WP. 16
27 September 2007

ENGLISH ONLY

UNITED NATIONS STATISTICAL COMMISSION and ECONOMIC COMMISSION FOR EUROPE

STATISTICAL OFFICE OF THE EUROPEAN COMMUNITIES (EUROSTAT)

CONFERENCE OF EUROPEAN STATISTICIANS

Joint Eurostat/UNECE Work Session on Demographic Projections
(Bucharest, 10-12 October 2007)

Agenda item 7: Household projections

PROBABILISTIC HOUSEHOLD PROJECTIONS BASED ON AN EXTENSION OF HEADSHIP RATES METHOD WITH APPLICATION TO THE CASE OF RUSSIA

Invited Paper

Submitted by Austria [1]

Summary

1. The paper presents a probabilistic method for projecting the number of households and their distribution by size. The method combines probabilistic population projection with probabilistic headship rates model. For distributing the households by size we use recently developed models for conditional proportions of households of different sizes among households of the same or bigger size. Models are approbated on the case of Russia with fertility scenario assuming considerable success of demographic policies recently introduced in the country. Parameters for household models are estimated from the 1994 microcensus sample using bootstrap procedures. Our results show significant changes in future distribution of private households in Russia. Also, despite overall decline in the number of households, they imply persisting shortage of housing infrastructure for four-person households. Typically these would be households of two parents with two kids, i.e., families put into the focus of recently introduced demographic policies.

1. Introduction

2. Household projections are important for planning purposes and also for analyzing the implications of population dynamics for consumption, labor, ecology, etc. (MacKellar, et al. 1995, O'Neill and Chen 2001, Perz 2001, Prskawetz, et.al. 2004). In some areas, like housing and urban planning, projections of distribution of households by size are of key importance, as

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The work presents results, which were in part obtained in the EU TACIS project. Grant Contract 2006/121-795.

they are relevant to decisions involving substantial long-term public and private expenditures. It is important for such applications to have better knowledge about trends expected in the future and also about *uncertainty* accompanying such trends. Understanding the uncertainty of households' prospects is important, as it is not always possible to easily and quickly adjust investment decisions to deviations from the projected trend. This issue is also important for developing demographic and taxation policies oriented on family and households' composition.

3. This work utilizes recent advances in modeling households' distribution by size and also in probabilistic projections to develop probabilistic household projections for Russia in 2005-2050. The case of Russia deserves special attention for several reasons. During the last century the country has passed through many dramatic social and economic disturbances, which imprinted deeply into the population age structure and have serious implications for demographic prospects (Ediev 2001). Almost inevitable depopulation of the country and changes in age structure may have significant and, some times, contradictive effects on prospects of households of different sizes. At the same time, the country is facing an urgent need for better planning and improving the infrastructure and living arrangements in particular. Hence the importance of understanding the prospects and uncertainty of household dynamics in future.

2. Methodology and data

4. This work is based on an extension of the conventional headship rates method (United States National Resources Panning Committee 1938, United Nations 1973). Several rationales support such choice. Firstly, these are the headship rates method and its extensions, which are widely used by governmental agencies, despite progress in more sophisticated modeling of households. Age-specific headship rates happen to be a remarkably stable indices, which vary only moderately despite significant demographic developments observed in many populations. Changes in fertility and mortality have only a limited effect on headship rates, and it is population age structure, which is a primary source of variations of households number and distribution (see the appendix for some analytical results in support for this view and implications for stable population). At older ages mortality and morbidity may play more significant role as a factor of headship rates' dynamics. Yet, this effect may be neglected in a study of the overall number of households, which this work is devoted to.

5. The need for probabilistic household forecasting has been acknowledged elsewhere (J.de Beer and M.Alders 1999[2], Leiwen and O'Neill 2004). De Beer and M.Alders forecast uncertainty in future number of households introducing a number of assumptions regarding institutional population, probability of changes in the age at leaving the parental home, assumption about the conditional probability of changes in the percentage of people living alone etc. In order to derive these assumptions a very good information base should be available. From the data available for Russia deriving such distribution would require too much of subjective judgment. Leiwen and O'Neill proposed an extension of the headship rates method introducing age- and household size-specific headship rates. The latter rates were proposed to be derived as functions of demographic indicators, such as propensity of leaving home, marriage, divorce, fertility rates, and mortality. Such an approach seems to be promising, as it is much less demanding data and model assumptions compared to micro-simulation approach and also allows addressing the role of demographic events in households' formation. In some applications, however, there might be not enough data for the model. Also, the extension to the method may require special reconciliation procedures in order to guarantee internal consistency of the

[2] Probabilistic population and household forecasts for the Netherlands Joop de Beer and Maarten Alders

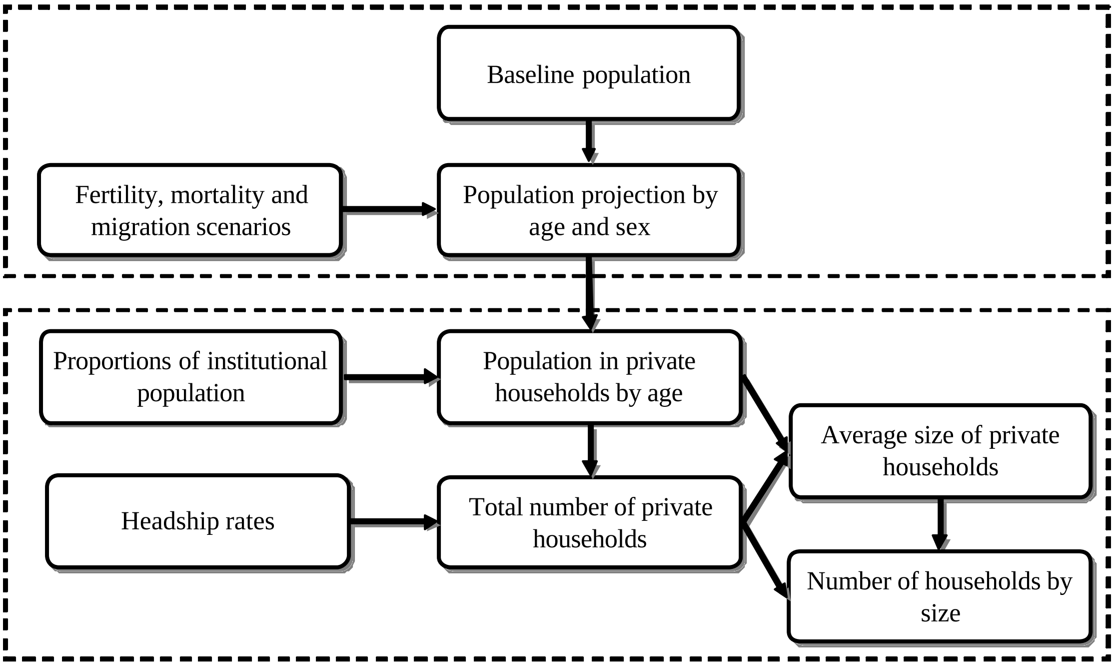
projection, which may limit its application especially in probabilistic projections. E.g., total population in private households obtained from their model distribution by size might be inconsistent with the size of actual population in private households. Another potential drawback is usage of parameters, which are quite volatile and involve non-trivial correlations between them. E.g., size/age-specific headship rates may vary considerably across time and regions, depending on prevailing fertility levels, while age-specific headship rates derived regardless household characteristics are usually much more stable, i.e., the former rates are negatively correlated. Usage of such model parameters may worsen performance and robustness of the probabilistic model and increase demand for data availability and quality.

6. Here we present an approach, which is also based on the extension of the headship rates method. The extension we use is based on deriving distribution of households by size from the overall average size of households, which, in turn, is derived from the conventional age-specific headship rates. The approach was proposed by Gisser (1986a, 1986b) and used in Austrian household projections since then. Advantageous feature of the approach is that the average size of households indirectly reflects demographic developments, even though headship rates might be insensitive to those developments. In particular, changes in fertility assumed in population projection will, in fact, affect population size and, thereby, the average size of households. Unfortunately, like many other extensions of the headship rates method, the approach may eventually result in inconsistent projections, and special reconciliation procedures are to be used, which somewhat limits its usage in probabilistic projections. E.g., sum of proportions of households of different sizes may deviate from one, or population totals obtained directly from age structure or from the distribution of households by size may differ considerably. Merits of the headship rates approach may be used to a wider extent, however, based on recent developments of models for conditional shares of households among households of the same or larger size and for average sizes of such households (Ediev 2007), see details further down in the text.

7. We use conventional age-specific headship rates (eventually, generated at random, however, as it is described below) to derive the number of households from the projected population by age and the average household size. Then we apply conditional shares approach to derive the number of households by size.

