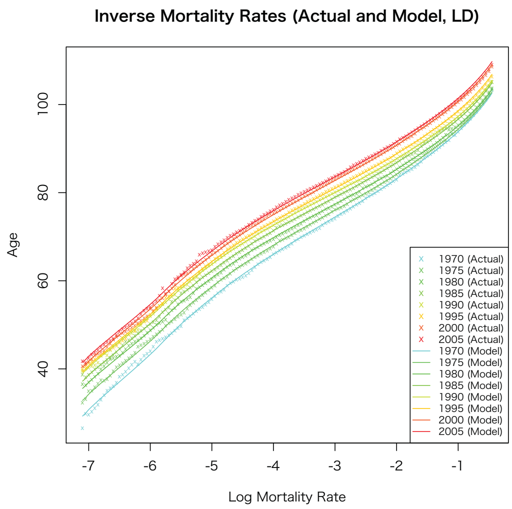


Figure 22 Inverse Mortality Rates (Actual and Model, LD)

Figure 23 Difference of Inverse Mortality Rates (Actual and Model, LD)

4.4 Comparison of the Models from a Statistical Viewpoint

In this section, we compare the LC and LD models from a statistical viewpoint to examine whether it is more plausible to understand the recent Japanese mortality as *declining* or *shifting*. Our approach is as follows.

1. The true mortality rates are assumed to be those that are estimated by models.
2. The number of deaths follows a binomial distribution $B(N_{x,t_c}, p_{x,t_c})$, where N_{x,t_c}: the number of the population and p_{x,t_c}: the death rate for age x and calendar year t_c.

3. N_{x,t_c} is approximated by the closest integer to E_{x,t_c}: exposure to risk.

Here, we took 0.01% as a critical value to construct the confidence intervals (CI), since $N_{x,t}$ would present too large value for the Japanese female population. Figure 24 shows the proportion where the log actual mortality rates are outside of the CIs for each age in the LC and LD models. This indicates that even though the proportions of LD are higher for certain ages, LD's performance would be considered as fairly better than LC's as a whole. This result suggests that *shift* is more strongly supported as recognition of the recent mortality improvement in Japan than *decline*.

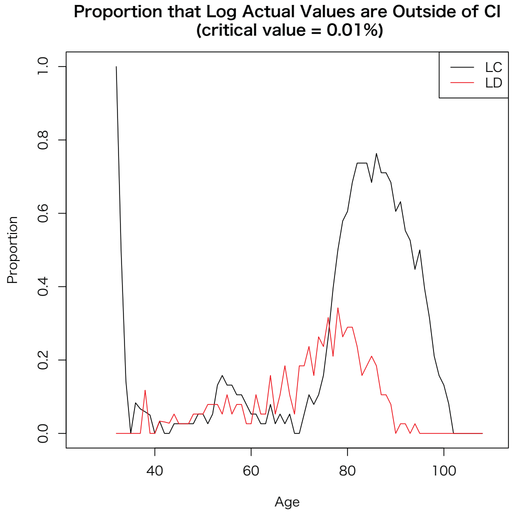


Figure 24 Proportion that Log Actual Values are Outside of CI (critical value = 0.01%)

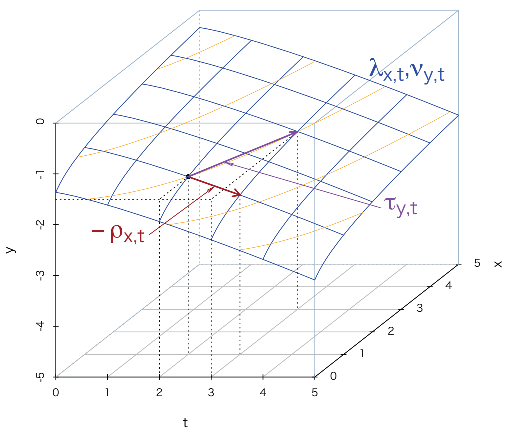


Figure 25 Log Mortality Surface and Two Differential Functions

4.5 Differential Forms and Age-transformations

Next, we consider the relationship between differential forms and age-transformations, and discuss how the LD model is related to the age-transformation approach.

In section 1, we defined $\rho_{x,t}$ and $\tau_{y,t}$ on the log mortality surface S. Then, the vectors

$$\boldsymbol{\rho}(x_0, t_0, y_0) = (0, 1, -\rho_{x_0,t_0})$$

$$\boldsymbol{\tau}(x_0, t_0, y_0) = (\tau_{y_0,t_0}, 1, 0)$$

are tangent vectors on S as shown in Figure 25. Each tangent vector defines a tangent vector field on S.

In general, an iso-transformed age map is defined by the projection of the integral curve induced by the tangent vector field onto a X-T plane. For example, the iso-transformed age map induced by $\boldsymbol{\rho}$ is an identity age-transformation, and one by $\boldsymbol{\tau}$ is an age-transformation that identifies the ages that yield the same log hazard rates. If we define another tangent vector field on S, then another iso-transformed age map is induced. Therefore, a tangent vector field on S is considered as another representation of an age-transformation.

Let us recall that the LD model is defined by a differential form that is a modeling of $\tau_{y,t}$. Therefore, the LD model defines an age-transformation through the vector field interpretation with a tangent vector $\boldsymbol{\tau}$. This relationship relates the LD model to the age-transformation approach.

Concluding Remarks

In this paper, we examined and proposed a new method for mortality projection for Japan as an application of the age-transformation approach.

We considered which is more plausible to understand mortality improvement in Japan as *decline* or *shift*. First, we described the definitions of the proportional hazard model and the Lee-Carter model, which are *decline*-type models. Then, we introduced the horizontal shifting model and the horizontal Lee-Carter model, which are *shift*-type models corresponding to the two *decline* type ones.

Next, we noticed that the actual τ_{y,t_c} for each year could be well-modeled by a linear function of y, and proposed the linear difference (LD) model. We observed that the LD model coincided quite well with the actual values.

Then, we compared the LC and LD models from a statistical viewpoint to examine whether it is more plausible to understand the recent Japanese mortality as a *decline* or *shift*. We observed that LD's performance would be considered advantageous over LC's as a whole. This result suggests that *shift* is more strongly supported as recognition of the recent mortality improvement in Japan than *decline*.

Finally, we considered the relationship between differential forms and age-transformations, and discussed how the LD model is related to the age-transformation approach. In general, an iso-transformed age map is defined by the projection of the integral curve induced by the tangent vector field onto $X - T$ plane. Therefore, a tangent vector field on S is considered as another representation of an age-transformation. The LD model is defined by a differential form that is a modeling of $\tau_{y,t}$. Therefore, the LD model defines an age-transformation through vector fields interpretation with tangent vector $\boldsymbol{\tau}$. This relationship relates the LD model to the age-transformation approach.

In this paper, we confirmed that the LD model is efficient, although we noted further points that should be examined. First, we discussed only the adult mortality model here, whereas the entire age model should be developed. Second, we focused on the modeling of the actual values in this paper, whereas we should consider how to project the parameters. These points should be studied in future.

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