8. General scheme of household projections adopted in this paper is presented at Diagram 1. Basically, two tasks are identified: making population projections and projecting number of households by size, based on population projections. In case of probabilistic projections this sequence is repeated a given number of times.

Diagram 1 Projection of the number of households



2.1 Population projections

9. Population projections were prepared using a probabilistic approach. This approach has been already applied successfully in many instances to project population at national, macro-region and global levels (Lutz et al. 1997; Lutz and Scherbov 1998; Keilman et al. 2002; Lutz et al. 2003; Lutz and Scherbov 2004, etc.). There are mainly three approaches to probabilistic projections that are proposed in the scientific literature. The first approach is based on the time-series analysis of past vital rates, the second approach is based on the analysis of past projection errors and the third one is based on expert opinion. A good overview of these approaches is given by Bongaarts and Bulatao (2000) and H. Booth (2006). Those three approaches are not mutually exclusive but often complementary. In particular, the expert judgement is implicitly or explicitly considered in all of them. The third approach, the one actually adopted here, uses expert's opinion explicitly. The expert-based population projections were first proposed in the scientific literature by Lutz et al. (1996). Further use and development of the method can be found in Lutz et al. (1997), Lutz and Scherbov (1998), Lutz et al. (1999, 2001, 2004).

10. There are many sources of uncertainty in the future developments of fertility, mortality and migration. Recent introduction of a new demographic policy in Russia makes situation even more uncertain. The main aim of the policy was to increase the number of second births. It is not clear what would be the reaction of population to the new measures aimed at fertility stimulation. In is not clear whether the number of second births will increase in cohorts of women or simply a shift in the birth calendar will occur without essential change in the completed fertility of cohorts.

11. In our projections we assumed that population policy will bring certain positive results. We assumed that this will lead, first, to shortening of interval between 1st and 2nd birth and, second, to increase of the number of second births by 50 percent. Those assumptions result in the increase of projected mean value of period TFR to 1.5 in 2008, peak at the value of 1.76 in 2014 and declines afterwards remaining constant at the level of 1.7 starting from 2027. The range of uncertainty in 2050 covers the interval of TFR from 1.25 to 2.15 children per woman.

12. In case of life expectancy we assumed that lower end of 90 percent range corresponds to no future increase of life expectancy both in case of males and females. The upper end corresponds to growth in life expectancy of about 2 years per decade for females and 2.8 years per decade for males thus decreasing the gap that exists between life expectancy of males and females. This result in mean predicted values of life expectancy in 2050 equal to 71.3 and 81.7 years of life for males and females correspondently.

13. Mean predicted value of the number of net migrants was considered constant and equal to 126 thousands people coming annually to Russia. The range between 0 and 256 thousands people assumes to cover 90 percent of all the future outcomes of net migration.

14. In order to generate the required distributions of the future path of fertility, mortality and migration, we adopt the method used by Lutz et al. (2001).

15. The starting year of projections was 2005. The data on population, fertility, mortality and migration for this year were utilized Age specific fertility rates were preliminary smoothed using mixed Gamma distribution function. Age-specific mortality rates are smoothed using Heligman-Pollard mortality schedule. Projections were made for single year age groups and are thus carried out on a yearly basis

2.2 Deriving the number of households by size

16. Next step after making population projections is to obtain the total number of private households. To do that we apply age-specific institutional population proportions to the projected population in order to obtain population living in private households. And next, we apply age specific headship rates to population living in private households in order to get the total number of private households (Diagram 1). Proportions of the institutional population were fixed at the level observed in census 2002[3]. Headship rates are derived from 1994 micro-census data using the bootstrap method. In order to avoid biases caused by artificial geographic compositions generated in bootstrap, we pull stratified samples, pooling together regions with similar average sizes of households. Two groups of regions were defined: those containing regions with average household size below 3 and above 3 members.

17. Based on the generated number of households, the average size is calculated as the ratio of the number of households to the population in private households, and the a -method from (Ediev 2007) is applied size after size:

$$n_{k/k+} = e^{-a_k \cdot h_k}, (1)$$

where $n_{k/k+}$ is the conditional share of households with k members among households of the same or larger size, h_k is average size of such households minus k, and a_k are model

[3] Population in institutional households comprise 1.6 percent of all the population. This percentage varies across age and sexes.

parameters. The parameters a_k are obtained from regressions against average size of the households, which are also derived from the bootstrap procedure based on stratified data of 1994 micro-census.

18. Procedure starts with smallest households, i.e., one-person households, and the average size for households of the next size is obtained recurrently by subtracting the number and the population residing in the households of the preceding size.

$$h_{k+1} = \frac{N_{k+} - k \cdot H_k}{H_{k+} - H_k} - (k+1) = \frac{h_k}{1 - n_{k/k+}} - 1, \quad (2)$$

here H_k and H_{k+} are the numbers of households of size k and of the same or larger size; N_{k+} is population residing in households with k or more members.

3. Results

19. We used two different approaches in developing our projections. First approach was based on directly applying headship rates obtained from Russian microcensus. Our probabilistic projection set contained 1000 simulations. For each simulation we stored age-specific population distributions for every projected year. Then, we applied fixed age specific headship rates obtained for Russia as a whole to each population composition, thus deriving the total number of households[4]. In the next step we calculated average size of the household for each simulation and distributed the total number of households by the number of households of each size. Since we used probabilistic age-specific population distributions, we also obtained probabilistic distribution of the number of households of different size.

20. In a second approach, the algorithm of the distribution of the number of households by size was similar, except we used random headship rates. They were obtained using the bootstrap procedure described above.

21. Resultant distributions of households by size were close in these two approaches with random headship rates approach having slightly higher variance. Thus we will present results only for a case when we applied random headship rates.

3.1. Population and Households: general overview of prospects

22. First of all let us look at the probabilistic population projection for Russia. At Figure 1 we present the fractals of this distribution. As we observe from this figure, there is virtually no chance for population growth in the future. This is predefined by a very low fertility and high mortality levels. Low fertility will have also implications for the total number of households. The total number of households is also projected to decline in a long term (Figure 2). In a short term, next 10-15 years, the median number of households is going to slightly increase even though fertility level is low. After that period a steady decline is expected and by 2050 the number of households may fall to 47 million from 52.5 million in 2005. The 95% uncertainty range will spread from 41.5 to 52.4 million of households in 2050.

[4] Since we are interested in population living in a private households, we adjusted projected population with proportion of people living in private households obtained from 2002 census.

Figure 1 Probabilistic population projection.

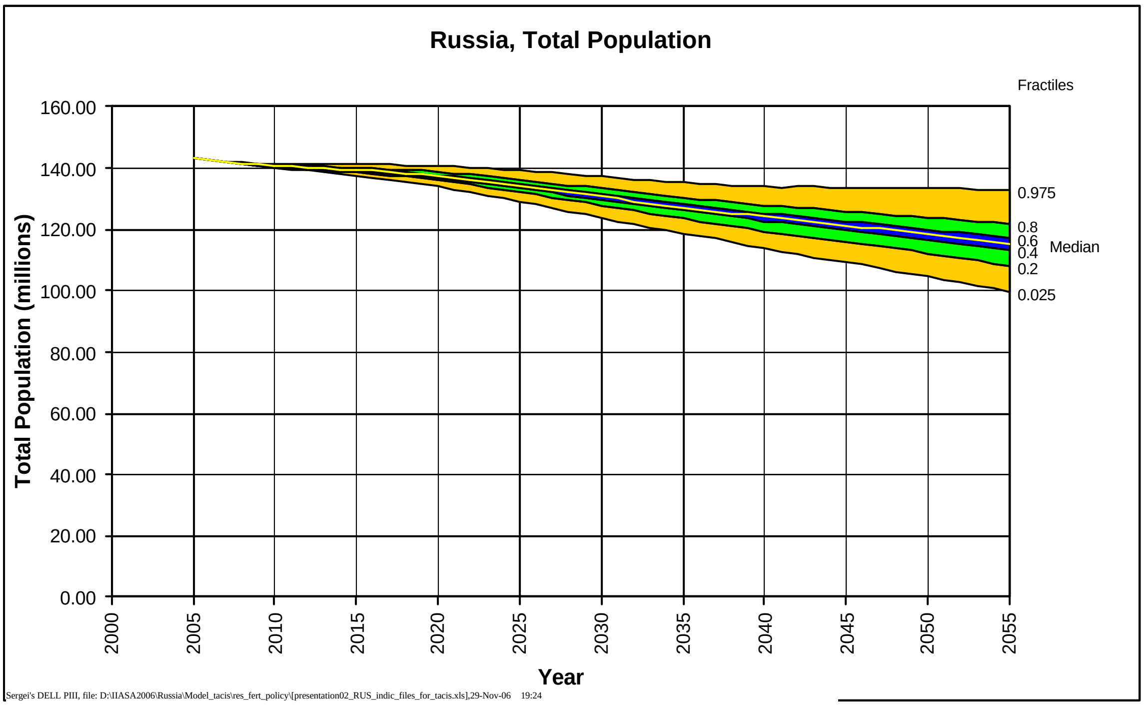
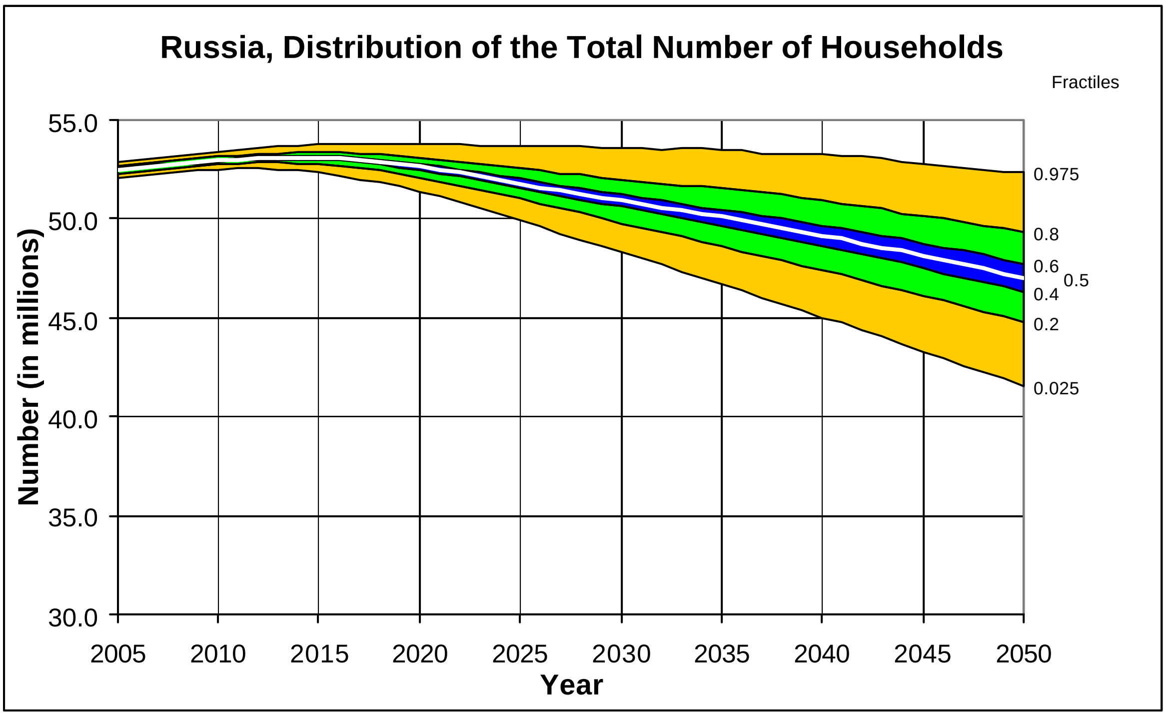
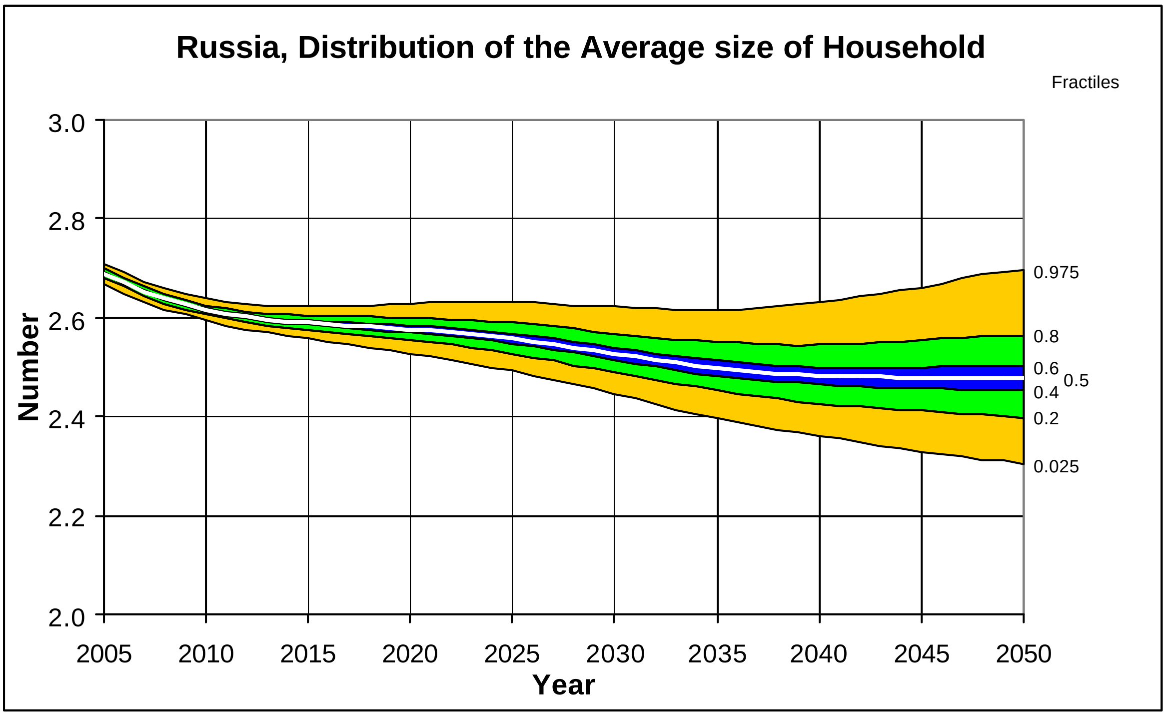


Figure 2 Distripution of the total number of households, projection 2005-2050, Russia



23. Not only the total number of households is projected to decline, but households will become smaller. (Figure 3). Median household size falls from 2.69 in 2005 to 2.5 in 2035 and after that stays almost constant. In 2035, 95% prediction interval includes households with sizes between with 2.4 and 2.6 members. In general the decline in fertility level leads to the decline of an average household size.

Figure 3 Distribution of the average household size, Russia, 2005-2005



24. Even though the total number of households is expected to decline, we may expect diverse trends if we study the dynamics of households of different sizes (figures 4-8). In the near future we may observe the rise in the number of households of size one from 11.5 million in 2005 to almost 13 million in 2035 (Figure 4). Households of size 2 and 3 show either no change or a very slight decrease in the near future with a moderate decrease by 2050 (figures 5 and 6). The strongest decline will be observed in households with four and more members (figures 7 and 8). We may expect that households of size four will decline by 20% and of size 5 by 60% by 2050. Typically that would be households consisting of two parents and two or three children.

Figure 4 Distribution of households of size one, Russia, 2005-2005

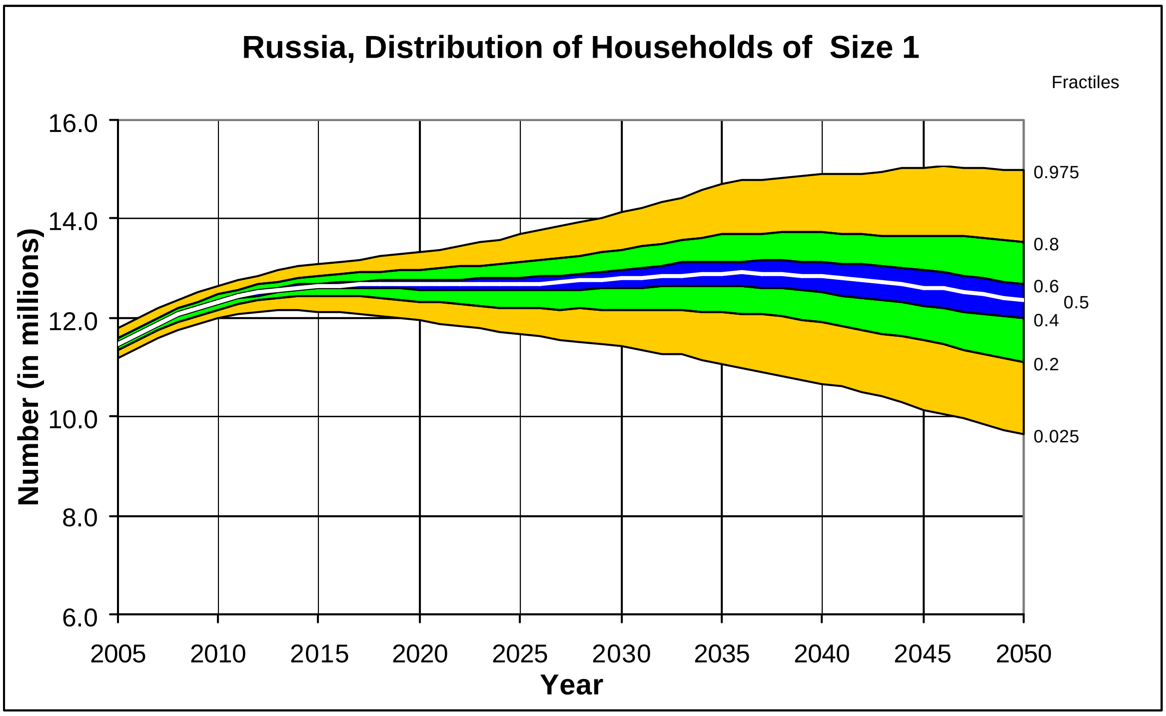


Figure 5 Distribution of households of size two, Russia, 2005-2005

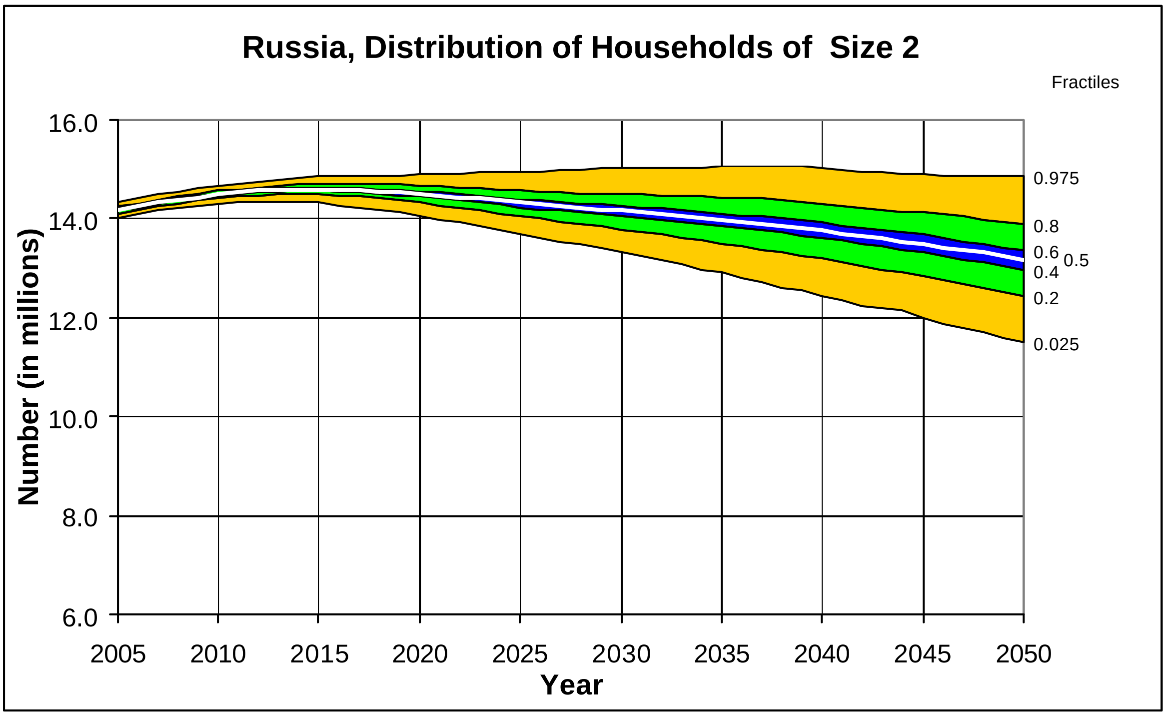


Figure 6 Distribution of households of sizethree, Russia, 2005-2005

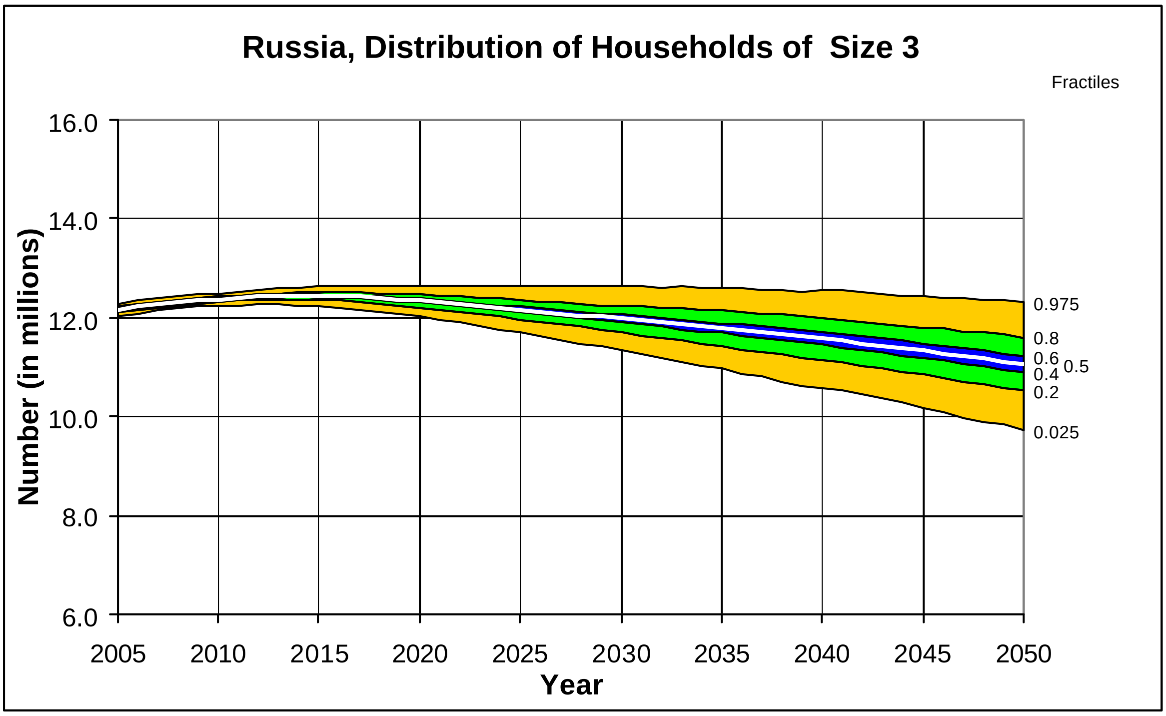


Figure 7 Distribution of households of size four, Russia, 2005-2005

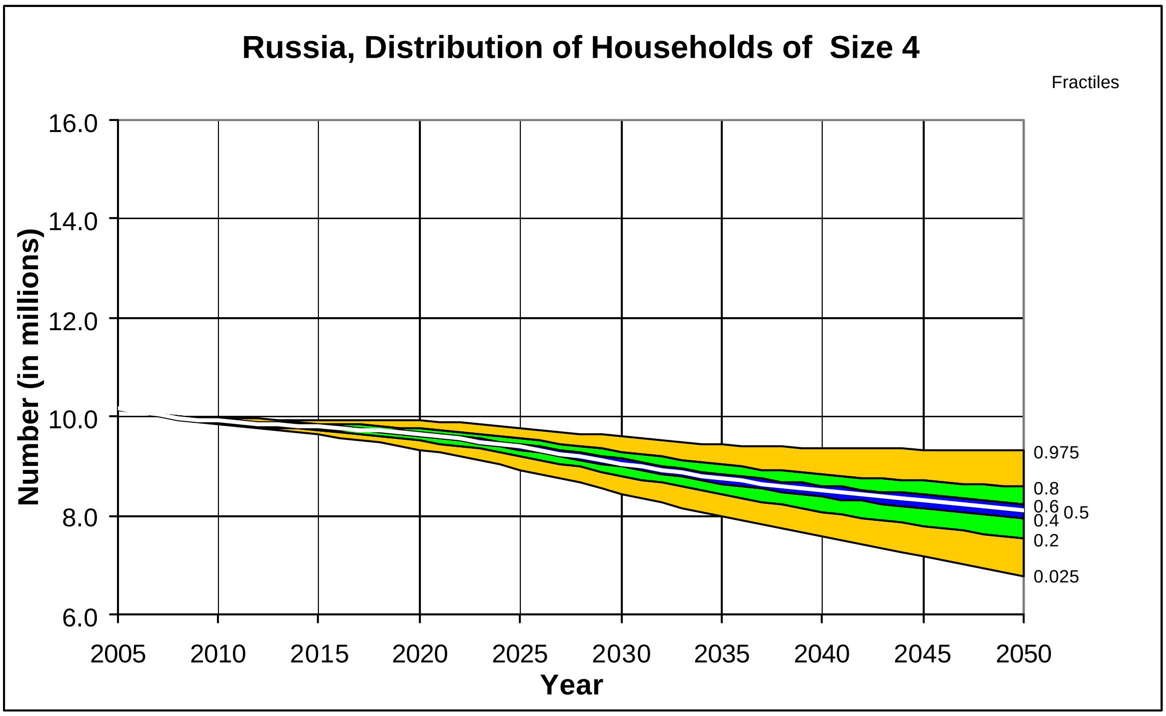
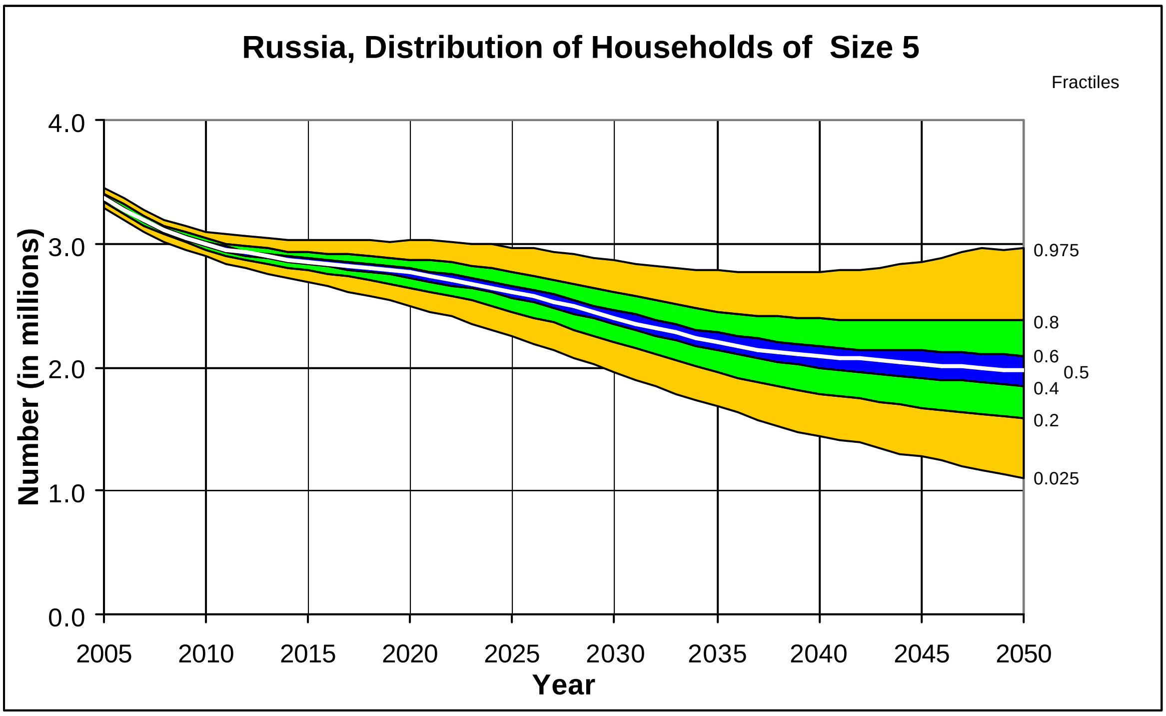


Figure 8 Distribution of households of size five, Russia, 2005-2005



25. Another way to look at the future distribution of households by size and to track the uncertainty associated with those distributions, is to present distribution of households by size for a particular time point (figures 9-10). From Figure 9 we may observe that there is virtually no chance that the number of households with four and more members will higher in 2025 than it was in 2005. In 2050 similar statement could be made regarding households with 3 and more members (Figure 10).

Figure 9 Distribution of households by size five, Russia, 2025

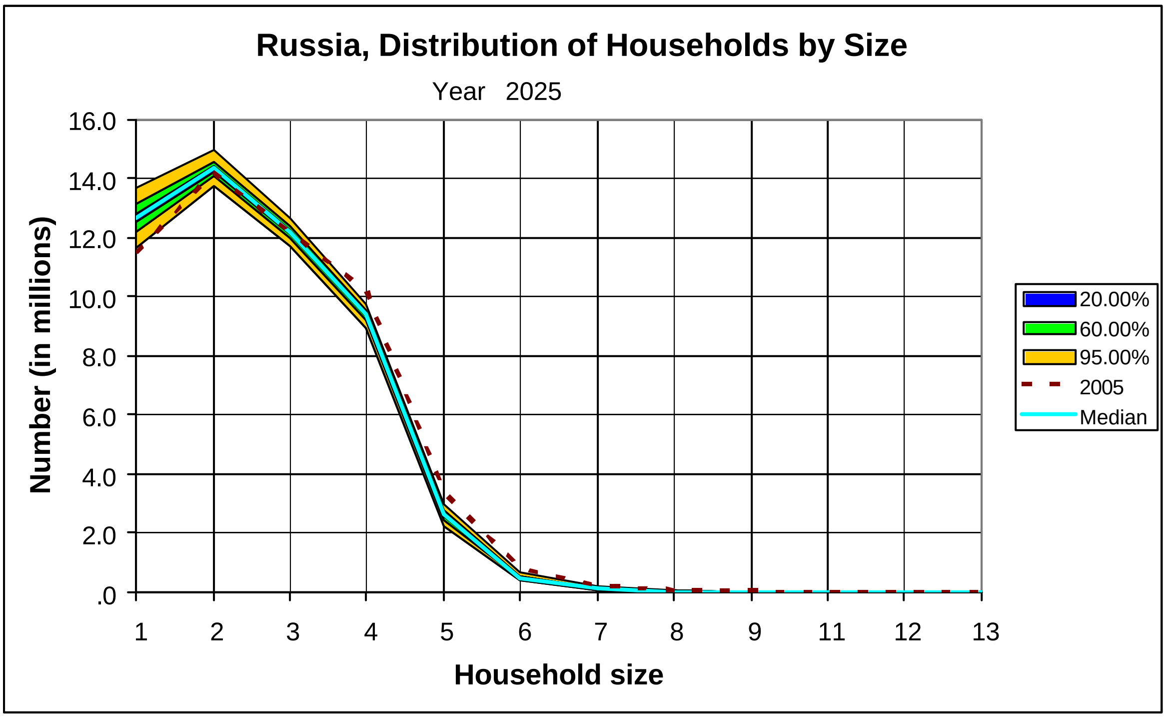
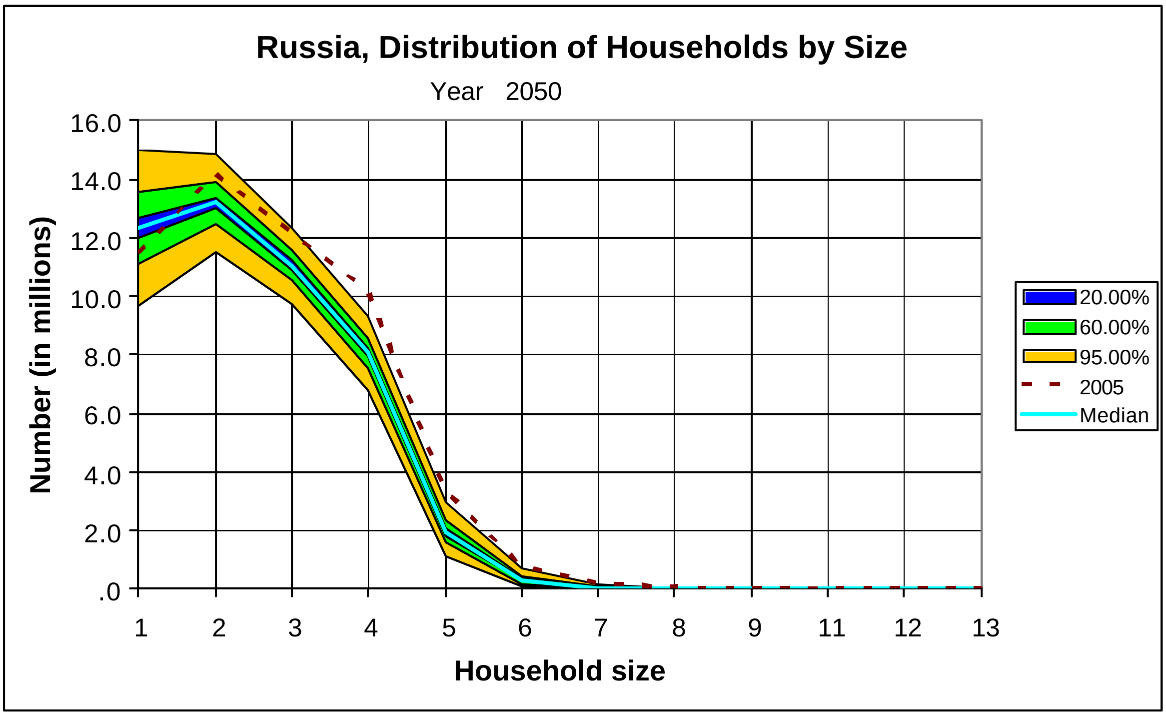


Figure 10 Distribution of households by size five, Russia, 2050



Conclusions

26. In this paper we presented the first probabilistic projections of the number of households in Russia. How this projections could be used? What type of questions are we able to answer having these results?

27. One of an extremely important issues in Russia today is availability of housing. Many families and households live in apartments where several people share one room. However, there exist social norms of housing per person, depending on the size of household[5]. Using those norms and assuming that they stay constant in the future, it is relatively easy to calculate the probabilistic demand for housing in Russia. At Figure 11 we present results of these calculation. With dotted line markers we designated existing availability (in 2002) of housing in millions of square meters that is occupied by households of different size. With vertical bars we present the demand for housing by household size calculated using social norms standards.

28. As we see from the graph, households with one or two members occupy even more housing space, then would correspond to social norms. There might be several reasons for that. First of all the distribution of housing is extremely uneven. Two households of the same size may live in a very different housing conditions. However one of the explanations of excessive available housing is that many of households of this type consist of elderly person living alone. Usually this person will have a bigger apartment, since at certain time he was living together with a spose and probably children. Children left home, spouse died and apartment or house (usually in rural area) is occupied by one person.

29. The most alarming situation is with availability of housing for households with four members. Typically that would be two patents living with two children. Since demographic policy adopted in Russia today is aimed at a second child, housing facilities for households consisting of four members, should be available. If the situation with housing availability does not improve, even though the number of households of size four is expected to decline, the shortage of housing will still be present, unless the policy will be developed to construct houses for households with four and more members (Figure 12). And even if in 2025 there might be enough of existing housing for households with 5 and more members, due to a very strong decline in the number of such households, the lack of housing for households with four members almost certain will be there if there is no considerable increase of housing stock for households with four member.

[5] Decree of the President of the Russian Federation Nr. 425 of April 28, 1997 "On Housing and Utility Sector Reform in the Russian Federation"

Figure 11 Demand and supply for housing, Russia, 2005

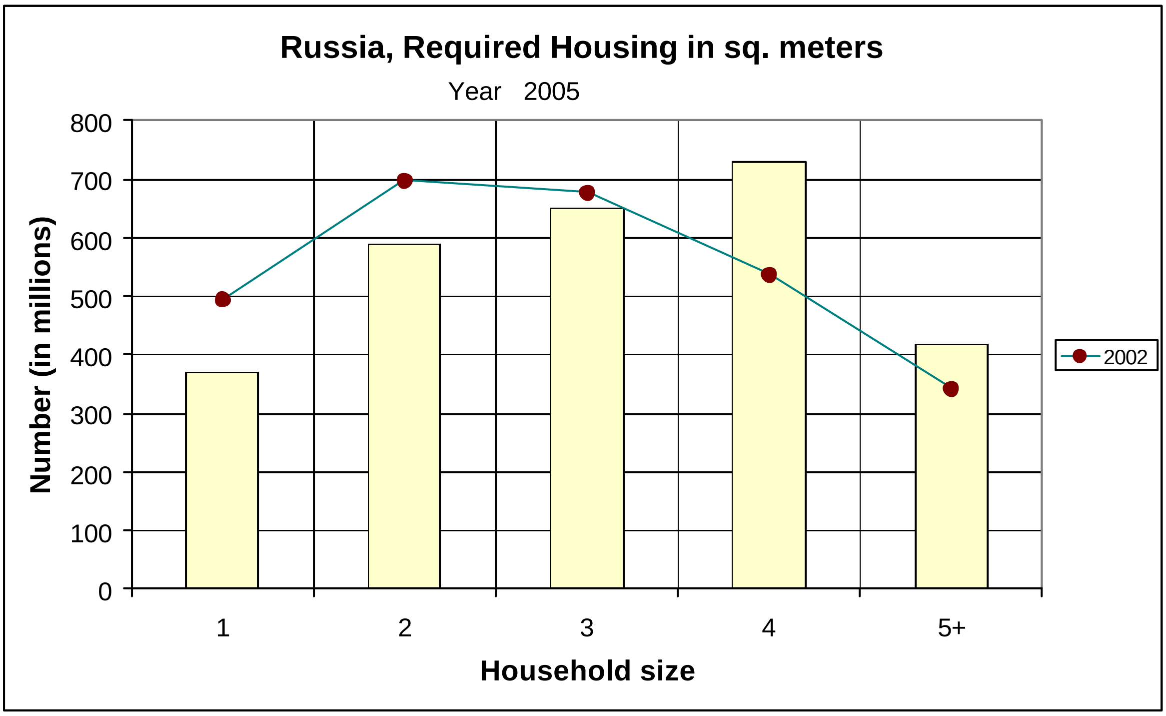
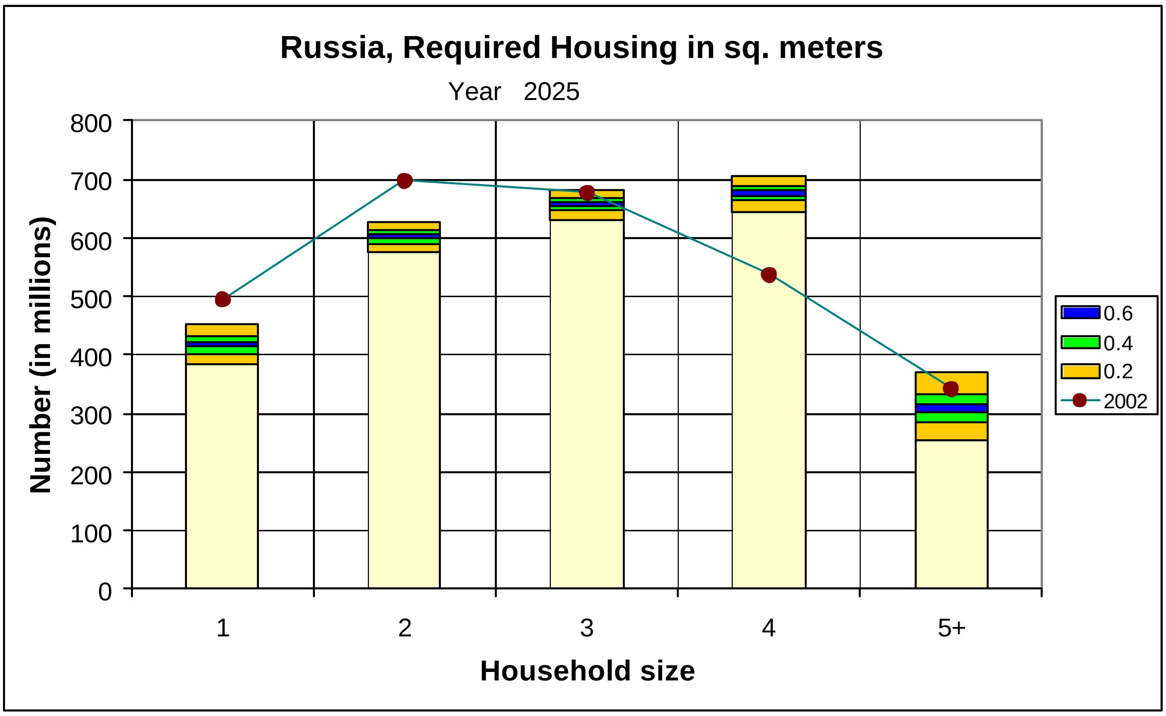


Figure 12   Demand and supply for housing, Russia, 2025



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Appendix. Private households by size in stable population

1. Headship rates

Let $P(x;t)$, $H(x;t)$, $M(x;t)=P(x;t)-H(x;t)$, and $h(x;t)=\frac{H(x;t)}{P(x;t)}$ be the population in private households, household heads, non-head members of households, and headship rates at age x[6]. We suppose the following simplified model determining evolvement of these functions. Dynamics of the number of heads is determined by mortality and also by formation of new households. Death of the head implies that all other household members move to other existing households, rather than forming a new household[7]. Secondly, formation of new households is through separation from existing households and happens at some fixed age-specific rates $g(x)$. Thirdly, we apply same age-specific death rates to both heads and non-head members. These simplifying assumptions allow separating the two processes and lead to the following differential equation for the population of non-head members of households:

$$\frac{\partial}{\partial t}M(x;t)+\frac{\partial}{\partial x}M(x;t)=-m(x;t)M(x;t)-g(x)M(x;t). \quad (A1)$$

From Eq. (A1), which is written in terms of the non-head population only, it is possible to derive the following relation for that population:

$$M(x;t)=M(0;t-x)e^{-\int_0^x (m(y;t-x+y)+g(y))dy}=P(x;t)e^{-\int_0^x g(y)dy}, \quad (A2)$$

where we suppose that there are no heads of age zero (i.e., $P(0;t)=M(0;t)$) and use the following traditional relation for the dynamics of the size of birth cohort:

$$P(x;t)=P(0;t-x)e^{-\int_0^x m(y;t-x+y)dy}. \quad (A2)$$

The population of heads may be obtained as the difference between the population total and the non-head population:

$$H(x;t)=P(x;t)-M(x;t)=P(x;t)\left(1-e^{-\int_0^x g(y)dy}\right). \quad (A3)$$

This allows obtaining the headship rates:

$$h(x;t)=\frac{H(x;t)}{P(x;t)}=1-e^{-\int_0^x g(y)dy}. \quad (A4)$$

Hence, headship rates are constant and do not directly depend on the reproduction regimen of the population as long as the age-specific rates of separating to new households are fixed. This result may be extended to the case of varying rate of new households' formation $g(x;t)$:

$$h(x;t)=1-e^{-\int_0^x g(y;t-x+y)dy}. \quad (A5)$$

In this more general case, again, mortality and fertility are not directly involved in headship rates. In the model proposed the reverse transitions from 'head' status to 'non-head' status were neglected. Hence, solutions (A4) and (A5)–under non-negative rates of transition from non-head status to the head status–are ever increasing by age. In real populations there is slight decrease in

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[6] For the sake of simplicity and also to avoid uncertainty related to the sex of the household head, we do not address sex, although it may be added to the study.

[7] In fact, emergence of new households due to the death of the head of existing household may indirectly be reflected in the model proposed through the age-specific rates of changing status from "non-head" to "head".

headship rates for oldest-old ages, as elderly may join households of their kin instead of continuing keeping their own household. However, this decline in headship rates may also reflect more options for stating the 'household head' in census in households with several generations cohabiting together and also reflect cohort effects on headship rates.

In any case, headship rates seem to be much less sensitive to variations in reproduction regimes compared to, say, population size and age structure. This explains the remarkable stability of headship rates in human populations and also provides a rational in support of the headship rates method. This point is also supported by empirical data (e.g., Leiwen and O'Neill 2004, Ediev 2007): age-specific headship rates are remarkably stable, when no details concerning the household size or type are concerned.

2. Average size of households

Due to relatively less sensitiveness of age-specific headship rates to changes in reproduction regimen, one may study consequences of stable populations' age structures for number, average size, and distribution of households by size assuming some fixed age profile of the headship rates.

Let $h(x)$ be the headship rate at age x, which we assume to be fixed for all populations to be considered. Average size of households, which–under the model proposed–determines their distribution by size, may be written as follows for the stable population:

$$n^s = \frac{\int_0^w Bl(x)e^{-rx}dx}{\int_0^w Bl(x)e^{-rx}h(x)dx} = \frac{\int_0^w l(x)e^{-rx}dx}{\int_0^w l(x)e^{-rx}h(x)dx}, \text{ (A6)}$$

here B are births in the stable population, $l(x)$ is the survivorship function, r is the Malthusian parameter (or Lotka's coefficient), and w is the maximum lifespan.

Headship rates are nil for children and grow rapidly to the level about 0.6 by age of 25-30. Therefore, one may use the following approximate for headship rates in (A6) in order to simplify the relation:

$$h(x) \approx \begin{cases} 0, \ x \le x_{\min}, \\ h^*, \ x > x_{\min}. \end{cases} \text{ (A7)}$$

Substituting this into (A6), we have:

$$n^s \approx \frac{\int_0^w l(x)e^{-rx}dx}{h^* \int_{x\min}^w l(x)e^{-rx}dx} = \frac{\int_0^{x\min} l(x)e^{-rx}dx + \int_{x\min}^w l(x)e^{-rx}dx}{h^* \int_{x\min}^w l(x)e^{-rx}dx} =$$

$$= \frac{1}{h^*}\left(1 + \frac{\int_0^{x\min} l(x)e^{-rx}dx}{\int_{x\min}^w l(x)e^{-rx}dx}\right). \text{ (A8)}$$

This expression indicates that there is a lower limit for average size of households of stable populations:

$$n^s \geq \frac{1}{h^*}. \text{ (A9)}$$

For usual case of headship rates of about 0.6 at most of adult ages, this implies, that *average size of households in stable population may not be lower than about* 1.67, which–given the models proposed for households distribution by size–has apparent implications for limiting the proportions of households of different sizes.

Expression in the right-hand side in (A8) depends on mortality and on reproduction regimen of the population. To make these relations more explicit, let us use the following simplifying approximation. Let us consider, that survivorship function may be approximated by a piece-wise constant function:

$$l(x) \approx \begin{cases} 1, & x \leq e_0, \\ 0, & x > e_0, \end{cases} \text{ (A10)}$$

here e_0 is the life expectancy at birth. Using this approximation, one may get from (A8):

$$n^s \approx \frac{1}{h^*}\left(1 + \frac{\int_0^{x\min} e^{-rx}dx}{\int_{x\min}^{e0} e^{-rx}dx}\right) = \frac{1}{h^*}\left(1 + \frac{1 - e^{-r\,x_{\min}}}{e^{-r\,x_{\min}} - e^{-r\,e_0}}\right) = \frac{1}{h^*}\frac{e^{r\,e_0} - 1}{e^{r(e_0 - x_{\min})} - 1}. \text{ (A11)}$$

For stable populations with reproduction close to simple replacement, i.e., with Lotka's coefficient close to zero, it is life expectancy at birth, which is the main factor of variations in average size of households:

$$n^s \approx \frac{1}{h^*}\frac{e_0}{e_0 - x_{\min}}, \text{ (A12)}$$

when $r\,e_0 << 1$.

For wider range of stable populations, one may use (A11) to study the variations of the average size of households[8]. Figure A1 presents results of calculations using $x_{\min} = 25$, $h^* = 0.6$. The figure shows explicitly that the main factors of declining average sizes of households were improvements in life expectancy and fertility decline – both processes tightly linked in the process of demographic transition. Hence, demographic transition itself–apart from cultural changes and reassessments of family values–has caused decline in average size of households. Note, however, that at the first stages of the transition, when mortality decline results in improvements of Lotka's r, average size of households might be relatively stable or even growing. Later on, however, decline in households' average size must follow.

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[8] Numerical simulations show that the approximation (A11) works pretty well and provides results very close to those obtained directly from (A6).

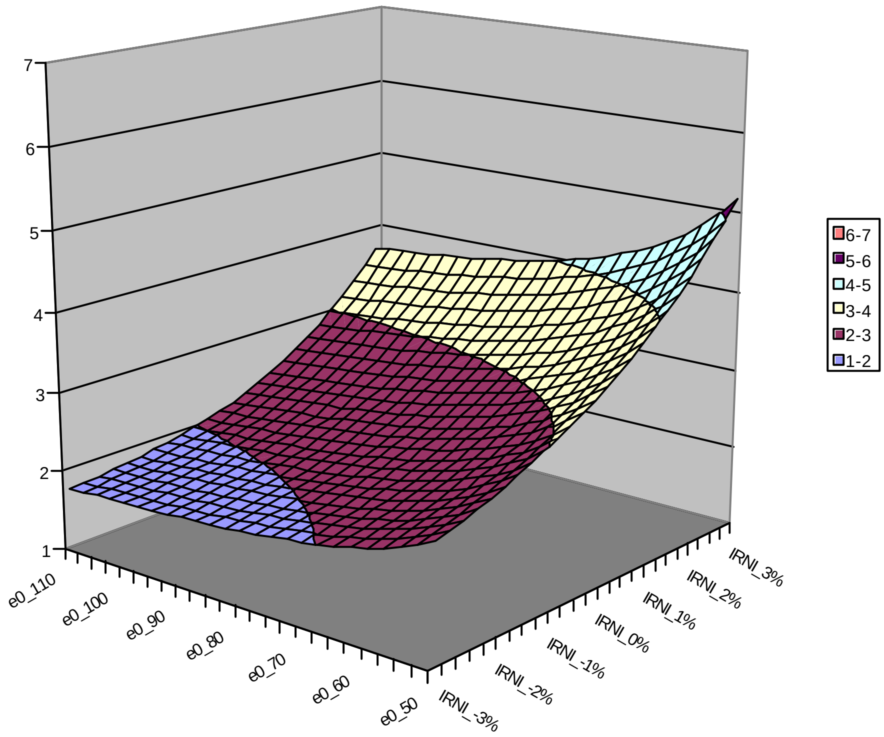

Fig. A1. Approximates of average size households in stable population as a function of life expectancy at birth and of Lotka's coefficient (IRNI).

3. Distributions by size

Distribution of households by size may be derived from their average size as it was proposed elsewhere (Ediev 2007) and is described in the paper.

Figure A2 presents results of estimating the proportions of households of different sizes in stable populations with varying fertility and with life expectancy fixed at the level of 80 years. Figure A3, on contrary, presents results for stable populations with varying mortality and with replacement fertility, i.e., in fact, for stationary populations. Changes in population age structure associated with fertility decline have negative effect on proportions of households with four and more members, and positive effect for proportions of one- and two-person households. Proportion of households with three persons, however, varies only moderately even within the remarkably wide range of fertility levels analyzed. Rise in life expectancy has nearly the same effect on households' distribution by size. Hence, simultaneous fall in fertility and rise in life expectancy, which was observed for many populations enhances effects of both processes on households' dynamics. In particular, it is the mere change in population age structure during the demographic transition processes, which seems to be the main factor of emergence of the modern distribution of households with declined share of large households and dramatic growth of the share of one-person households. This is illustrated on figure A4, which presents proportions of one-person households for stable populations with different combinations of fertility and life expectancy.

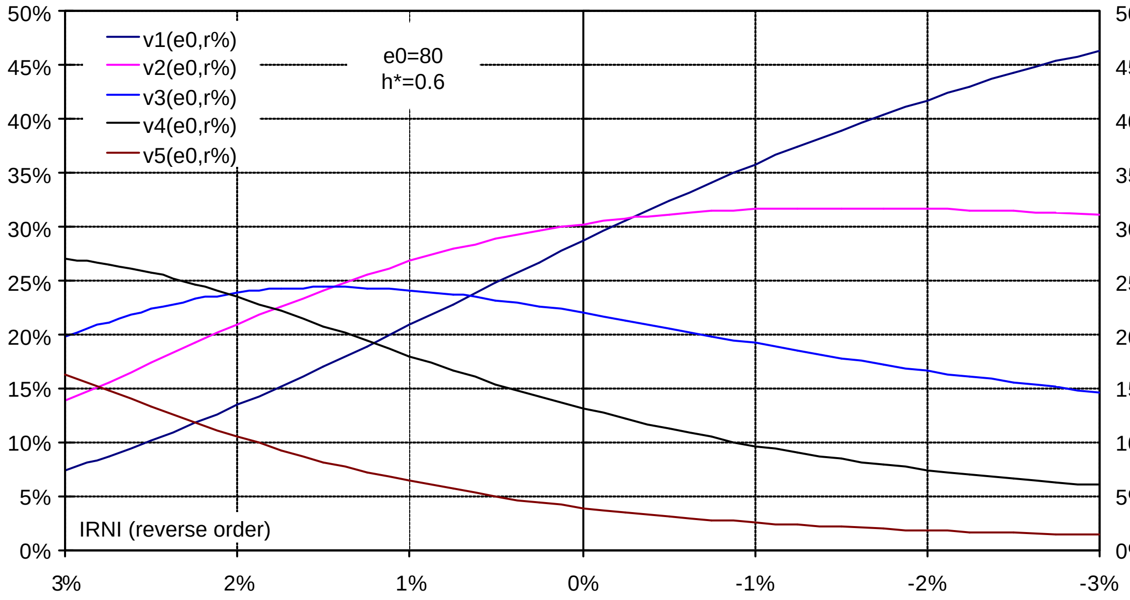

Fig. A2. Proportions of private households of sizes 1 to 5 in stable population as a function of Lotka's coefficient (IRNI) under life expectancy at birth fixed at 80 years.

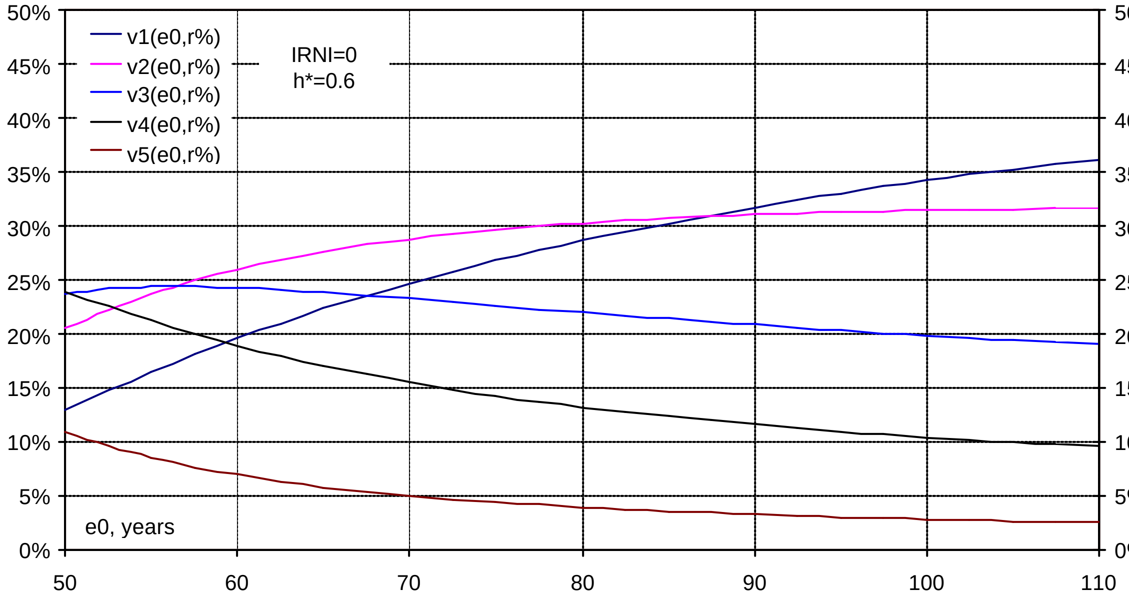

Fig. A3. Proportions of private households of sizes 1 to 5 in stable population as a function of life expectancy at birth under the replacement fertility.

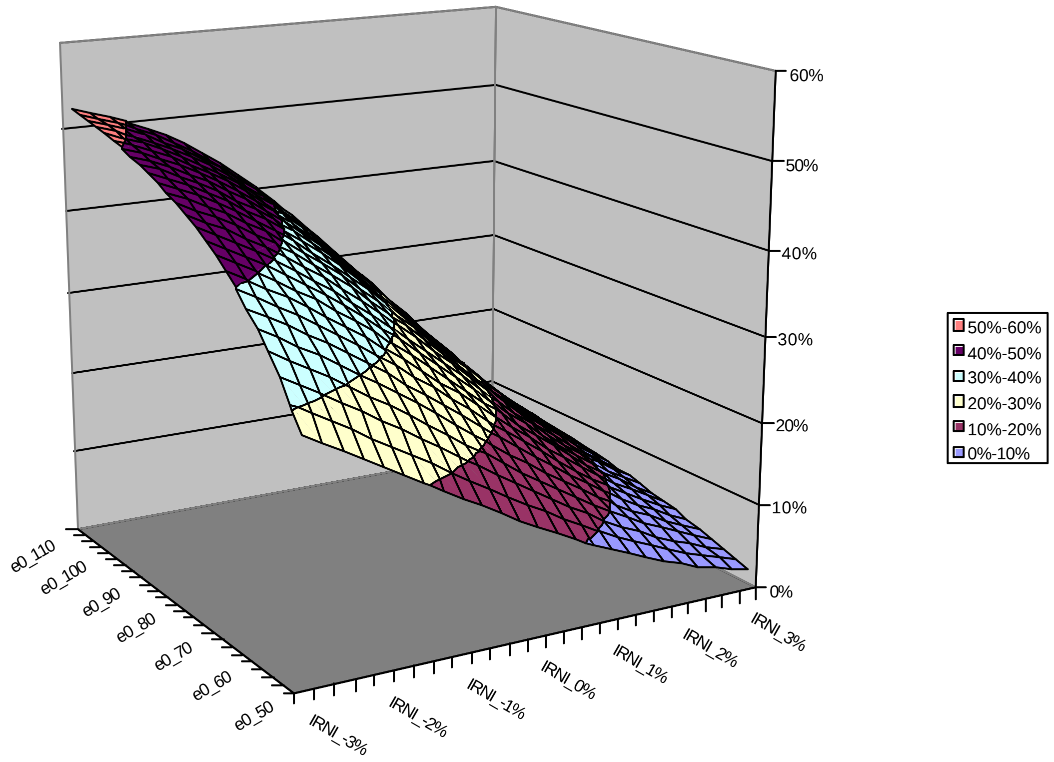

Fig. A4. Proportion of single-person households in stable population as a function of life expectancy at birth and of Lotka's coefficient (IRNI).

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